6th Grade Common Core State Standards

Flip Book

This document is intended to show the connections to the Standards of Mathematical Practices and the content standards and to get detailed information and instructional strategies at each level. Resources used: CCSS, Arizona DOE, Ohio DOE and North Carolina DOE. This "Flip Book" is intended to help teachers understand what each standard means in terms of what students must know and be able to do. It provides only a *sample* of instructional strategies and examples. The goal of every teacher should be to guide students in understanding & making sense of mathematics.

Construction directions:

Print on cardstock. Cut the tabs on each page starting with page 2. Cut the bottom off of this top cover to reveal the tabs for the subsequent pages. Staple or bind the top of all pages to complete your flip book.

Compiled by Melisa Hancock (Send feedback to: melisa@ksu.edu)

1. **Make sense of problems and persevere in solving them.**
   Mathematically proficient students interpret and make meaning of the problem looking for starting points. They analyze what is given to find the meaning of the problem. They plan a solution pathway instead of jumping to a solution. These students can monitor their progress and change the approach if necessary. They see relationships between various representations. They relate current situations to concepts or skills previously learned and connect mathematical ideas to one another. In grade 6, students solve problems involving ratios and rates and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
2. **Reason abstractly and quantitatively.**
   Mathematically proficient students make sense of quantities and their relationships. They are able to decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships. Younger students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities, not just how to compute them. In grade 6, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
3. **Construct viable arguments and critique the reasoning of others.**
   Mathematically proficient students analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments. First graders construct arguments using concrete referents, such as objects, pictures, drawings, and actions. In grade 6, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.

4. **Model with mathematics.**
   Mathematically proficient students understand that models are a way to reason quantitatively and abstractly (able to decontextualize and contextualize). In grade 6, students model problem situations symbolically, graphically, tabular, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students begin to explore covariance and represent two quantities simultaneously. Students use number lines to compare numbers and represent inequalities. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences about and make comparisons between data sets. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.
5. **Use appropriate tools strategically.**
   Mathematically proficient students use available tools recognizing the strengths and limitations of each. Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 6 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Additionally, students might use physical objects or applets to construct nets and calculate the surface area of three-dimensional figures.
6. **Attend to precision.**
   Mathematically proficient students communicate precisely with others and try to use clear mathematical language when discussing their reasoning. They understand meanings of symbols used in mathematics and can label quantities appropriately. In grade 6, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to rates, ratios, geometric figures, data displays, and components of expressions, equations or inequalities.
7. **Look for and make use of structure. (Deductive Reasoning)**
   Mathematically proficient students apply general mathematical rules to specific situations. They look for the overall structure and patterns in mathematics. Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables recognizing both the additive and multiplicative properties. Students apply properties to generate equivalent expressions (i.e. $6 + 2x = 2(3 + x)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$, $2c = 12$ by subtraction property of equality; $c=6$ by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving area and volume.
8. **Look for and express regularity in repeated reasoning. (Inductive Reasoning)**
   Mathematically proficient students see repeated calculations and look for generalizations and shortcuts. In grade 6, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. Students connect place value and their prior work with operations to understand algorithms to fluently divide multi-digit numbers and perform all operations with multi-digit decimals. Students informally begin to make connections between covariance, rates, and representations showing the relationships between quantities.

Math Practice Standards					

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
1. Make sense of problems and persevere in solving them. • Interpret and make meaning of the problem to find a starting point. Analyze what is given in order to explain to themselves the meaning of the problem. • Plan a solution pathway instead of jumping to a solution. • Monitor their progress and change the approach if necessary. • See relationships between various representations. • Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another. • Continually ask themselves, "Does this make sense?" Can understand various approaches to solutions.	How would you describe the problem in your own words? How would you describe what you are trying to find? What do you notice about...? What information is given in the problem? Describe the relationship between the quantities. Describe what you have already tried. What might you change? Talk me through the steps you've used to this point. What steps in the process are you most confident about? What are some other strategies you might try? What are some other problems that are similar to this one? How might you use one of your previous problems to help you begin? How else might you organize...represent... show...?
2. Reason abstractly and quantitatively. • Make sense of quantities and their relationships. • Decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships. • Understand the meaning of quantities and are flexible in the use of operations and their properties. • Create a logical representation of the problem. • Attends to the meaning of quantities, not just how to compute them.	What do the numbers used in the problem represent? What is the relationship of the quantities? How is _____ related to _____? What is the relationship between _____and _____? What does_____mean to you? (e.g. symbol, quantity, diagram) What properties might we use to find a solution? How did you decide in this task that you needed to use...? Could we have used another operation or property to solve this task? Why or why not?
3. Construct viable arguments and critique the reasoning of others. • Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments. • Justify conclusions with mathematical ideas. • Listen to the arguments of others and ask useful questions to determine if an argument makes sense. • Ask clarifying questions or suggest ideas to improve/revise the argument. • Compare two arguments and determine correct or flawed logic.	What mathematical evidence would support your solution? How can we be sure that...? / How could you prove that...? Will it still work if...? What were you considering when...? How did you decide to try that strategy? How did you test whether your approach worked? How did you decide what the problem was asking you to find? (What was unknown?) Did you try a method that did not work? Why didn't it work? Would it ever work? Why or why not? What is the same and what is different about...? How could you demonstrate a counter-example?
4. Model with mathematics. • Understand this is a way to reason quantitatively and abstractly (able to decontextualize and contextualize). • Apply the mathematics they know to solve everyday problems. • Are able to simplify a complex problem and identify important quantities to look at relationships. • Represent mathematics to describe a situation either with an equation or a diagram and interpret the results of a mathematical situation. • Reflect on whether the results make sense, possibly improving/revising the model. • Ask themselves, "How can I represent this mathematically?"	What number model could you construct to represent the problem? What are some ways to represent the quantities? What is an equation or expression that matches the diagram, number line.., chart..., table..? Where did you see one of the quantities in the task in your equation or expression? How would it help to create a diagram, graph, table...? What are some ways to visually represent...? What formula might apply in this situation?

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
5. Use appropriate tools strategically. • Use available tools recognizing the strengths and limitations of each. • Use estimation and other mathematical knowledge to detect possible errors. • Identify relevant external mathematical resources to pose and solve problems. • Use technological tools to deepen their understanding of mathematics.	What mathematical tools could we use to visualize and represent the situation? What information do you have? What do you know that is not stated in the problem? What approach are you considering trying first? What estimate did you make for the solution? In this situation would it be helpful to use...a graph..., number line..., ruler..., diagram..., calculator..., manipulative? Why was it helpful to use...? What can using a _____ show us that _____may not? In what situations might it be more informative or helpful to use...?
6. Attend to precision. • Communicate precisely with others and try to use clear mathematical language when discussing their reasoning. • Understand the meanings of symbols used in mathematics and can label quantities appropriately. • Express numerical answers with a degree of precision appropriate for the problem context. • Calculate efficiently and accurately.	What mathematical terms apply in this situation? How did you know your solution was reasonable? Explain how you might show that your solution answers the problem. What would be a more efficient strategy? How are you showing the meaning of the quantities? What symbols or mathematical notations are important in this problem? What mathematical language...,definitions..., properties can you use to explain...? How could you test your solution to see if it answers the problem?
7. Look for and make use of structure. • Apply general mathematical rules to specific situations. • Look for the overall structure and patterns in mathematics. • See complicated things as single objects or as being composed of several objects.	What observations do you make about...? What do you notice when...? What parts of the problem might you eliminate..., simplify...? What patterns do you find in...? How do you know if something is a pattern? What ideas that we have learned before were useful in solving this problem? What are some other problems that are similar to this one? How does this relate to...? In what ways does this problem connect to other mathematical concepts?
8. Look for and express regularity in repeated reasoning. • See repeated calculations and look for generalizations and shortcuts. • See the overall process of the problem and still attend to the details. • Understand the broader application of patterns and see the structure in similar situations. • Continually evaluate the reasonableness of their intermediate results	Explain how this strategy work in other situations? Is this always true, sometimes true or never true? How would we prove that...? What do you notice about...? What is happening in this situation? What would happen if...? Is there a mathematical rule for...? What predictions or generalizations can this pattern support? What mathematical consistencies do you notice ?

Critical Areas for Mathematics in 6th Grade

In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

(2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

(3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

(4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

Domain: **Ratios and Proportional Relationships (RP)**

Cluster: Understand ratio concepts and use ratio reasoning to solve problems.

Standard: **6.RP.1**. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

Standards for Mathematical Practice (MP):

MP.2. Reason abstractly and quantitatively.
MP.6. Attend to precision

Connections:

This cluster is connected to the Grade 6 Critical Area of Focus #1, **Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems.**

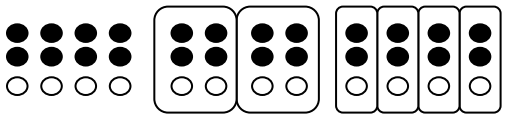
In Grade 6, students develop the foundational understanding of ratio and proportion that will be extended in Grade 7 to include scale drawings, slope and real-world percent problems.

Explanations and Examples

6.RP.1 A ratio is the comparison of two quantities or measures. The comparison can be part-to-whole (ratio of guppies to all fish in an aquarium) or part-to-part (ratio of guppies to goldfish). Students need to understand each of these ratios when expressed in the following forms: , 6 to 15 or 6:15. These values can be *simplified to , 2 to 5 or 2:5; however, students would need to understand how the simplified valu*es relate to the original numbers.

A rate is a ratio where two measurements are related to each other. When discussing measurement of different units, the word rate is used rather than ratio. Understanding rate, however, is complicated and there is no universally accepted definition. When using the term rate, contextual understanding is critical. Students need many opportunities to use models to demonstrate the relationships between quantities before they are expected to work with rates numerically.

A comparison of 8 black circles to 4 white circles can be written as the ratio of 8:4 and can be regrouped into 4 black circles to 2 white circles (4:2) and 2 black circles to 1 white circle (2:1).



Students should be able to identify all these ratios and describe them using "For every…., there are …"

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Instructional Strategies - 6.RP.1-3

Proportional reasoning is a process that requires instruction and practice. It does not develop over time on its own. Grade 6 is the first of several years in which students develop this multiplicative thinking. Examples with ratio and proportion must involve measurements, prices and geometric contexts, as well as rates of miles per hour or portions per person within contexts that are relevant to sixth graders. Experience with proportional and nonproportional relationships, comparing and predicting ratios, and relating unit rates to previously learned unit fractions will facilitate the development of proportional reasoning. Although algorithms provide efficient means for finding solutions, the cross-product algorithm commonly used for solving proportions will not aid in the development of proportional reasoning. Delaying the introduction of rules and algorithms will encourage thinking about multiplicative situations instead of indiscriminately applying rules.

Students develop the understanding that ratio is a comparison of two numbers or quantities. Ratios that are written as part-to-whole are comparing a specific part to the whole. Fractions and percents are examples of part-to-whole ratios. Fractions are written as the part being identified compared to the whole amount. A percent is the part identified compared to the whole (100). Provide students with multiple examples of ratios, fractions and percents of this type. For example, the number of girls in the class (12) to the number of students in the class (28) is the ratio 12 to 28.

Percents are often taught in relationship to learning fractions and decimals. This cluster indicates that percents are to be taught as a special type of rate. Provide students with opportunities to find percents in the same ways they would solve rates and proportions.

Part-to-part ratios are used to compare two parts. For example, the number of girls in the class (12) compared to the number of boys in the class (16) is the ratio the ratio 12 to 16. This form of ratios is often used to compare the event that can happen to the event that cannot happen. Rates, a relationship between two units of measure, can be written as ratios, such as miles per hour, ounces per gallon and students per bus. For example, 3 cans of pudding cost $2.48 at Store A and 6 cans of the same pudding costs $4.50 at Store B. Which store has the better buy on these cans of pudding? Various strategies could be used to solve this problem:

- A student can determine the unit cost of 1 can of pudding at each store and compare.
- A student can determine the cost of 6 cans of pudding at Store A by doubling $2.48.
- A student can determine the cost of 3 cans of pudding at Store B by taking ½ of $4.50.

Using ratio tables develops the concept of proportion. By comparing equivalent ratios, the concept of proportional thinking is developed and many problems can be easily solved.

Store A	
3 cans	6 cans
$2.48	$4.96

Store B	
6 cans	3 cans
$4.50	$2.50

Students should also solve real-life problems involving measurement units that need to be converted. Representing these measurement conversions with models such as ratio tables, t-charts or double number line diagrams will help students internalize the size relationships between same system measurements and relate the process of converting to the solution of a ratio.

