Algebra I

Sample Assessments for Big Ideas, per the CPS Algebra I Planning Guide, 2012-2013

Contents

Big Idea: Foundations for Algebra 2

Big Idea: Introduction to Functions and Their Rules 3

Big Idea: Linear Equations and Inequalities 4

Big Idea: Modeling with Linear Functions 5

Big Idea: Solving Systems of Equations and Inequalities 7

Big Idea: Non-linear Functions and Equations 8

Copyright information: Unless otherwise noted, the tasks included in this document have been created by CPS teachers. Others are used with permission.

Big Idea: Foundations for Algebra

Assessment: Central Park

Central Park is the largest park in New York City with a perimeter of 6 miles and an area of 1.25 square miles.

a. If the park is in the shape of a rectangle and the length of the park is 2 miles longer than its width, write an equation that relates the width of the park, w, to its perimeter, P.

b. Explain in a sentence or two what values might be in the domain of the variable w and what values would not be in the domain.

c. Using the equation from Part a, complete the table of values below. Then, in the standard coordinate plane, graph the relationship between the variables representing the width, w, and the perimeter, P, of Central Park.

w	P
0	
.5	
1	
1.5	
2	

d. Which of the values of w in the table above makes no sense in this problem? Explain your answer.

e. From the results you calculated in the Table for Part c, what is the width of Central Park? What is the length?

f. Solve the equation you wrote in Part a to arrive at the same answers you got in Part e.

g. It is generally accepted that in New York City, one mile is the equivalent of 20 city blocks. Express the dimensions of Central Park in blocks.

h. Bonus: Land is very valuable in Manhattan, the borough in New York City where Central Park is located. Suppose the designers of the park had wanted to maximize the area of the park without changing its perimeter. What dimensions of Central Park would result in a maximum area

Big Idea: Introduction to Functions and Their Rules

Assessment: Cell Phones

http://illustrativemathematics.org/

Let $f(t)$ be the number of people, in millions, who own cell phones t years after 1990.

Explain the meaning of the following statements.

1. $f(10)=100.3$
2. $f(a)=20$
3. $f(20)=b$
4. $n=f(t)$

Commentary:

This simple task assesses whether students can interpret function notation. The four parts of the task provide a logical progression of exercises for advancing understanding of function notation and how to interpret it in terms of a given context.

Solution: Cell phones

1. The number of people who own cell phones in the year 2000 is 100,300,000 .
2. There are 20,000,000 people who own cell phones a years after 1990.
3. There will be b million people who own cell phones in the year 2010.
4. The number n is the number of people (in millions) who own cell phones t years after 1990.

Big Idea: Linear Equations and Inequalities

Assessment: The Road Trip!

You have decided to go on a road trip over summer with some friends. You have three separate trips you want to take during the 2 months of summer break.

One is to a town a distance of 90 miles to see an old friend.

Another trip is to a shopping mall 120 miles away.

And the third is to see a concert in a city 250 miles away.

Finances are tight and you need to hire a car for the trip. There are three car hire companies for you to choose from.

- Bill's Car Hire charges a base fee of $50 and then 20c per mile
- Crazy Johns Car Hire charges a base fee of $20 and then 30c per mile
- Cheap Drive Car Hire charges 40c per mile with no base fee

Question 1 Construct a table with the costs for each company for distances up to 500 miles. You should include at least 5 different distances but more would be helpful.

Question 2 a) Write a mathematical equation to show the cost of hiring a car for each company.

b) What are the slopes and y-intercepts of these of these linear equations?

Question 3 Which car hire place would you choose for each of your trips? Explain your choice in light of your answers to Tasks 1 & 2.

(Hint: For what distances is Bill's Car Hire the cheapest? What about Crazy John? What about Cheap Drive?)

Question 4 At what trip distance would Bill's Car Hire and Crazy John cost the same?

Question 5 At what trip distance would Crazy John and Cheap Drive cost the same?

Question 6 At what trip distance would Bill's Car Hire and Cheap Drive cost the same?

Question 7 On the same set of axes, draw a graph for each car hire company. The horizontal axis is distance and the vertical axis is cost. Make sure your graph is large and that the horizontal axis extends to at least 500 miles.

Question 8 Find the coordinates of the intersection of the lines and label them.