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Multiplicative reasoning is used when finding the missing element in a proportion. For example, use 2 cups of syrup to 5 cups of water to make fruit punch. If 6 cups of syrup are used to make punch, how many cups of water are needed?

2/5= 6/x Recognize that the relationship between 2 and 6 is 3 times; 2 · 3 = 6
To find x, the relationship between 5 and x must also be 3 times. 3 · 5 = x, therefore, x = 15

2/5 = 6/15 The final proportion.

Other ways to illustrate ratios that will help students see the relationships that follow. Begin written representation of ratios with the words "out of" or "to" before using the symbolic notation of the colon and then the fraction bar; for example, 3 out of 7, 3 to 5, 6:7 and then 4/5.
Use skip counting as a technique to determine if ratios are equal.
Labeling units helps students organize the quantities when writing proportions.

$$\frac{3\ eggs}{2\ cups\ of\ flour} = \frac{z\ eggs}{8\ cups\ of\ flour}$$

Using hue/color intensity is a visual way to examine ratios of part-to-part. Students can compare the intensity of the color green and relate that to the ratio of colors used. For example, have students mix green paint into white paint in the following ratios: 1 part green to 5 parts white, 2 parts green to 3 parts white, and 3 parts green to 7 parts white. Compare the green color intensity with their ratios.

<u>Common Misconceptions:</u>

Fractions and ratios may represent different comparisons. Fractions always express a part-to-whole comparison, but ratios can express a part-to-part comparison which can be written as: a to b, $\frac{a}{b}$, or $a{:}b$.

Even though ratios and fractions express a part-to-whole comparison, the addition of ratios and the addition of fractions are distinctly different procedures. When adding ratios, the parts are added, the wholes are added and then the total part is compared to the total whole. For example, (2 out of 3 parts) + (4 out of 5 parts) is equal to six parts out of 8 total parts (6 out of 8) if the parts are equal. When dealing with fractions, the procedure for addition is based on a common denominator: (2/3) + (4/5) = (10/15) + (12/15) which is equal to (22/15). Therefore, the addition process for ratios and for fractions is distinctly different.
Often there is a misunderstanding that a percent is always a natural number less than or equal to 100. Provide examples of percent amounts that are greater than 100%, and percent amounts that are less 1%.

Arizona, Ohio & NC DOE

Domain: **Ratios and Proportional Reasoning (RP)**

Cluster: Understand ratio concepts and use ratio reasoning to solve problems.

Standard: **6.RP.2**
Understand the concept of a unit rate a/b associated with a ratio a:b with b 0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."(*Expectations for unit rates in this grade are limited to non-complex fractions).

Standards for Mathematical Practice (MP):
MP.2 Reason abstractly and quantitatively.
MP.6 Attend to precision.

Connections:
See 6.RP.1

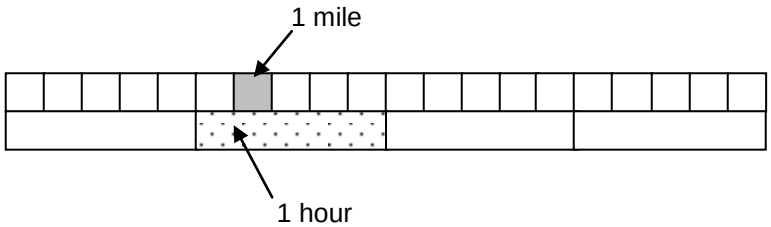
Explanations and Examples:
6.RP.2 A unit rate expresses a ratio as part-to-one or one unit of another quantity. For example, if there are 2 cookies for 3 students, each student receives 2/3 of a cookie, so the unit rate is 2/3:1. If a car travels 240 miles in 4 hours, the car travels 60 miles per hour (60:1). Students understand the unit rate from various contextual situations.

Students will often use unit rates to solve missing value problems. Cost per item or distance per time unit are common unit rates, however, students should be able to flexibly use unit rates to name the amount of either quantity in terms of the other quantity. Students will begin to notice that related unit rates are reciprocals as in the first example. It is not intended that this be taught as an algorithm or rule because at this level, students should primarily use reasoning to find these unit rates.

In Grade 6, students are not expected to work with unit rates expressed as complex fractions. Both the numerator and denominator of the original ratio will be whole numbers.

Examples:

- On a bicycle you can travel 20 miles in 4 hours. What are the unit rates in this situation, (the distance you can travel in 1 hour and the amount of time required to travel 1 mile)?
  Solution: You can travel 5 miles in 1 hour written as $\frac{5\,mi}{1\,hr}$ and it takes $\frac{1}{5}$ of a hour to travel each mile written as $\frac{\frac{1}{5}\text{hr}}{1\text{mi}}$. Students can represent the relationship between 20 miles and 4 hours.



A simple modeling clay recipe calls for 1 cup corn starch, 2 cups salt, and 2 cups boiling water. How many cups of corn starch are needed to mix with each cup of salt?

Common Misconceptions:
See **6.RP.1**

Domain: **Ratios and Proportional Reasoning (RP)**

Cluster: Understand ratio concepts and use ration reasoning to solve problems.

Standard: **6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Standards for Mathematical Practice (MP):

MP.1 Make sense of problems and persevere in solving them.
MP.2 Reason abstractly and quantitatively.
MP.4 Model with mathematics.
MP.5 Use appropriate tools strategically.
MP.7 Look for and make use of structure.

Connections:

See **6.RP.1**

Explanations and Examples:

6.RP.3

- Using the information in the table, find the number of yards in 24 feet.

Feet	3	6	9	15	24
Yards	1	2	3	5	?

There are several strategies that students could use to determine the solution to this problem.

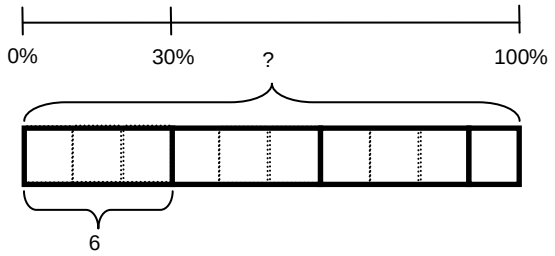
- Add quantities from the table to total 24 feet (9 feet and 15 feet); therefore the number of yards must be 8 yards (3 yards and 5 yards).
- Use multiplication to find 24 feet: 1) 3 feet x 8 = 24 feet; therefore 1 yard x 8 = 8 yards, or 2) 6 feet x 4 = 24 feet; therefore 2 yards x 4 = 8 yards.

- Compare the number of black to white circles. If the ratio remains the same, how many black circles will you have if you have 60 white circles?

● ● ● ● ○ ○ ○

Black	4	40	20	60	?
White	3	30	15	45	60

- If 6 is 30% of a value, what is that value? (Solution: 20)



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- A credit card company charges 17% interest on any charges not paid at the end of the month. Make a ratio table to show how much the interest would be for several amounts. If your bill totals $450 for this month, how much interest would you have to pay if you let the balance carry to the next month? Show the relationship on a graph and use the graph to predict the interest charges for a $300 balance.

Charges	$1	$50	$100	$200	$450
Interest	$0.17	$8.50	$17	$34	?

Examples for 6.RP.3 a:
Ratios and rates can be used in ratio tables and graphs to solve problems. Previously, students have used additive reasoning in tables to solve problems. To begin the shift to proportional reasoning, students need to begin using multiplicative reasoning. To aid in the development of proportional reasoning the cross-product algorithm is ***not*** expected at this level. When working with ratio tables and graphs, whole number measurements are the expectation for this standard.
For example, At Books Unlimited, 3 paperback books cost $18. What would 7 books cost? How many books could be purchased with $54. To find the price of 1 book, divide $18 by 3. One book is $6. To find the price of 7 books, multiply $6 (the cost of one book times 7 to get $42. To find the number of books that can be purchased with $54, multiply $6 times 9 to get $54 and then multiply 1 book times 9 to get 9 books.
Students use ratios, unit rates and multiplicative reasoning to solve problems in various contexts, including measurement, prices, and geometry. Notice in the table below, a multiplicative relationship exists between the numbers both horizontally and vertically. (Red numbers indicate solutions.)

Number of Books	**Cost**
1	6
3	18
7	42
9	54

Students use tables to compare ratios. Another bookstore offers paperback books at the prices below. Which bookstore has the best buy? Explain how you determined your answer.
To help understand the multiplicative relationship between the number of books and cost, students write equations to express the cost of any number of books. Writing equations is foundational for work in 7th grade. For example, the equation for the first table would be $C = 6n$.

Number of Books	**Cost**
4	20
8	40

The numbers in the table can be expressed as ordered pairs (number of books, cost) and plotted on a coordinate plane. Students should be able to plot ratios as ordered pairs.

Examples for 6.RP.3-b next page

Examples for 6.RP.3-b:
Students recognize the use of ratios, unit rate and multiplication in solving problems, which could allow for the use of fractions and decimals.
The ratio tables above use unit rate by determining the cost of one book. However, ratio tables can be used to solve problems without the use of a unit rate. For example, in trail mix, the ratio of cups of peanuts to cups of chocolate candies is 3 to 2. How many cups of chocolate candies would be needed for 9 cups of peanuts. (Show in a table)
One possible way to solve this problem is to recognize that 3 cups of peanuts times 3 will give 9 cups. The amount of chocolate will also increase at the same rate (3 times) to give 6 cups of chocolate Students could also find the number of cups of chocolate candies for 1 cup of peanuts by dividing both sides of the table by 3, giving 2/3 cup of chocolate for each cup of peanuts. To find the amount of chocolate needed for 9 cups of peanuts, students multiply the unit rate by nine (9 • 2/3), giving 6 cups of chocolate.

Examples for 6.RP.3 c -d:
This is the students' first introduction to percents. Percentages are a rate per 100. Models, such as percent bars or 10 x 10 grids should be used to model percents.
Students use percentages to find the part when given the percent, by recognizing that the whole is being divided into 100 parts and then taking a part of them (the percent). For example, to find 40% of 30, students could use a 10 x 10 grid to represent the whole (or 30). If the 30 is divided into 100 parts, the rate for one block is 0.3. Forty percent would be 40 of the blocks, or 40 x 0.3, which equals 12.
Student also find the whole, given a part and the percent. For example, if 25% of the students in Mrs. Rutherford's class like chocolate ice cream, then how many students are in Mrs. Rutherford's class if 6 like chocolate ice cream? Students can reason that if 25% is 6 and 100% is 4 times the 25%, then 6 times 4 would give 24 students in Mrs. Rutherford's class.

A ratio can be used to compare measures of two different types, such as inches per foot, milliliters per liter and centimeters per inch. Students recognize that a conversion factor is a fraction equal to 1 since the quantity described in the numerator and denominator is the same. For example $\frac{12 \text{ inches}}{1 \text{ foot}}$ is a conversion factor since the numerator and denominator name the same amount. Since the ratio is equivalent to 1, the identity property of multiplication allows an amount to be multiplied by the ratio. Also, the value of the ratio can also be expressed as $\frac{1 \text{ foot}}{12 \text{ inches}}$ allowing for the conversion ratios to be expressed in a format so that units will "cancel".

Students use ratios as conversion factors and the identity property for multiplication to convert ratio units. For example, how many centimeters are in 7 feet, given that 1 inch = 2.54 cm.

$$7 \text{ feet} \times \frac{12 \text{ inches}}{1 \text{ foot}} \times \frac{2.54 \text{ cm}}{1 \text{ inch}} = 7 \text{ feet} \times \frac{12 \text{ inches}}{1 \cancel{\text{foot}}} \times \frac{2.54 \text{ cm}}{1 \cancel{\text{inch}}} = 7 \times 12 \times 2.54 \text{ cm} = 213.36 \text{ cm}$$

Note: Conversion factors will be given. Conversions can occur both between and across the metric and English system. Estimates are not expected.

Common Misconceptions:
See 6.RP.1

Extended Common Core State Standards
Mathematics 6-8

The *Alternate Achievement Standards for Students With the Most Significant Cognitive Disabilities Non-Regulatory Guidance* states, "…materials should show a clear link to the content standards for the grade in which the student is enrolled, although the grade-level content may be reduced in complexity or modified to reflect pre-requisite skills." Throughout the Standards descriptors such as, describe, count, identify, etc, should be interpreted to mean that the students will be taught and tested according to their mode of communication. (NC DOE)

Sixth Grade Mathematics Ratios and Proportional Reasoning (RP)				
Common Core State Standards		**Essence**	**Extended Common Core**	
Understanding ratio concepts and use ratio reasoning to solve problems.		Understand ratios	**Understand ratio concepts**	
Cluster	1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* 2. Understand the concept of a unit rate a/b associated with a ratio $a{:}b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* 3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.		**Cluster**	1. Compare part-part and part-whole relationships (i.e., how many pieces of fruit? How many are apples how many are oranges?). 2. Write ratios to represent relationships between two quantities.

Domain: **The Number System (NS)**

Cluster:
Apply and extend previous understands of multiplication and division to divide fractions by fractions.

Standard: **6.NS.1**
Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?

Standards for Mathematical Practice (MP):

MP 1 Make sense of problems and persevere in solving them.
MP 2 Reason abstractly and quantitatively.
MP 3 Construct viable arguments and critique the reasoning of others.
MP 4 Model with mathematics.
MP 7 Look for and make use of structure.
MP 8 Look for and express regularity in repeated reasoning.

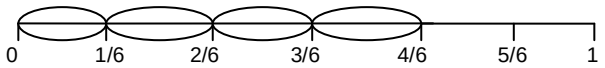
Connections:
This cluster is also connected to the Grade 6 Critical Area of Focus #2, **Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems.** This cluster connects to the other Grade 6 clusters within The Number System Domain. It marks the final opportunity for students to demonstrate fluency with the four operations with whole numbers and decimals. Grade 7 will extend these learning in The Number System and in Expressions and Equations.

Explanations and Examples:

6.NS.1 In 5th grade students divided whole numbers by unit fractions. Students continue this understanding by using visual models and equations to divide whole numbers by fractions and fractions by fractions to solve word problems. Students understand that a division problem such as 3 ÷ 2/5 is asking, "how many 2/5 are in 3?" One possible visual model would begin with three whole and divide each into fifths. There are 7 groups of two-fifths in the three wholes. However, one-fifth remains. Since one-fifth is half of a two-fifths group, there is a remainder of ½. Therefore, 3 ÷ 2/5 = 7 ½, meaning there are 7 ½ groups of two-fifths. Students interpret the solution, explaining how division by fifths can result in an answer with halves.

Students should also write contextual problems for fraction division problems. For example, the problem, 2/3 ÷ 1/6 can be illustrated with the following word problem:
Susan has 2/3 of an hour left to make cards. It takes her about 1/6 of an hour to make each card. About how many can she make?
This problem can be modeled using a number line.
1. Start with a number line divided into thirds.

2. The problem wants to know how many sixths are in two-thirds. Divide each third in half to create sixths.



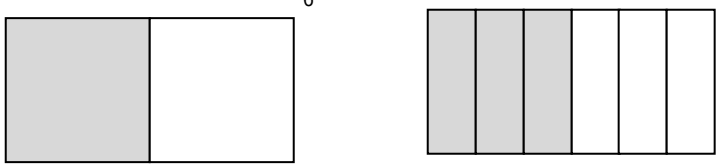
3. Each circled part represents 1/6. There are four sixths in two-thirds; therefore, Susan can make 4 cards.

Contexts and visual models can help students to understand quotients of fractions and begin to develop the relationship between multiplication and division. Model development can be facilitated by building from familiar scenarios with whole or friendly number dividends or divisors. Computing quotients of fractions build upon and extends student understandings developed in Grade 5. Students make drawings, model situations with manipulatives, or manipulate computer generated models.

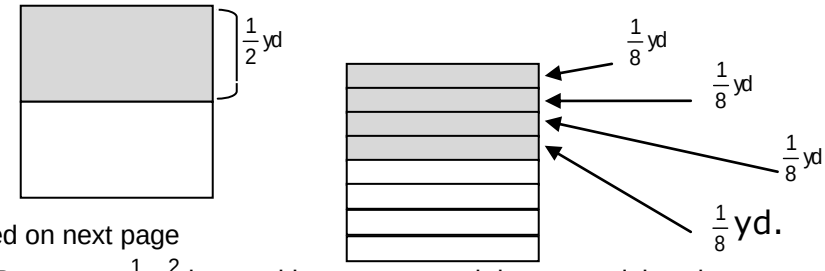
Examples:

- 3 people share $\frac{1}{2}$ pound of chocolate. How much of a pound of chocolate does each person get?

  Solution: Each person gets $\frac{1}{6}$ lb. of chocolate.



- Manny has $\frac{1}{2}$ yard of fabric to make book covers. Each book is made from $\frac{1}{8}$ yard of fabric. How many book covers can Manny make? Solution: Manny can make 4 book covers.



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- Represent $\frac{1}{2} \div \frac{2}{3}$ in a problem context and draw a model to show your solution.

Context: You are making a recipe that calls for $\frac{2}{3}$ cup of yogurt. You have $\frac{1}{2}$ cup of yogurt from a snack pack. How much of the recipe can you make?

Explanation of Model:

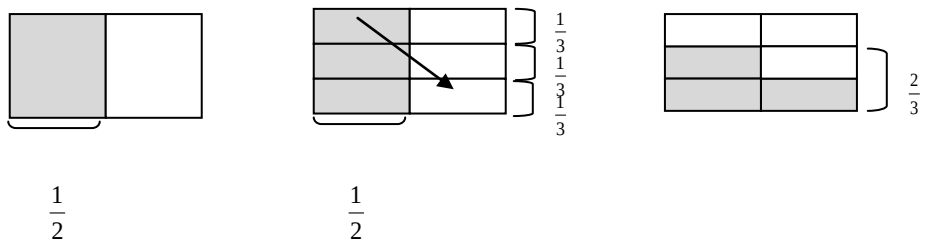
The first model shows $\frac{1}{2}$ cup. The shaded squares in all three models show $\frac{1}{2}$ cup.

The second model shows $\frac{1}{2}$ cup and also shows $\frac{1}{3}$ cups horizontally.

The third model shows $\frac{1}{2}$ cup moved to fit in only the area shown by $\frac{2}{3}$ of the model.

$\frac{2}{3}$ is the new referent unit (whole) .

3 out of the 4 squares in the $\frac{2}{3}$ portion are shaded. A $\frac{1}{2}$ cup is only $\frac{3}{4}$ of a $\frac{2}{3}$ cup portion, so you can only make ¾ of the recipe.



Instructional Strategies continued next page

Instructional Strategies

Computation with fractions is best understood when it builds upon the familiar understandings of whole numbers and is paired with visual representations. Solve a simpler problem with whole numbers, and then use the same steps to solve a fraction divided by a fraction. Looking at the problem through the lens of "How many groups?" or "How many in each group?" helps visualize what is being sought.

For example: 12÷3 means; How many groups of three would make 12? Or how many in each of 3 groups would make 12? Thus 7/2 ÷ 1/4 can be solved the same way. How many groups of 1/4 make 7/2 ? Or, how many objects in a group when 7/2 fills one fourth?

Creating the picture that represents this problem makes seeing and proving the solutions easier.

Set the problem in context and represent the problem with a concrete or pictorial model.

5/4 ÷ 1/2 5/4 cups of nuts fills 1/2 of a container. How many cups of nuts will fill the entire container?

Teaching "invert and multiply" without developing an understanding of why it works first leads to confusion as to when to apply the shortcut.

Learning how to compute fraction division problems is one part, being able to relate the problems to real-world situations is important. Providing opportunities to create stories for fraction problems or writing equations for situations is needed.

Common Misconceptions:

Students may believe that dividing by 1/2 is the same as dividing in half. Dividing by half means to find how many 1/2s there are in a quantity, whereas, dividing in half means to take a quantity and split it into two equal parts. Thus 7 divided by 1/2 = 14 and 7 divided in half equals 3 1/2.

6.NS.1					

Domain: **The Number System (NS)**

Cluster: Compute fluently with multi-digit numbers and find common factors and multiples.

Standard: **6.NS.2**
Fluently divide multi-digit numbers using the standard algorithm.

Standards for Mathematical Practice (MP):
MP 2 Reason abstractly and quantitatively.
MP 7 Look for and make use of structure.
MP 8 Look for and express regularity in repeated reasoning.

Connections:
This cluster is connected to the Grade 6 Critical Area of Focus #2, **Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems.**

This cluster connects to the other Grade 6 clusters within The Number System Domain. It marks the final opportunity for students to demonstrate fluency with the four operations with whole numbers and decimals. Grade 7 will extend these learning's in The Number System and in Expressions and Equations.

Explanations and Examples:
6.NS.2 Procedural fluency is defined by the Common Core as "skill in carrying out procedures **flexibly**, accurately, efficiently and appropriately". In the elementary grades, students were introduced to division through concrete models and various strategies to develop an understanding of this mathematical operation (limited to 4-digit numbers divided by 2-digit numbers). In 6th grade, students become fluent in the use of the standard division algorithm. This understanding is foundational for work with fractions and decimals in 7th grade.
Students are expected to fluently and accurately divide multi-digit whole numbers. Divisors can be any number of digits at this grade level.
As students divide they should continue to use their understanding of place value to describe what they are doing. When using the standard algorithm, students' language should reference place value. For example, when dividing 32 into 8456, as they write a 2 in the quotient they should say, "there are 200 thirty-twos in 8456 " and could write 6400 beneath the 8456 rather than only writing 64.

$\begin{array}{r} 2 \\ 32\overline{)8456} \end{array}$	There are 200 thirty twos in 8456.
$\begin{array}{r} 2 \\ 32\overline{)8456} \\ -\underline{6400} \\ 2056 \end{array}$	200 times 32 is 6400. 8456 minus 6400 is 2056.
$\begin{array}{r} 26 \\ 32\overline{)8456} \\ -\underline{6400} \\ 2056 \end{array}$	There are 60 thirty twos in 2056. **Continued next page**

26 32)8456 −6400 2056 −1920 136	60 times 32 is 1920. 2056 minus 1920 is 136.
264 32)8456 −6400 2056 −1920 136 −128	There are 4 thirty twos in 136. 4 times 32 is 128.
264 32)8456 −6400 2056 −1920 136 −128 8	The remainder is 8. There is not a full thirty two in 8; there is only part of a thirty two in 8. This can also be written as $\frac{8}{32}$ or $\frac{1}{4}$. There is ¼ of a thirty two in 8. 8456 = 264 * 32 + 8

<u>Instructional Strategies</u> (NS. 2 through 4)

As students study whole numbers in the elementary grades, a foundation is laid in the conceptual understanding of each operation. Discovering and applying multiple strategies for computing creates connections which evolve into the proficient use of standard algorithms. Fluency with an algorithm denotes an ability that is efficient, accurate, appropriate and flexible. Division was introduced in Grade 3 conceptually, as the inverse of multiplication. In Grade 4, division continues using place-value strategies, properties of operations, and the relationship with multiplication, area models, and rectangular arrays to solve problems with one digit divisors. In Grade 6, fluency with the algorithms for division and all operations with decimals is developed.

Fluency is something that develops over time; practice should be given over the course of the year as students solve problems related to other mathematical studies. Opportunities to determine when to use paper pencil algorithms, mental math or a computing tool is also a necessary skill and should be provided in problem solving situations.

Greatest common factor and *least common multiple* are usually taught as a means of combining fractions with unlike denominators. This cluster builds upon the previous learning of the **multiplicative structure** of whole numbers, as well as prime and composite numbers in Grade 4. Although the process is the same, the point is to become aware of the relationships between numbers and their multiples. For example, consider answering the question: "If two numbers are multiples of four, will the sum of the two numbers also be a multiple of four?" Being able to see and write the relationships between numbers will be beneficial as further algebraic understandings are developed. Another focus is to be able to see how the GCF is useful in expressing the numbers using the distributive property, (36 + 24) = 12(3+2), where 12 is the GCF of 36 and 24. This concept will be extended in Expressions and Equations as work progresses from understanding the number system and solving equations to simplifying and solving algebraic equations in Grade 7.

Domain: **The Number System (NS)**

Cluster: Compute fluently with multi-digit numbers and find common factors and multiples.

Standard: **6.NS.3**
Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Standards for Mathematical Practice (MP):
MP 2 Reason abstractly and quantitatively.
MP 7 Look for and make use of structure.
MP 8 Look for and express regularity in repeated reasoning.

Connections:
See 6.NS.2

Explanations and Examples:
6.NS.3 Procedural fluency is defined by the Common Core as "skill in carrying out procedures flexibly, accurately, efficiently and appropriately". In 4th and 5th grades, students added and subtracted decimals. Multiplication and division of decimals was introduced in 5th grade (decimals to the hundredth place). At the elementary level, these operations were based on concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. In 6th grade, students become fluent in the use of the standard algorithms of each of these operations.