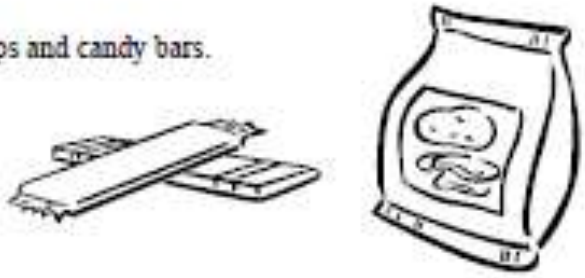
Question 9 Comment of the significance of these points.

Enjoy your trip!

Big Idea: Modeling with Linear Functions

Assessment: Buying Chips and Candy

Ralph and Jody go to the shop to buy potato chips and candy bars.



Ralph buys 3 bags of potato chips and 4 candy bars. He spends $3.75.

Jody buys 4 bags of potato chips and 2 candy bars. She spends $3.00.

Later Clancy joins Ralph and Jody and asks to buy one bag of potato chips and one candy bar from them. They need to work out how much he should pay.

Ralph writes $3p + 4b = 375$

1. If p stands for the cost, in cents, of a bag of potato chips and b stands for the cost, in cents, of a candy bar, what does the 375 in Ralph's equation mean?

_____

_____

_____

2. Write a similar equation, using p and b, for the items Jody bought.

_____

Copyright © 2011 by Mathematics Assessment Resource Service. All rights reserved.

Assessment: Buying Chips and Candy (continued)

3. Use the two equations to figure out the price of a bag of potato chips and the price of a candy bar.

Potato chips_____

Candy bar _____

Show your work.

4. Clancy has just $1. Does he have enough money to buy a bag of potato chips and a candy bar?

_____

Explain your answer by showing your calculation.

_____

_____

_____

Copyright © 2011 by Mathematics Assessment Resource Service. All rights reserved.

Big Idea: Solving Systems of Equations and Inequalities

Assessment: Passenger Jet

1. A passenger jet took four hours to fly 2000 miles in the direction of the jet stream. The return trip against the jet stream took five hours. What is the jet's speed in still air and the jet stream's speed? (Extension-Why is it always shorter flying East vs. flying West? If you have a meeting in San Francisco at 2 p.m. and need to book a flight out of Chicago, approximately 2000 miles away, what time do you need to book your flight?)
2. A landscaping company placed two orders with a nursery. The first order was for 11 bushes and 5 trees, and totaled $1422. The second order was for 5 bushes and 3 trees, and totaled $722. The bills do not list the per-item price. What were the costs of one bush and of one tree? (What other costs are associated with building a garden? How can these be incorporated into the problem?)
3. The admission fee at a small fair is $4.00 for children and $6.00 for adults. On a certain day, 2000 people enter the fair and $9600 is collected. How many children and how many adults attended? (Would buying a family pack for $19 be a good idea? Explain)

Big Idea: Non-linear Functions and Equations

Assessment: Summer Olympics

In 2008, the Summer Olympics are in Beijing, China. In order to make sure they are ready for the world, the organizers hired you, a mathematical analyst, to oversee certain aspects of the Olympics.

Task #1 – Monitor Air Quality

Beijing is under a lot of scrutiny when it comes to air quality. In particular, there are two sites in the Olympic Village where you are asked to test the air quality.

You monitored the amount of pollutants in the air in parts per billion (ppb) over a span of 8 hours from 8 am to 4 pm.

Here are your observations and results:

- At Site #1, the weather tends to be foggy in the morning and by mid-afternoon the fog lifts.
- At Site #2, it tends to be an area that is high in traffic at certain times of the day.

Site #1	
Time (h)	**Amount of Pollution (ppb)**
0	250.80
2	200.53
4	150.26
6	99.99
8	49.72

Site #2	
Time (h)	**Amount of Pollution (ppb)**
0	115.0
2	143.5
4	153.0
6	143.5
8	115.0

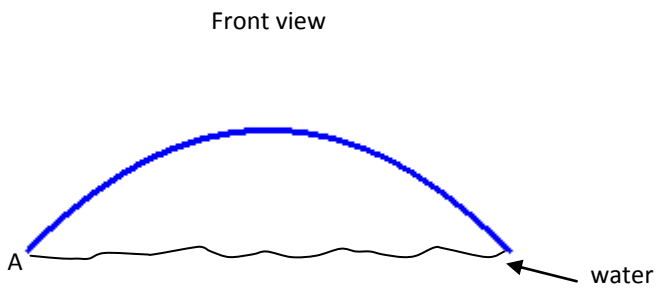
1. Would you model the data with a linear relationship, quadratic relationship or neither for Site #1? Justify your answer.