The use of estimation strategies supports student understanding of operating on decimals.

Example:

- First, students estimate the sum and then find the exact sum of 14.4 and 8.75. An estimate of the sum might be 14 + 9 or 23. Students may also state if their estimate is low or high. They would expect their answer to be greater than 23. They can use their estimates to self-correct.

  Answers of 10.19 or 101.9 indicate that students are not considering the concept of place value when adding (adding tenths to tenths or hundredths to hundredths) whereas answers like 22.125 or 22.79 indicate that students are having difficulty understanding how the four-tenths and seventy-five hundredths fit together to make one whole and 25 hundredths.

Students use the understanding they developed in 5th grade related to the patterns involved when multiplying and dividing by powers of ten to develop fluency with operations with multi-digit decimals.

Instructional Strategies
See NS. 2

Domain: **The Number System (NS)**

Cluster: Compute fluently with multi-digit numbers and find common factors and multiples.

Standard: **6.NS.4**
Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).

Standards for Mathematical Practice (MP):
MP 7 Look for and make use of structure

Connections:
See NS. 2

Explanations and Examples:
Students will find the greatest common factor of two whole numbers less than or equal to 100. For example, the greatest common factor of 40 and 16 can be found by
1) listing the factors of 40 (1, 2, 4, 5, 8, 10, 20, 40) and 16 (1, 2, 4, 8, 16), then taking the greatest common factor (8). Eight (8) is also the largest number such that the other factors are **relatively prime** (two numbers with no common factors other than one). For example, 8 would be multiplied by 5 to get 40; 8 would be multiplied by 2 to get 16. Since the 5 and 2 are relatively prime, then 8 is the greatest common factor. If students think 4 is the greatest, then show that 4 would be multiplied by 10 to get 40, while 16 would be 4 times 4. Since the 10 and 4 are not relatively prime (have 2 in common), the 4 cannot be the greatest common factor.
2) listing the prime factors of 40 (2 • 2 • 2 • 5) and 16 (2 • 2 • 2 • 2) and then multiplying the common factors (2 • 2 • 2 = 8).

Students also understand that the greatest common factor of two prime numbers will be 1.

Students use the greatest common factor and the distributive property to find the sum of two whole numbers. For example, 36 + 8 can be expressed as 4 (9 + 20 = 4 (11).

Students find the least common multiple of two whole numbers less than or equal to twelve. For example, the least common multiple of 6 and 8 can be found by
1) listing the multiplies of 6 (6, 12, 18, 24, 30, …) and 8 (8, 26, 24, 32, 40…), then taking the least in common from the list (24); or
2) using the prime factorization.

Step 1: find the prime factors of 6 and 8.
6 = 2 • 3
8 = 2 • 2 • 2

Step 2: Find the common factors between 6 and 8. In this example, the common factor is 2
Step 3: Multiply the common factors and any extra factors: 2 • 2 • 2 • 3 or 24 (one of the twos is in common; the other twos and the three are the extra factors.

More examples continued on next page

Examples:

- What is the greatest common factor (GCF) of 24 and 36? How can you use factor lists or the prime factorizations to find the GCF?
  Solution: $2^2 * 3 = 12$. Students should be able to explain that both 24 and 36 have 2 factors of 2 and one factor of 3, thus 2 x 2 x 3 is the greatest common factor.)
- What is the least common multiple (LCM) of 12 and 8? How can you use multiple lists or the prime factorizations to find the LCM?
  Solution: $2^3 * 3 = 24$. Students should be able to explain that the least common multiple is the smallest number that is a multiple of 12 and a multiple of 8. To be a multiple of 12, a number must have 2 factors of 2 and one factor of 3 (2 x 2 x 3). To be a multiple of 8, a number must have 3 factors of 2 (2 x 2 x 2). Thus the least common multiple of 12 and 8 must have 3 factors of 2 and one factor of 3 (2 x 2 x 2 x 3).
- Rewrite 84 + 28 by using the distributive property. Have you divided by the largest common factor? How do you know?
- Given various pairs of addends using whole numbers from 1-100, students should be able to identify if the two numbers have a common factor. If they do, they identify the common factor and use the distributive property to rewrite the expression. They prove that they are correct by simplifying both expressions.
  - 27 + 36 = 9 (3 + 4)
    63 = 9 x 7
    63 = 63
  - 31 + 80

There are no common factors. I know that because 31 is a prime number, it only has 2 factors, 1 and 31. I know that 31 is not a factor of 80 because 2 x 31 is 62 and 3 x 31 is 93.

Instructional Strategies

See NS. 2

Common Misconceptions:

See NS. 2

Arizona, Ohio & NC DOE

Sixth Grade Mathematics
The Number System (NS)

Common Core State Standards	Essence	Extended Common Core
Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	**Understand fractions**	**Extend previous understandings of fractions.**
Cluster 1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc). How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4- cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*		**Cluster** 1. Compare the relationships between the unit fractions (1/2, 1/3, ¼, 1/5, 1/6, 1/8,1/10). 2. Add fractions with like denominators to make a whole (halves, thirds, fourths, fifths, sixths, eighths, and tenths).
Compute fluently with multi-digit numbers and find common factors and multiples.	**Understand Multiplication**	**Multiply with numbers 0-10.**
Cluster 2. Fluently divide multi-digit numbers using the standard algorithm. 3. Fluently add, subtract, multiply, and divide multidigit decimals using the standard algorithm for each operation. 4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).		**Cluster** 3. Solve multiplication problems when groups and size of groups is known but the whole is unknown (a x b=). NC CCSS Extended

EXTENDED 6.NS.1-4

Domain: **The Number System (NS)**

Cluster: Apply and extend previous understandings of numbers to the system of rational numbers.

Standard: **6.NS.5**
Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

Standards for Mathematical Practice (MP):
MP 1 Make sense of problems and persevere in solving them.
MP 2 Reason abstractly and quantitatively.
MP 4 Model with mathematics.

Connections:
This cluster does not directly address one of the Grade 6 Critical Areas of Focus. However, it is the foundation for working with rational numbers, algebraic expressions and equations, functions, and the coordinate plane in subsequent grades.

Explanations and Examples:
6.NS.5 Students use rational numbers (fractions, decimals, and integers) to represent real-world contexts and understand the meaning of 0 in each situation.

Example 1:
a) Use an integer to represent 25 feet below sea level
b) Use an integer to represent 25 feet above sea level
c) What would 0 (zero) represent in the scenario above?

Solution:
a) -25
b) +25
c) 0 would represent sea level

Instructional Strategies NS. 5 through 8
The purpose of this cluster (NS 5-8) is to begin study of the existence of negative numbers, their relationship to positive numbers, and the meaning and uses of absolute value. Starting with examples of having/owing and above/below zero sets the stage for understanding that there is a mathematical way to describe opposites. Students should already be familiar with the counting numbers (positive whole numbers and zero), as well as with fractions and decimals (also positive). They are now ready to understand that all numbers have an opposite. These special numbers can be shown on vertical or horizontal number lines, which then can be used to solve simple problems. Demonstration of understanding of positives and negatives involves translating among words, numbers and models: given the words "7 degrees below zero," showing it on a thermometer and writing -7; given -4 on a number line, writing a real-life example and mathematically -4. Number lines also give the opportunity to model absolute value as the distance from zero.
Simple comparisons can be made and order determined. Order can also be established and written mathematically: -3° C > -5° C or -5° C < -3° C. Finally, absolute values should be used to relate contextual problems to their meanings and solutions.
Using number lines to model negative numbers, prove the distance between opposites, and understand the meaning of absolute value easily transfers to the creation and usage of four-quadrant coordinate grids. Points can now be plotted in all four quadrants of a coordinate grid. Differences between numbers can be found by counting the distance between numbers on the grid. Actual computation with negatives and positives is handled in Grade 7.

Domain: **The Number System (NS)**

Cluster: Apply and extend previous understandings of numbers to the system of rational numbers.

Standard: **6.NS.6**

Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite.
b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

Standards for Mathematical Practice (MP):

MP 1 Make sense of problems and persevere in solving them.
MP 2 Reason abstractly and quantitatively.
MP 4 Model with mathematics.

Connections:

This cluster does not directly address one of the Grade 6 Critical Areas of Focus. However, it is the foundation for working with rational numbers, algebraic expressions and equations, functions, and the coordinate plane in subsequent grades.

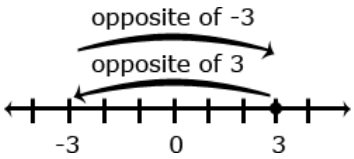
Explanations and Examples:

6.NS.6 In elementary school, students worked with positive fractions, decimals and whole numbers on the number line. In 6th grade, students extend the number line to represent all rational numbers and recognize that number lines may be either horizontal or vertical (i.e. thermometer). Students recognize that a number and its opposite are equidistance from zero (reflections about the zero). The opposite sign (–) shifts the number to the opposite side of 0. For example, – 4 could be read as "the opposite of 4" which would be negative 4. The following example, – (–6.4) would be read as "the opposite of the opposite of 6.4" which would be 6.4. Zero is its own opposite.

Students worked with Quadrant I in elementary school. As the *x*-axis and *y*-axis are extending to include negatives, students begin with the Cartesian Coordinate system. Students recognize the point where the *x*-axis and *y*-axis intersect as the origin. Students identify the four quadrants and are able to identify the quadrant for an ordered pair based on the signs of the coordinates. For example, students recognize that in Quadrant II, the signs of all ordered pairs would be (–, +).
Students understand the relationship between two ordered pairs differing only by signs as reflections across one or both axes. For example, in the ordered pairs (-2, 4) and (-2, -4), the *y*-coordinates differ only by signs, which represents a reflection across the *x*-axis. A change is the *x*-coordinates from (-2, 4) to (2, 4), represents a reflection across the *y*-axis. When the signs of both coordinates change, [(2, -4) changes to (-2, 4)], the ordered pair has been reflected across both axes.
Students are able to plot all rational numbers on a number line (either vertical or horizontal) or identify the values of given points on a number line. For example, students are able to identify where the following numbers would be on a number line: –4.5, 2, 3.2, –3 3/5, 0.2, –2, 11/2.

More examples continued on next page

Number lines can be used to show numbers and their opposites. Both 3 and -3 are 3 units from zero on the number line. Graphing points and reflecting across zero on a number line extends to graphing and reflecting points across axes on a coordinate grid. The use of both horizontal and vertical number line models facilitates the movement from number lines to coordinate grids.



Example 1:

- Graph the following points in the correct quadrant of the coordinate plane. If you reflected each point across the x-axis, what are the coordinates of the reflected points? What similarities do you notice between coordinates of the original point and the reflected point?

$\left(\frac{1}{2}, -3\frac{1}{2}\right)$ $\left(-\frac{1}{2}, -3\right)$ $(0.25, -0.75)$

Solution:
The coordinates of the reflected points would be (1/2, 3 1/2) (-1/2, 3) (0.25, -.75)
Note that the y-coordinates are opposite.

Example 2:
What is the opposite of 2 ½? Explain your answer?

Solution:
-2 ½ because it is the same distance from 0 on the opposite side.

Students place the following numbers would be on a number line: -4.5, 2, 3.2, -3 3/5, 0.2, -2, 11/2. Based on number line placement, numbers can be placed in order.

Solution:
The numbers in order from least to greatest are:
-4.4, -3 3/5, -2, 0.2, 2, 3.2, 11/2.
Students place each of these numbers on a number line to justify this order.

Instructional Strategies **See NS. 5-8**

Domain: **The Number System (NS)**

Cluster: Apply and extend previous understandings of numbers to the system of rational numbers.

Standard: **6.NS.7**

Understand ordering and absolute value of rational numbers.

a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*
b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 °C > –7 °C to express the fact that –3 °C is warmer than –7 °C.*
c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.*
d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*

Standards for Mathematical Practice (MP):

MP 1 Make sense of problems and persevere in solving them.
MP 2 Reason abstractly and quantitatively.
MP 4 Model with mathematics.

Connections:

Same as NS. 5

Explanations and Examples:

6.NS.7

Students identify the absolute value of a number as the distance from zero but understand that although the value of -7 is less than -3, the absolute value (distance) of -7 is greater than the absolute value (distance) of -3. Students use inequalities to express the relationship between two rational numbers, understanding that the value of numbers is smaller moving to the left on a number line. For example, –4 ½ < –2 because –4 ½ is located to the left of –2 on the number line.

b. Students write statements using < or > to compare rational number in context. However, explanations should reference the context rather than "less than" or "greater than". For example, the balance in Sue's checkbook was –12.55. The balance in Ron's checkbook was –10.45. Since –12.55 < –10.45, Sue owes more than Ron. The interpretation could also be "Ron owes less than Sue".

c. Students understand absolute value as the distance from zero and recognize the symbols | | as representing absolute value. For example, | –7 | can be interpreted as the distance –7 is from 0 which would be 7. Likewise | 7 | can be interpreted as the distance 7 is from 0 which would also be 7. In real-world contexts, the absolute value can be used to describe size or magnitude. For example, for an ocean depth of –900 feet, write | –900| = 900 to describe the distance below sea level.

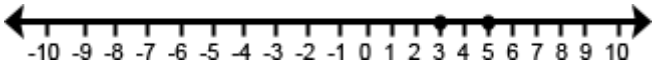
d. When working with positive numbers, the absolute value (distance from zero) of the number and the value of the number is the same; therefore, ordering is not problematic. However, negative numbers have a distinction that students need to understand. As the negative number increases (moves to the left on a number line), the value of the number decreases. For example, –24 is less than –14 because –24 is located to the left of –14 on the number line. However, absolute value is the distance from zero. In terms of absolute value (or distance) the absolute value of –24 is greater than –14. For negative numbers, as the absolute value increases, the value of the number decreases.

6.NS.7-a Explanation
Student use inequalities to express the relationship between two rational numbers, understanding the value of numbers is smaller moving to the left on a number line.

Common models to represent and compare integers include number line models, temperature models and the profit-loss model. On a number line model, the number is represented by an arrow drawn from zero to the location of the number on the number line; the absolute value is the length of this arrow. The number line can also be viewed as a thermometer where each point of on the number line is a specific temperature. In the profit-loss model, a positive number corresponds to profit and the negative number corresponds to a loss. Each of these models is useful for examining values but can also be used in later grades when students begin to perform operations on integers.

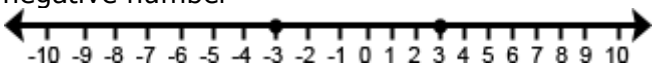
In working with number line models, students internalize the order of the numbers; larger numbers on the right or top of the number line and smaller numbers to the left or bottom of the number line. They use the order to correctly locate integers and other rational numbers on the number line. By placing two numbers on the same number line, they are able to write inequalities and make statements about the relationships between the numbers.

Case 1: Two positive numbers



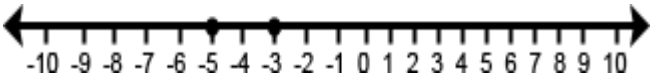
5 > 3
5 is greater than 3

Case 2: One positive and one negative number



3 > -3
positive 3 is greater than negative 3
negative 3 is less than positive 3

Case 3: Two negative numbers



-3 > -5
negative 3 is greater than negative 5
negative 5 is less than negative 3

Example:
Write a statement to compare – 4 . and –2. Explain your answer.

Solution:
– 4 . < –2 because – 4 . is located to the left of –2 on the number line

Students recognize the distance from zero as the absolute value or magnitude of a rational number. Students need multiple experiences to understand the relationships between numbers, absolute value, and statements about order.

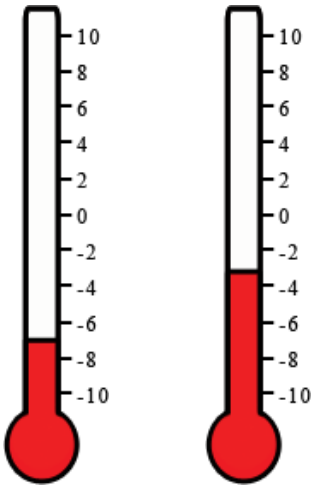
Comparative statements generate informal experience with operations and lay the foundation for formal work with operations on integers in grade 7.

6.NS.7b Explanation
Students write statements using < or > to compare rational number in context. However, explanations should reference the context rather than "less than" or "greater than".

Example1:
The balance in Sue's checkbook was –$12.55. The balance in Ron's checkbook was –$10.45. Write an inequality to show the relationship between these amounts. Who owes more?

Solution: –12.55 < –10.45, Sue owes more than Ron. The interpretation could also be Ron owes less than Sue".

Example 2:
- One of the thermometers shows -3°C and the other shows -7°C. Which thermometer shows which temperature? Which is the colder temperature? How much colder? Write an inequality to show the relationship between the temperatures and explain how the model shows this relationship.



Solution:
• The thermometer on the left is -7; right is -3
• The left thermometer is colder by 4 degrees
• Either -7 < -3 or -3 > -7

Although 6.NS.7a is limited to two numbers, this part of the standard expands the ordering of rational numbers to more than two numbers in context.

Example 3:
A meteorologist recorded temperatures in four cities around the world. List these cities in order from coldest temperature to warmest temperature:

Albany 5°
Anchorage -6°
Buffalo -7°
Juneau -9°
Reno 12°

Solution:
Juneau -9°
Buffalo -7°
Anchorage -6°
Albany 5°
Reno 12° *Solution:*

6.NS.7c Explanation

Students recognize the distance from zero as the absolute value or magnitude of a rational number and recognize the symbols | | as representing absolute value.

Students need multiple experiences to understand the relationships between numbers, absolute value, and statements about order.

Example 1:
Which numbers have an absolute value of 7
Solution: 7 and −7 since both numbers have a distance of 7 units from 0 on the number line.

Example 2:
What is the | −3 ½| ?

Solution: 3 1/2

In real-world contexts, the absolute value can be used to describe size or magnitude. For example, for an ocean depth of 900 feet, write | −900| = 900 to describe the distance below sea level.

Example 3:
The Great Barrier Reef is the world's largest reef system and is located off the coast of Australia. It reaches from the surface of the ocean to a depth of 150 meters. Students could represent this value as less than 150 meters or a depth no greater than 150 meters below sea level.

6.NS.7d Explanation
When working with positive numbers, the absolute value (distance from zero) of the number and the value of the number is the same; therefore, ordering is not problematic. However, negative numbers have a distinction that students need to understand. As the negative number increases (moves to the left on a number line), the value of the number decreases. For example, −24 is less than −14 because −24 is located to the left of −14 on the number line. However, absolute value is the distance from zero. In terms of absolute value (or distance) the absolute value of −24 is greater than the absolute value of −14. For negative numbers, as the absolute value increases, the value of the negative number decreases.

Instructional Strategies
See NS.5

Common Misconceptions:
See NS.5

Arizona**, Ohio & NC DOE**

Domain: **The Number System (NS)**

Cluster: Apply and extend previous understandings of numbers to the system of rational numbers.

Standard: **6.NS.8.**
Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

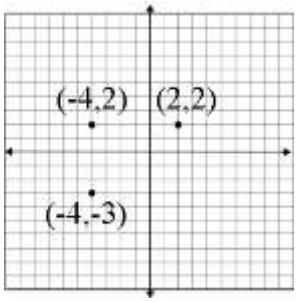
Standards for Mathematical Practice (MP):
MP 1 Make sense of problems and persevere in solving them.
MP 2 Reason abstractly and quantitatively.
MP 4 Model with mathematics.
MP 5 Use appropriate tools strategically.
MP 7 Look for and make use of structure.

Connections:
See NS.5

Explanations and Examples:
6.NS.8 Students find the distance between points whose ordered pairs have the same x-coordinate (vertical) or same y-coordinate (horizontal).
For example, the distance between (–5, 2) and (–9, 2) would be 4 units. This would be a horizontal line since the *y*-coordinates are the same. In this scenario, both coordinates are in the same quadrant. The distance can be found by using a number line to find the distance between –5 and –9. Students could also recognize that –5 is 5 units from 0 (absolute value) and that –9 is 9 units from 0 (absolute value). Since both of these are in the same quadrant, the distance can be found by finding the difference between 9 and 5. (| 9 | - | 5 |).

Coordinates could also be in two quadrants. For example, the distance between (3, –5) and (3, 7) would be 12 units. This would be a vertical line since the *x*-coordinates are the same. The distance can be found by using a number line to count from –5 to 7 or by recognizing that the distance (absolute value) from –5 to 0 is 5 units and the distance (absolute value) from 0 to 7 is 7 units so the total distance would be 5 + 7 or 12 units. Students graph coordinates for polygons and find missing vertices based on properties of triangles and quadrilaterals.

Example:
- If the points on the coordinate plane below are the three vertices of a rectangle, what are the coordinates of the fourth vertex? How do you know? What are the length and width of the rectangle?



To determine the distance along the x-axis between the point (-4, 2) and (2, 2) a student must recognize that -4 is $|-4|$ or 4 units to the left of 0 and 2 is $|2|$ or 2 units to the right of zero, so the two points are total of 6 units apart along the x-axis. Students should represent this on the coordinate grid and numerically with an absolute value expression, $|-4| + |2|$.
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Sixth Grade Mathematics Number System (NS)				
Common Core State Standards		**Essence**	**Extended Common Core**	
Apply and extend previous understandings of numbers to the system of rational numbers.		Extend Number knowledge	**Apply and extend previous understandings of numbers to the system of rational numbers.**	
Cluster	5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. 6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. 7. Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*		**Cluster**	4. Understand that the order of the digits determines the given number and use this understanding to compare sets and numbers (i.e., 24 and 42, 24 is less than 42 because it contains 2 tens and 42 contains 4 tens). 5. Compare temperatures including negatives (use a non-digital thermometer). NC CCSS Extended

Sixth Grade Mathematics Number System (NS) continued				
Common Core State Standards		**Essence**	**Extended Common Core**	
Apply and extend previous understandings of numbers to the system of rational numbers.		Extend Number knowledge	**Apply and extend previous understandings of numbers to the system of rational numbers.**	
Cluster	b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write –3 o C > –7 o C to express the fact that –3 o C is warmer than –7 o C. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of –30 dollars, write |-30| = 30 to describe the size of the debt in dollars. d. Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars. 8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.		**Cluster**	NC CCSS Extended

Domain: **Expressions and Equations (EE)**
Cluster: Apply and extend previous understanding of arithmetic to algebraic expressions.
Standard: **6.EE.1** Write and evaluate numerical expression involving whole-number exponents.
Standards for Mathematical Practice (MP): MP 2 Reason abstractly and quantitatively.
Connections: This cluster (EE.1-4) is connected to the Grade 6 Critical Area of Focus #3, **Writing, interpreting and using expressions, and equations**. The learning in this cluster is foundational in the transition to algebraic representation and problem solving which is extended and formalized in Grade 7, the Number System and Expressions and Equations.

Explanations and Examples:

6.EE.1 Students demonstrate the meaning of exponents to write and evaluate numerical expressions with whole number exponents. The base can be a whole number, positive decimal or a positive fraction (i.e. $\frac{1}{2}^4$ can be written $\frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2}$ which has the same value as $\frac{1}{16}$).

Students recognize that an expression with a variable represents the same mathematics (i.e. x^4 can be written as $x \bullet x \bullet x \bullet x$) and write algebraic expressions from verbal expressions.

Examples:

- Write the following as a numerical expressions using exponential notation.
  - The area of a square with a side length of 8 m (Solution: $8^2 m^2$)
  - The volume of a cube with a side length of 5 ft.: (Solution: $5^3 ft^3$)
  - Yu-Lee has a pair of mice. The mice each have 2 babies. The babies grow up and have two babies of their own: (Solution: 2^3 mice)
- Evaluate:
  - 4^3 (Solution: 64)
  - $5+2^4 \bullet 6$ (Solution: 101)
  - $7^2 - 24 \div 3 + 26$ (Solution: 67)

Instructional Strategies

The skills of reading, writing and evaluating expressions are essential for future work with expressions and equations, and are a Critical Area of Focus for Grade 6. In earlier grades, students added grouping symbols ( ) to reduce ambiguity when solving equations. Now the focus is on using ( ) to denote terms in an expression or equation. Students should now focus on what terms are to be solved first rather than invoking the PEMDAS rule. Likewise, the division symbol (3 ÷ 5) was used and should now be replaced with a fraction bar (3/5). Less confusion will occur as students write algebraic expressions and equations if *x* represents only variables and not multiplication. The use of a dot or parentheses between number terms is preferred.