2. Would you model the data with a linear relationship, quadratic relationship or neither for Site #2? Justify your answer.

Assessment: Summer Olympics (continued)

Task #2 – Sight-Seeing Venue

Visitors to the Olympics will spend some time sight-seeing. A popular place to visit is one of the many arch bridges in China. The tourism department for the Olympics wants to build boats that will carry people under the bridge. Before they build these boats, they want you to check if these boats will fit underneath the bridge.



The shape of the bridge can be modeled by the relation $h = -0.05w^2 + w$ where *h* is the height in metres above the water, and *w* is the horizontal distance from the point A in the diagram.

1. How wide is the river?

2. What is the highest point under the bridge?

3. The proposed design of the boats has a width of 12 m and a height of 2.5 m. Will these boats fit under the bridge? Justify your answer. Round your answer to two decimal places.

Assessment: Summer Olympics (continued)

Task #3 – Marketing the Olympics

In order to promote the Olympics, the marketing division has decided to create a few contests. In order to claim the prize, contestants need to answer skill-testing questions. You need to create the solutions for these skill-testing questions.

1. Expand and Simplify.

a) $(x-5)(x+7)$

b) $(2x-1)^2$

2. Factor fully.

a) $4m^2n^3 - 2mn^2$

b) $x^2 - 3x - 28$

c) $4x^2 - 1$

d) $6x^2 + 7x - 5$

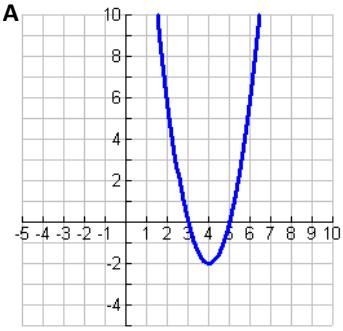
e) $9x^2 - 30x + 25$

Assessment: Summer Olympics (continued)

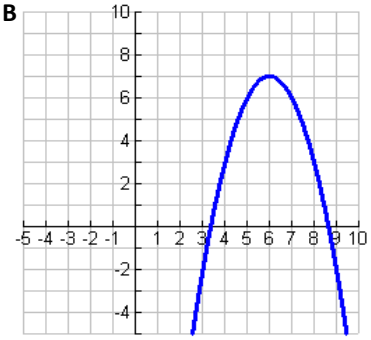
3. Match the following equations to its correct graph.

Equation	Letter of the Graph (A, B, C, D, E, F, G or H)
$y = -(x+5)(x+3)$	
$y = -(x-6)^2 + 7$	
$y = -\frac{1}{2}(x+5)(x-3)$	
$y = (x+6)^2 - 2$	
$y = 2(x-5)(x-3)$	
$y = (x+6)^2 - 7$	