Provide opportunities for students to write expressions for numerical and real-world situations. Write multiple statements that represent a given algebraic expression. For example, the expression *x* – 10 could be written as "ten less than a number," "a number minus ten," "the temperature fell ten degrees,"', "I scored ten fewer points than my brother," etc. Students should also read an algebraic expression and write a statement.

Through modeling, encourage students to use proper mathematical vocabulary when discussing terms, factors, coefficients, etc.

Provide opportunities for students to write equivalent expressions, both numerically and with variables. For example, given the expression $x + x + x + x + 4 \bullet 2$, students could write $2x + 2x + 8$ or some other equivalent expression. Make the connection to the simplest form of this expression as $4x + 8$. Because this is a foundational year for building the bridge between the concrete concepts of arithmetic and the abstract thinking of algebra, using hands-on materials (such as algebra tiles, counters, unifix cubes, "Hands on Algebra") to help students translate between concrete numerical representations and abstract symbolic representations is critical. Provide expressions and formulas to students, along with values for the variables so students can evaluate the expression. Evaluate expressions using the order of operations with and without parentheses. Include whole-number exponents, fractions, decimals, etc. Provide a model that shows step-by-step thinking when simplifying an expression. This demonstrates how two lines of work maintain equivalent algebraic expressions and establishes the need to have a way to review and justify thinking.

Provide a variety of expressions and problem situations for students to practice and deepen their skills. Start with simple expressions to evaluate and move to more complex expressions. Likewise start with simple whole numbers and move to fractions and decimal numbers. The use of negatives and positives should mirror the level of introduction in Grade 6 The Number System; students are developing the concept and not generalizing operation rules.

The use of technology can assist in the exploration of the meaning of expressions. Many calculators will allow you to store a value for a variable and then use the variable in expressions. This enables the student to discover how the calculator deals with expressions like $x2$, $5x$, xy, and $2(x + 5)$.

Common Misconceptions:

Many of the misconceptions when dealing with expressions stem from the misunderstanding/reading of the expression. For example, knowing the operations that are being referenced with notation like, $x3$, $4x$, $3(x + 2y)$ is critical. The fact that x^3 means $x \cdot x \cdot x$, means x times x times x, not $3x$ or 3 times x; $4x$ means 4 times x or $x+x+x+x$, not forty-something. When evaluating $4x$ when $x = 7$, substitution does not result in the expression meaning 47. Use of the "x" notation as both the variable and the operation of multiplication can complicate this understanding.

Arizona, Ohio & NC DOE

Domain: **Expressions and Equations (EE)**

Cluster: Apply and extend previous understanding of arithmetic to algebraic expressions.

Standard: **6.EE.2**

Write, read, and evaluate expressions in which letters stand for numbers.

a. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as 5 – y.

b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.

c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole- number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas V = s3 and A = 6 s2 to find the volume and surface area of a cube with sides of length s = ½.*

Standards for Mathematical Practice (MP):

MP 1 Make sense of problems and persevere in solving them.
MP 2 Reason abstractly and quantitatively.
MP 3 Construct viable arguments and critique the reasoning of others.
MP 4 Model with mathematics.
MP 6 Attend to precision.

Connections:

See EE.1

Explanations and Examples:

6.EE.2a-c Students write expressions from verbal descriptions using letters and numbers. Students understand order is important in writing subtraction and division problems. Students understand that the expression "5 times any number, *n*" could be represented with 5*n* and that a number and letter written together means to multiply.

Students use appropriate mathematical language to write verbal expressions from algebraic expressions. Students can describe expressions such as 3 (2 + 6) as the product of two factors: 3 and (2 + 6). The quantity (2 + 6) is viewed as one factor consisting of two terms.

Students evaluate algebraic expressions, using order of operations as needed. Given an expression such as 3*x* + 2 *y*, find the value of the expression when *x* is equal to 4 and *y* is equal to 2.4. This problem requires students to understand that multiplication is understood when numbers and variables are written together and to use the order of operations to evaluate.

3 • 4 + 2 • 2.4
12 + 4.8
16.8

Given a context and the formula arising from the context, students could write an expression and then evaluate for any number. For example, it costs $100 to rent the skating rink plus $5 per person. The cost for any number (*n*) of people could be found by the expression, 100 + 5*n*. What is the cost for 25 people?

Examples continue on next page

It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number.

- r + 21 as "some number plus 21 as well as "r plus 21"
- n • 6 as "some number times 6 as well as "n times 6"
- $\frac{s}{6}$ and s ÷ 6 as "as some number divided by 6" as well as "s divided by 6"

Students should identify the parts of an algebraic expression including variables, coefficients, constants, and the names of operations (sum, difference, product, and quotient). Development of this common language helps students to understand the structure of expressions and explain their process for simplifying expressions.

Terms are the parts of a sum. When the term is an explicit number, it is called a constant. When the term is a product of a number and a variable, the number is called the coefficient of the variable.

Variables are letters that represent numbers. There are various possibilities for the numbers they can represent; students can substitute these possible numbers for the letters in the expression for various different purposes.

Consider the following expression:
$x^2 + 5y + 3x + 6$

The variables are x and y.
There are 4 terms, x^2, 5y, 3x, and 6.
There are 3 variable terms, x^2, 5y, 3x. They have coefficients of 1, 5, and 3 respectively. The coefficient of x^2 is 1, since x^2 = 1 x^2. The term 5y represent 5 y's or 5 * y.
There is one constant term, 6.
The expression shows a sum of all four terms.

Examples:

- 7 more than 3 times a number (Solution: $3x+7$)
- 3 times the sum of a number and 5 (Solution: $3(x+5)$
- 7 less than the product of 2 and a number (Solution: $2x-7$)
- Twice the difference between a number and 5 (Solution: $2(z-5)$)
- Evaluate *5(n* + 3) – 7*n*, when $n = \frac{1}{2}$.
- The expression c + 0.07c can be used to find the total cost of an item with 7% sales tax, where *c* is the pre-tax cost of the item. Use the expression to find the total cost of an item that cost $25.

The perimeter of a parallelogram is found using the formula *p* = 2*l* + 2*w.* What is the perimeter of a rectangular picture frame with dimensions of 8.5 inches by 11 inches.

Instructional Strategies

See EE.1

Common Misconceptions:

See EE.1

Arizona, Ohio & NC DOE

Domain: **Expressions and Equations (EE)**

Cluster: Apply and extend previous understanding of arithmetic to algebraic expressions.

Standard: **6.EE.3** Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.

Standards for Mathematical Practice (MP):

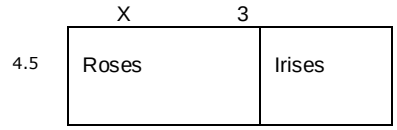
MP 2 Reason abstractly and quantitatively.
MP 3 Construct viable arguments and critique the reasoning of others.
MP 4 Model with mathematics.
MP 6 Attend to precision.
MP 7 Look for and make use of structure.

Connections:

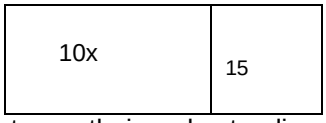
See EE.1

Explanations and Examples:

6.EE.3 Students use the distributive property to write equivalent expressions. For example, area models from elementary can be used to illustrate the distributive property with variables. Given that the width is 4.5 units and the length can be represented by x + 2, the area of the flowers below can be expressed as 4.5(x + 3) or 4.5x + 13.5.

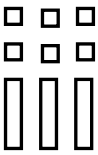


When given an expression representing area, students need to find the factors. For example, the expression 10x + 15 can represent the area of the figure below. Students find the greatest common factor (5) to represent the width and then use the distributive property to find the length (2x + 3). The factors (dimensions) of this figure would be 5(2x + 3).



Students use their understanding of multiplication to interpret *3 (2 + x). For example, 3 groups of (2 + x).* They use a model to represent x, and make an array to show the meaning of *3(2 + x).* They can explain why it makes sense that *3(2 + x)* is equal to *6 + 3x.*

An array with 3 columns and x + 2 in each column:



Students interpret y as referring to one y. Thus, they can reason that one y plus one y plus one y ***must be*** *3y.* They also the distributive property, the multiplicative identity property of 1, and the commutative property for multiplication to prove that $y + y + y = 3y$:

$y + y + y$ = y x 1 + y x 1 + y x 1 = y x (1 + 1 + 1) = y x 3 = 3y

Instructional Strategies

See EE.1

Common Misconceptions:

See EE.1

Arizona, Ohio & NC DOE

Domain: **Expressions and Equations (EE)**

Cluster: Apply and extend previous understanding of arithmetic to algebraic expressions.

Standard: **6.EE.4** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

Standards for Mathematical Practice (MP):

MP 2 Reason abstractly and quantitatively.
MP 3 Construct viable arguments and critique the reasoning of others.
MP 4 Model with mathematics.
MP 6 Attend to precision.
MP 7 Look for and make use of structure.

Connections:

See EE.1

Explanations and Examples:

6.EE.4 Students demonstrate an understanding of like terms as quantities being added or subtracted with the same variables and exponents. For example, 3x + 4x are like terms and can be combined as 7x; however, 3x + 4x2 are not.

This concept can be illustrated by substituting in a value for *x*. For example, 9*x* – 3*x* = 6*x* not 6. Choosing a value for *x*, such as 2, can prove non-equivalence.

9(2) – 3(2) = 6(2)	however	9(2) – 3(2) = 6
18 – 6 = 12		18– 6 = 6
12 = 12		12 ≠ 6

Students connect their experiences with finding and identifying equivalent forms of whole numbers and can write expressions in various forms. Students generate equivalent expressions using the associative, commutative, and distributive properties. They can prove that the expressions are equivalent by simplifying each expression into the same form.

Example:

- Are the expressions equivalent? How do you know?

4m + 8 *4(m+2)* *3m + 8 + m* *2 + 2m + m + 6 + m*

Solution:

Expression	Simplifying the Expression	Explanation
4m + 8	*4m + 8*	Already in simplest form
4(m+2)	*4(m+2)* *4m + 8*	*Distributive property*
3m + 8 + m	*3m + 8 + m* *3m + m + 8* *(3m + m) + 8* *4m + 8*	*Combined like terms*
2 + 2m + m + 6 + m	*2 + 2m + m + 6 + m* *2 + 6 + 2m + m + m* *(2 + 6) + (2m + m + m)* *8 + 4m* *4m + 8*	*Combined like terms*

Instructional Strategies and Common Misconceptions:

See EE.1

6th Grade Mathematics Expressions and Equations (EE)				
Common Core State Standards		**Essence**	**Extended Common Core**	
Apply and extend previous understandings of arithmetic to algebraic expressions.		**Addition and subtraction of algebraic expressions**	**Apply and extend previous understandings of arithmetic to algebraic expressions**	
Cluster	1. Write and evaluate numerical expressions involving whole number exponents. 2. Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as 5 – y. b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms. c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas V = s3 and A = 6 s2 to find the volume and surface area of a cube with sides of length s = 1/2. 3. Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y. 4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.		**Cluster**	1. Write, read, and evaluate addition and subtraction expressions in which letters stand for numbers; i.e., 2 numbers with one number being represented by one letter (fixed variable 7+X=9 where x can only be one number).

		Extended 6.EE.1-4			

Domain: Expressions and Equations (EE)

Cluster: Reason about and solve one-variable equations and inequalities.

Standard: **6.EE.5** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Standards for Mathematical Practice (MP):

MP 2 Make sense of problems and persevere in solving them.
MP 2 Reason abstractly and quantitatively.
MP 4 Model with mathematics.
MP 7 Look for and make use of structure.

Connections:

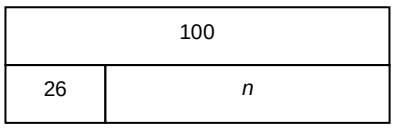
This cluster is connected to the Grade 6 Critical Area of Focus #3, **Writing, interpreting and using expressions, and equations.**

Explanations and Examples:

Beginning experiences in solving equations should require students to understand the meaning of the equation as well as the question being asked. Solving equations using reasoning and prior knowledge should be required of students to allow them to develop effective strategies such as using reasoning, fact families, and inverse operations. Students may use balance models in representing and solving equations and inequalities.

Consider the following situation: Joey had 26 papers in his desk. His teacher gave him some more and now he has 100. How many papers did his teacher give him?
This situation can be represented by the equation $26 + n = 100$ where n is the number of papers the teacher gives to Joey. This equation can be stated as "some number was added to 26 and the result was 100". Students ask themselves "What number was added to 26 to get 100?" to help them determine the value of the variable that makes the equation true. Students could use several different strategies to find a solution to the problem.

- Reasoning: 26 + 70 is 96. 96 + 4 is 100, so the number added to 26 to get 100 is 74.
- Use knowledge of fact families to write related equations:
  $n + 26 = 100$, $100 - n = 26$, $100 - 26 = n$. Select the equation that helps you find n easily.
- Use knowledge of inverse operations: Since subtraction "undoes" addition then subtract 26 from 100 to get the numerical value of n
- Scale model: There are 26 blocks on the left side of the scale and 100 blocks on the right side of the scale. All the blocks are the same size. 74 blocks need to be added to the left side of the scale to make the scale balance.
- Bar Model: Each bar represents one of the values. Students use this visual representation to demonstrate that 26 and the unknown value together make 100.



Examples continued next page

Examples:

- The equation $0.44s = 11$ where *s* represents the number of stamps in a booklet. The booklet of stamps costs 11 dollars and each stamp costs 44 cents. How many stamps are in the booklet? Explain the strategies you used to determine your answer. Show that your solution is correct using substitution.
- Twelve is less than 3 times another number can be shown by the inequality $12 < 3n$. What numbers could possibly make this a true statement?

Students identify values from a specified set that will make an equation true. For example, given the expression x + 2 1/2 which of the following value(s) for x would make x + 2 ½ = 6.

{0, 3 ½, 4}

By using substitution, students identify 3 ½ as the value that will make both sides of the equation equal.

The solving of inequalities is limited to choosing values from a specified set that would make the inequality true. For example, find the value(s) of x that will make $x + 3.5 \geq 9$.

{5, 5.5, 6, 15/2, 10.2,15}

Using substitution, students identify 5.5, 6, 15/2,10.2, and 15 as the values that make the inequality true. NOTE: If the inequality had been $x + 3.5 > 9$, then 5.5 would not work since 9 is not greater than 9.

This standard is foundational to **6.EE.7** and **6.EE.8**

<u>Instructional Strategies</u> EE. 5-8

In order for students to **understanding** equations: The skill of solving an equation must be developed *conceptually* before it is developed *procedurally*. This means that students should be thinking about what numbers could possibly be a solution to the equation before solving the equation. For example, in the equation $x + 21 = 32$ students know that 21 + 9 = 30 therefore the solution must be 2 more than 9 or 11, so $x = 11$.

Provide multiple situations in which students must determine if a single value is required as a solution, or if the situation allows for multiple solutions. This creates the need for both types of equations (single solution for the situation) and inequalities (multiple solutions for the situation). Solutions to equations should not require using the rules for operations with negative numbers since the conceptual understanding of negatives and positives is being introduced in Grade 6. When working with inequalities, provide situations in which the solution is not limited to the set of positive whole numbers but includes rational numbers. This is a good way to practice fractional numbers and introduce negative numbers. Students need to be aware that numbers less than zero could be part of a solution set for a situation. As an extension to this concept, certain situations may require a solution between two numbers. For example, a problem situation may have a solution that requires more than 10 but not greater than 25. Therefore, the exploration with students as to what this would look like both on a number line and symbolically is a reasonable extension.

The process of translating between mathematical phrases and symbolic notation will also assist students in the writing of equations/inequalities for a situation. This process should go both ways; Students should be able to write a mathematical phrase for an equation. Additionally, the writing of equations from a situation or story does not come naturally for many students. A strategy for assisting with this is to give students an equation and ask them to come up with the situation/story that the equation could be referencing.

Domain: **Expressions and Equations (EE)**

Cluster: Reason about and solve one-variable equations and inequalities.

Standard: **6.EE.6**
Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

Standards for Mathematical Practice (MP):

MP 2 Reason abstractly and quantitatively.
MP 3 Construct viable arguments and critique the reasoning of others.
MP 4 Model with mathematics.
MP 6 Attend to precision.
MP 7 Look for and make use of structure.

Connections:

See EE.5

Explanations and Examples:

6.EE.6. Students write expressions to represent various real-world situations. For example, the expression $a + 3$ could represent Susan's age in three years, when a represents her present age. The expression $2n$ represents the number of wheels on any number of bicycles. Other contexts could include age (Johnny's age in 3 years if a represents his current age) and money (value of any number of quarters)

Given a contextual situation, students define variables and write an expression to represent the situation. For example, the skating rink charges $100 to reserve the place and then $5 per person. Write an expression to represent the cost for any number of people.

N = the number of people
100 + 5n

Connecting writing expressions with story problems and/or drawing pictures will give students a context for this work. It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number.

Examples:

- Maria has three more than twice as many crayons as Elizabeth. Write an algebraic expression to represent the number of crayons that Maria has.
  (Solution: $2c + 3$ where c represents the number of crayons that Elizabeth has.)
- An amusement park charges $28 to enter and $0.35 per ticket. Write an algebraic expression to represent the total amount spent.
  (Solution: $28 + 0.35t$ where t represents the number of tickets purchased)
- Andrew has a summer job doing yard work. He is paid $15 per hour and a $20 bonus when he completes the yard. He was paid $85 for completing one yard. Write an equation to represent the amount of money he earned.
  (Solution: $15h + 20 = 85$ where h is the number of hours worked)
- Describe a problem situation that can be solved using the equation
  $2c + 3 = 15$; where c represents the cost of an item
- Bill earned $5.00 mowing the lawn on Saturday. He earned more money on Sunday. Write an expression that shows the amount of money Bill has earned. (Solution: $5.00 + n)

Instructional Strategies

See EE.5

Common Misconceptions:

See .EE.5

Domain: **Expressions and Equations (EE)**

Cluster: Reason about and solve one-variable equations and inequalities.

Standard: **6.EE.7**
Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

Standards for Mathematical Practice (MP):
MP 1 Make sense of problems and persevere in solving them.
MP 2 Reason abstractly and quantitatively.
MP 3 Construct viable arguments and critique the reasoning of others.
MP 4 Model with mathematics.
MP 7 Look for and make use of structure.

Connections:
See EE.5

Explanations and Examples:
6.EE.7 Students have used algebraic expressions to generate answers given values for the variable. This understanding is now expanded to equations where the value of the variable is unknown but the outcome is known. For example, in the expression, $x + 4$, any value can be substituted for the x to generate a numerical answer; however, in the equation $x + 4 = 6$, there is only one value that can be used to get a 6. Problems should be in context when possible and use only one variable.
Students write equations from real-world problems and then use inverse operations to solve one-step equations. Equations may include fractions and decimals with non-negative solutions.

Students create and solve equations that are based on real world situations. It may be beneficial for students to draw pictures that illustrate the equation in problem situations. Solving equations using reasoning and prior knowledge should be required of students to allow them to develop effective strategies.

Example:

- Meagan spent $56.58 on three pairs of jeans. If each pair of jeans costs the same amount, write an algebraic equation that represents this situation and solve to determine how much one pair of jeans cost.

$56.58		
J	J	J

Sample Solution: Students might say: "I created the bar model to show the cost of the three pairs of jeans. Each bar labeled J is the same size because each pair of jeans costs the same amount of money. The bar model represents the equation $3J = \$56.58$. To solve the problem, I need to divide the total cost of 56.58 between the three pairs of jeans. I know that it will be more than $10 each because 10 x 3 is only 30 but less than $20 each because 20 x 3 is 60. If I start with $15 each, I am up to $45. I have $11.58 left. I then give each pair of jeans $3. That's $9 more dollars. I only have $2.58 left. I continue until all the money is divided. I ended up giving each pair of jeans another $0.86. Each pair of jeans costs $18.86 (15+3+0.86). I double check that the jeans cost $18.86 each because $18.86 x 3 is $56.58."

Continued on next page

- Julio gets paid $20 for babysitting. He spends $1.99 on a package of trading cards and $6.50 on lunch. Write and solve an equation to show how much money Julio has left. (Solution: 20 = 1.99 + 6.50 + *x*, *x* = $11.51)

20		
1.99	6.50	money left over (m)

Instructional Strategies:

See EE.5

Common Misconceptions:

See EE.5

Domain: **Expressions and Equations (EE)**

Cluster: Reason about and solve one-variable equations and inequalities.

Standard: **6.EE.8**
Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

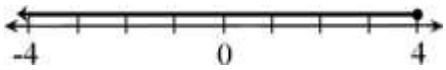
Standards for Mathematical Practice (MP):
MP 1 Make sense of problems and persevere in solving them.
MP 2 Reason abstractly and quantitatively.
MP 3 Construct viable arguments and critique the reasoning of others.
MP 4 Model with mathematics.
MP 7 Look for and make use of structure.

Connections:
See EE.5

Explanations and Examples:
6.EE.8 Many real-world situations are represented by inequalities. Students write an inequality and represent solutions on a number line for various contextual situations.
For example, the class must raise at least $80 to go on the field trip. If *m* represents money, then the inequality $m \geq$ to $80. Students recognize that possible values can include too many decimal values to name. Therefore, the values are represented on a number line by shading.

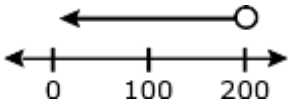
A number line diagram is drawn with an open circle when an inequality contains a < or > symbol to show solutions that are less than or greater than the number but not equal to the number. The circle is shaded, as in the example above, when the number is to be included. Students recognize that possible values can include fractions and decimals, which are represented on the number line by shading. Shading is extended through the arrow on a number line to show that an inequality has an infinite number of solutions.

Examples:

- Graph $x \leq 4$. 

- Jonas spent more than $50 at an amusement park. Write an inequality to represent the amount of money Jonas spent. What are some possible amounts of money Jonas could have spent? Represent the situation on a number line.

- Less than $200.00 was spent by the Flores family on groceries last month. Write an inequality to represent this amount and graph this inequality on a number line.

Solution*:* $200 > x$ 

Instructional Strategies & Common Misconceptions:
See EE.5

Domain: **Expressions and Equations (EE)**

Cluster: Represent and analyze quantitative relationships between dependent and independent variables.

Standard: **6.EE.9**
Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

Standards for Mathematical Practice (MP):

MP 1 Make sense of problems and persevere in solving them.
MP 2 Reason abstractly and quantitatively.
MP 3 Construct viable arguments and critique the reasoning of others.
MP 4 Model with mathematics.
MP 7 Look for and make use of structure.
MP 8 Look for and express regularity in repeated reasoning.

Connections:

This cluster is connected to the Grade 6 Critical Area of Focus #3, **Writing, interpreting and using expressions, and equations**.

This cluster, Expressions and Equations, is closely tied to Ratios and Proportional Relationships, allowing the ideas in each to be connected and taught together.

Explanations and Examples:

6.EE.9 The purpose of this standard is for students to understand the **relationship** between two variables, which begins with the distinction between dependent and independent variables. The independent variable is the variable that can be changed; the dependent variable is the variable that is affected by the change in the independent variable. Students recognize that the independent variable is graphed on the x-axis; the dependent variable is graphed on the y-axis. Students recognize that not all data should be graphed with a line. Data that is discrete would be graphed with coordinates only. Discrete data is data that would not be represented with fractional parts such as people, tents, records, etc. For example, a graph illustrating the cost per person would be graphed with points since part of a person would not be considered. A line is drawn when both variables could be represented with fractional parts.
Students are expected to recognize and explain the impact on the dependent variable when the independent variable changes (As the *x* variable increases, how does the *y* variable change?) Relationships should be proportional with the line passing through the origin. Additionally, students should be able to write an equation from a word problem and understand how the coefficient of the dependent variable is related to the graph and /or a table of values.

Students can use many forms to represent relationships between quantities. Multiple representations include describing the relationship using language, a table, an equation, or a graph. Translating between multiple representations helps students understand that each form represents the same relationship and provides a different perspective on the function.

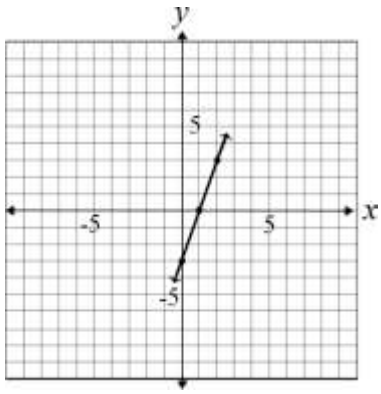
Examples on next page

Examples:

- What is the relationship between the two variables? Write an expression that illustrates the relationship.

x	1	2	3	4
y	2.5	5	7.5	10

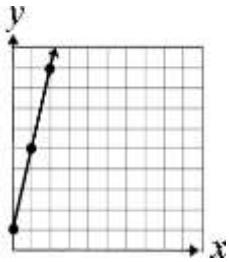
- Use the graph below to describe the change in y as x increases by 1.