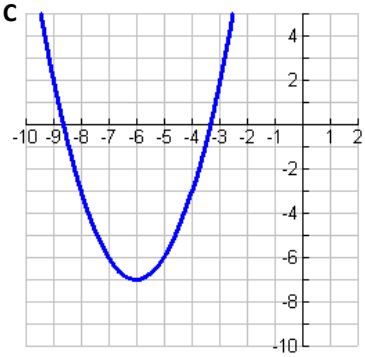
A



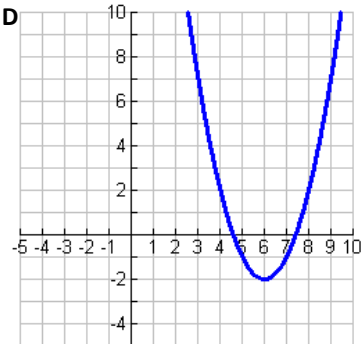
B



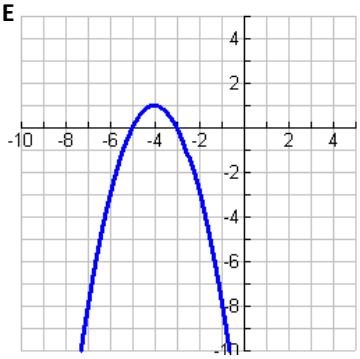
C



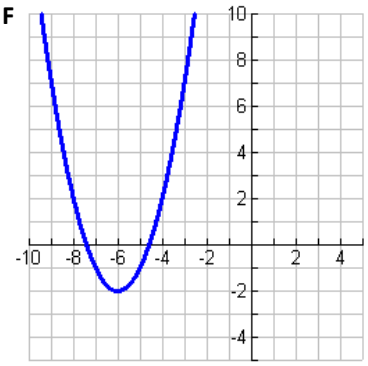
D



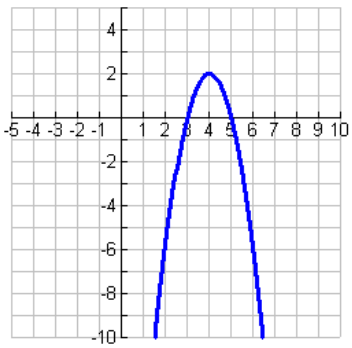
E



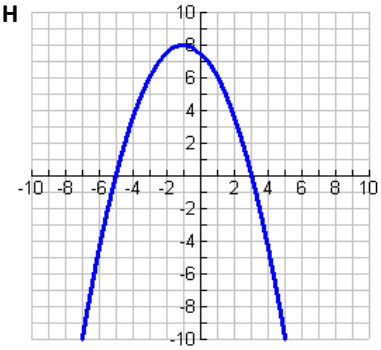
F



G



H



Assessment: Summer Olympics (continued)

Task #4 – Maximizing Ticket Sales

The financing department wanted to make sure they sold the tickets to the events at a reasonable price that would both fill all the stands and maximize revenue.

They want you to determine the optimal ticket selling price to the opening ceremonies for the games. Based on the previous games, they know that if they sold general admission tickets for $300, they would sell 2000 tickets. For every $10 decrease they will gain 200 sales.

1. At what price should the ticket be sold at to maximize revenue?

2. What is the maximum revenue?

Task #5 – Attending a Game

For all your hard work, they authorities have given you free tickets to see one event in Athletics: Shot Put.

Shot put is a sport that requires athletes to throw a metal sphere from inside a circular ring of 2.135 m in diameter, as far as they can into a marked area. For men, the shot put is usually between 11 – 13 cm in diameter and it weighs 7.260 kg.

All eyes are on the Canadian Olympian, Dylan Armstrong. He recently broke his own record *and* the Canadian Shot Put record, throwing 20.92 m.

You arrive early to the venue to watch the practice sessions. From your observations, the path of the shot put can be modeled by the equation:

$$h = -0.015d^2 + 0.216d + 2.1$$

where h is the height of the shot put in metres, and d is the horizontal distance traveled by the shot put, in metres.

1. At what height did Dylan release the shot put?

2. Based on this model, was Dylan able to break his own record? Justify your answer. Round your answer to two decimal places.

Bonus Assessment: Taxes and Sales

http://illustrativemathematics.org/illustrations/677

Judy is working at a retail store over summer break. A customer buys a $50 shirt that is on sale for 20% off. Judy computes the discount, then adds sales tax of 10%, and tells the customer how much he owes. The customer insists that Judy first add the sales tax and then apply the discount. He is convinced that this way he will save more money because the discount amount will be larger.

1. Is the customer right?
2. Does your answer to part (a) depend on the numbers used or would it work for any percentage discount and any sales tax percentage? Find a convincing argument using algebraic expressions and/or diagrams for this more general scenario.

Commentary:

This task is not about computing the final price of the shirt but about using the structure in the computation to make a general argument. The key underlying idea is that multiplication is commutative, which we often just take for granted and don't feel needs any explanation. In this case, the context of the problem makes it not obvious at all that we can switch the order of the two computations, but it becomes quite obvious after observing that the application of both the discount and the sales tax are just instances of multiplication. Since the order in which we multiply is irrelevant, the answer must be the same regardless of which we apply first.

The solution presents both an algebraic approach to the general result in part (b), and also a diagram that illustrates the same result graphically.

This task presents a good opportunity for students to construct a viable argument and critique the reasoning of others (MP3).

Solution:

1. Judy first takes 20% off which gives a new price of $50(0.80) = $40 . She then adds the 10% sales tax for a final price of $40(1.10)=$44 .

   The customer first adds 10% for a new price (including tax) of $50(1.10) = $55 . He then takes 20% off for a final price of $55(0.80)=$44

   The customer is right to say that the <u>discount amount</u> will be larger, it is $11 opposed to $10 with his method. But the additional $1 just gets subtracted from the tax amount that was added in the first step. So the final price is the same in both cases.

   It does not matter in which order the discount and tax are computed.

2. If we don't actually perform the computations but just record them we find the following:

   Judy: $50(0.80)(1.10) = 44$

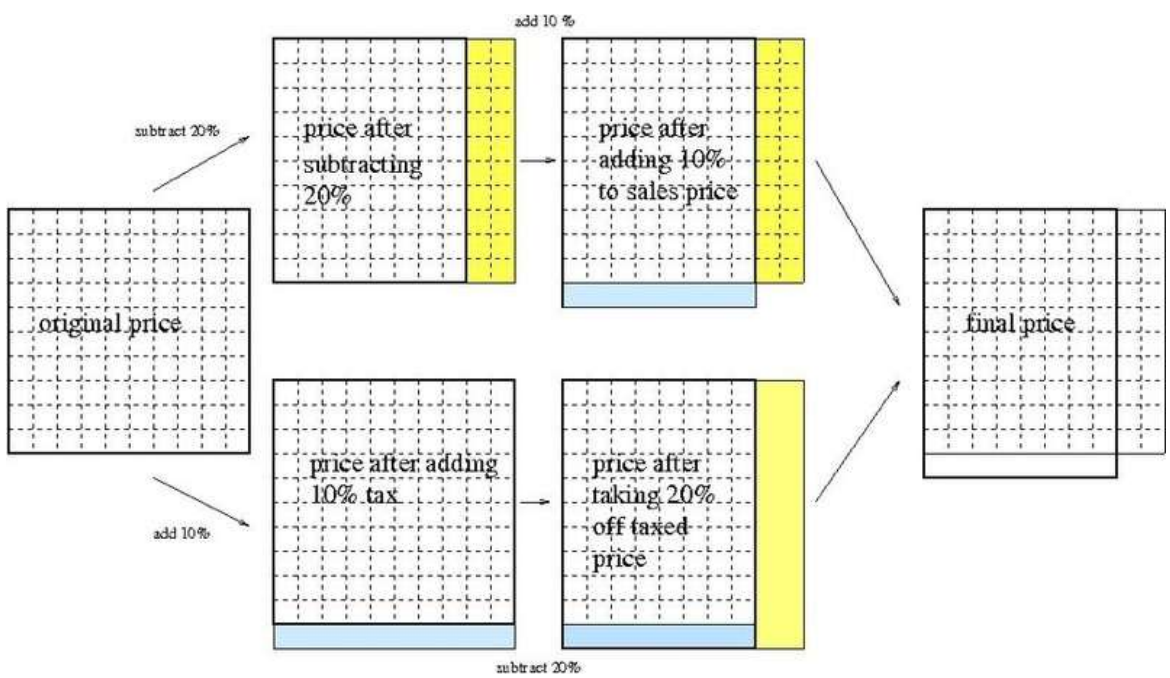
   customer: $50(1.10)(0.80) = 44$

Bonus Assessment: Taxes and Sales (continued)

We see that it is not surprising that both computations get the same answer, since $(0.80)\cdot(1.10) = (1.10)\cdot(0.80)$.

This result will generalize if we replace \$50, 20%, 10% by any other numbers. If we let P stand for the original price, s for the sales percentage and t for the tax percentage, we have

$$P(1-s\,100)(1+t\,100) = P(1+t\,100)(1-s\,100)$$

We see that changing the order in which the sale and the tax are applied does not matter.

3. We can also visualize this with the following diagram. Yellow represents the action of subtracting 20% and blue represents the action of adding 10%. We see that both paths result in the same final answer. Even though the diagram uses the numbers from the problem, we can see from the structure in the diagram that both paths will result in the same final price even if the yellow and blue areas are altered.