- Susan started with $1 in her savings. She plans to add $4 per week to her savings. Use an equation, table and graph to demonstrate the relationship between the number of weeks that pass and the amount in her savings account.
  - Language: Susan has $1 in her savings account. She is going to save $4 each week.
  - Equation: $y = 4x + 1$
  - Table:

x	***y***
0	1
1	5
2	9

  - Graph:



Instructional Strategies next page

Instructional Strategies:

The goal is to help students connect the pieces together. This can be done by having students use multiple representations for the mathematical relationship. Students need to be able to translate freely among the story, words (mathematical phrases), models, tables, graphs and equations. They also need to be able to start with any of the representations and develop the others.

Provide multiple situations for the student to analyze and determine what unknown is dependent on the other components. For example, how far I travel is dependent on the time and rate that I am traveling.

Throughout the expressions and equations domain in Grade 6, students need to have an understanding of how the expressions or equations relate to situations presented, as well as the process of solving them.

The use of technology, including computer apps, CBLs, and other hand-held technology allows the collection of real-time data or the use of actual data to create tables and charts. It is valuable for students to realize that although real-world data often is not linear, a line sometimes can model the data .

Common Misconceptions:

Students may misunderstand what the graph represents in context. For example, that moving up or down on a graph does not necessarily mean that a person is moving up or down.

Domain: **Geometry (G)**

Cluster: Solve real-world and mathematical problems involving area, surface area, and volume.

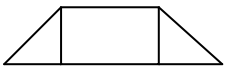
Standard: **6.G.1**
Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

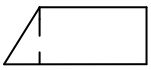
Standards for Mathematical Practice (MP):
MP 1 Make sense of problems and persevere in solving them.
MP 2 Reason abstractly and quantitatively.
MP 3 Construct viable arguments and critique the reasoning of others.
MP 4 Model with mathematics.
MP 5 Use appropriate tools strategically.
MP 6 Attend to precision.
MP 7 Look for and make use of structure.
MP 8 Look for and express regularity in repeated reasoning.

Connections:
This cluster does not directly relate to one of the Grade 6 Critical Areas of Focus. This cluster focuses on additional content for development. Students in Grade 6 build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume.
An understanding of how to find the area, surface area and volume of an object is developed in Grade 5 and should be built upon in Grade 6 to facilitate understanding of the formulas found in Measurement and Data and when to use the appropriate formula.

The use of floor plans and composite shapes on dot paper is a foundational concept for scale drawing and determining the actual area based on a scale drawing Grade 7 (Geometry and Ratio and Proportional Relationships).

Explanations and Examples:
6.G.1 Students continue to understand that area is the number of squares needed to cover a plane figure. Finding the area of triangles is introduced in relationship to the area of rectangles – a rectangle can be decomposed into two congruent triangles. Therefore, the area of the triangle is ½ the area of the rectangle. The area of a rectangle can be found by multiplying base x height; therefore, the area of the triangle is ½ bh or (b x h)/2. Students decompose shapes into rectangles and triangles to determine the area. For example, a trapezoid can be decomposed into triangles and rectangles (see figure below). Using the trapezoid's dimensions, the area of the individual triangle(s) and rectangle can be found and then added together.


Isosceles trapezoid

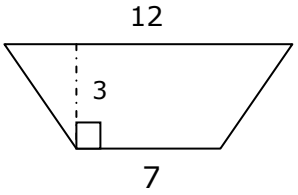

Right trapezoid

Students should know the formulas for rectangles and triangles. "Knowing the formula" does not mean memorization of the formula. To "know" means to have an understanding of ***why*** the formula works and how the formula relates to the measure (area) and the figure. This understanding should be for *all* students.

Special quadrilaterals include rectangles, squares, parallelograms, trapezoids, rhombi, and kites. Students can use tools such as the Isometric Drawing Tool on NCTM's Illuminations site to shift, rotate, color, decompose and view figures in 2D or 3D
Examples next page

Examples:

- Find the area of a triangle with a base length of three units and a height of four units.
- Find the area of the trapezoid shown below using the formulas for rectangles and triangles.



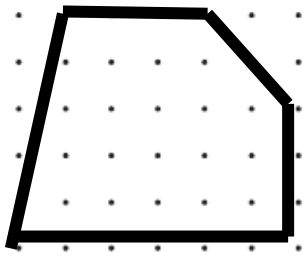
- A rectangle measures 3 inches by 4 inches. If the lengths of each side double, what is the effect on the area?
- The area of the rectangular school garden is 24 square units. The length of the garden is 8 units. What is the length of the fence needed to enclose the entire garden?
- The sixth grade class at Hernandez School is building a giant wooden H for their school. The H will be 10 feet tall and 10 feet wide and the thickness of the block letter will be 2.5 feet.
  - How large will the H be if measured in square feet?
  - The truck that will be used to bring the wood from the lumber yard to the school can only hold a piece of wood that is 60 inches by 60 inches. What pieces of wood (how many pieces and what dimensions) are needed to complete the project?

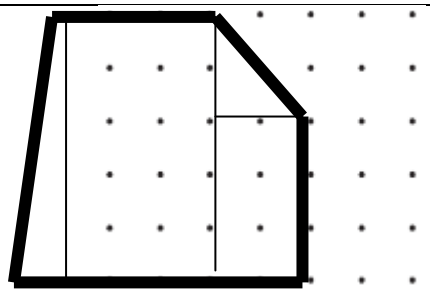
Instructional Strategies: G.1-4

It is very important for students to continue to physically manipulate materials and make connections to the symbolic and more abstract aspects of geometry. Exploring possible nets should be done by taking apart (unfolding) three-dimensional objects. This process is also foundational for the study of surface area of prisms. Building upon the understanding that a net is the two-dimensional representation of the object, students can apply the concept of area to find surface area. The surface area of a prism is the sum of the areas for each face.

Multiple strategies can be used to aid in the skill of determining the area of simple two-dimensional composite shapes. A beginning strategy should be using rectangles and triangles, building upon shapes for which they can already determine area to create composite shapes. This process will reinforce the concept that composite shapes are created by joining together other shapes, and that the total area of the two-dimensional composite shape is the sum of the areas of all the parts.

A follow-up strategy is to place a composite shape on grid or dot paper. This aids in the decomposition of a shape into its foundational parts. Once the composite shape is decomposed, the area of each part can be determined and the sum of the area of each part is the total area.





Fill prisms with cubes of different edge lengths (including fractional lengths) to explore the relationship between the length of the repeated measure and the number of units needed. An essential understanding to this strategy is the volume of a rectangular prism does not change when the units used to measure the volume changes. Since focus in Grade 6 is to use fractional lengths to measure, if the same object is measured using one centimeter cubes and then measured using half centimeter cubes, the volume will appear to be eight times greater with the smaller unit. However, students need to understand that the value or the number of cubes is greater but the volume is the same.

Online dot paper: http://illuminations.nctm.org/lessons/DotPaper.pdf#search=%22dot paper%22

Common Misconceptions:

Students may believe that the orientation of a figure changes the figure. In Grade 6, some students still struggle with recognizing common figures in different orientations. For example, a square rotated 45° is no longer seen as a square and instead is called a diamond.

This impacts students' ability to decompose composite figures and to appropriately apply formulas for area. Providing multiple orientations of objects within classroom examples and work is essential for students to overcome this misconception.

Domain: **Geometry (G)**

Cluster: Solve real-world and mathematical problems involving area, surface area, and volume.

Standard: **6.G.2**

Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

Standards for Mathematical Practice (MP):

MP 1 Make sense of problems and persevere in solving them.
MP 2 Reason abstractly and quantitatively.
MP 3 Construct viable arguments and critique the reasoning of others.
MP 4 Model with mathematics.
MP 5 Use appropriate tools strategically.
MP 6 Attend to precision.
MP 7 Look for and make use of structure.
MP 8 Look for and express regularity in repeated reasoning.

Connections:

See G.1

Explanations and Examples:

6.G.2 Previously students calculated the volume of right rectangular prisms (boxes) using whole number edges. The unit cube was 1 x 1 x 1. In 6th grade the unit cube will have fractional edge lengths. (i.e. ½ • ½ • ½) Students find the volume of the right rectangular prism with these unit cubes. For example, the right rectangular prism below has edges of 1¼", 1" and 1½". The volume can be found by recognizing that the unit cube would be ¼" on all edges, changing the dimensions to 5/4", 4/4" and 6/4". The volume is the number of unit cubes making up the prism (5 x 4 x 6), which is 120 unit cubes each with a volume of 1/64 (¼" x ¼" x ¼"). This can also be expressed as 5/4 x 6/4 x 4/4 or 120/64.

"Know the formula" does not mean memorization of the formula. To "know" means to have an understanding of ***why*** the formula works and how the formula relates to the measure (volume) and the figure. This understanding should be for *ALL* students.

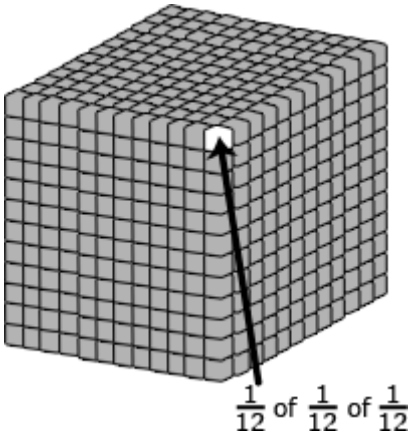
Students need multiple opportunities to measure volume by filling rectangular prisms with blocks and looking at the relationship between the total volume and the area of the base. Through these experiences, students derive the volume formula (volume equals the area of the base times the height). Students can explore the connection between filling a box with unit cubes and the volume formula using interactive applets such as the Cubes Tool on NCTM's Illuminations (See http://illuminations.nctm.org/ActivityDetail.aspx?ID=6).

In addition to filling boxes, students can draw diagrams to represent fractional side lengths, connecting with multiplication of fractions. This process is similar to composing and decomposing two dimensional shapes.

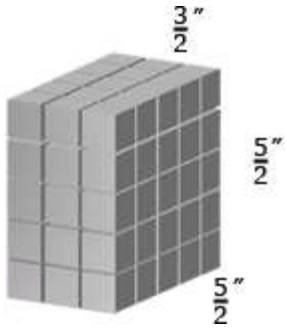
Examples next page

Examples:

- The model shows a cubic foot filled with cubic inches. The cubic inches can also be labeled as a fractional cubic unit with dimensions of $\frac{1}{12}$ ft^3.



- The models show a rectangular prism with dimensions 3/2 inches, 5/2 inches, and 5/2 inches. Each of the cubic units in the model is $\frac{1}{2}$ in^3**.** Students work with the model to illustrate 3/2 x 5/2 x 5/2 = (3 x 5 x 5) x 1/8. Students reason that a small cube has volume 1/8 because 8 of them fit in a unit cube.
-



Instructional Strategies:
See G.1

Common Misconceptions:
See G.1

Arizona, Ohio & NC DOE

Domain: **Geometry (G)**

Cluster**: Solve real-world and mathematical problems involving area, surface**

Standard: **6.G.3** Draw polygons in the coordinate plane given coordinates for the vertices; use to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Standards for Mathematical Practice (MP):

MP 1 Make sense of problems and persevere in solving them.
MP 2 Reason abstractly and quantitatively.
MP 3 Construct viable arguments and critique the reasoning of others.
MP 4 Model with mathematics.
MP 5 Use appropriate tools strategically.
MP 6 Attend to precision.
MP 7 Look for and make use of structure.
MP 8 Look for and express regularity in repeated reasoning.

Connections:

See G.1

Explanations and Examples:

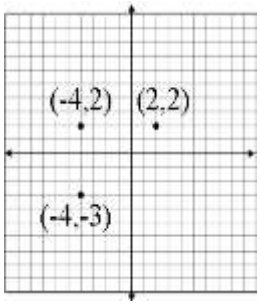
Students are given the coordinates of polygons to draw in the coordinate plane. If both *x*-coordinates are the same (2, -1) and (2, 4), then students recognize that a vertical line has been created and the distance between these coordinates is the distance between -1 and 4, or 5. If both the *y*-coordinates are the same (-5, 4) and (2, 4), then students recognize that a horizontal line has been created and the distance between these coordinates is the distance between -5 and 2, or 7. Using this understanding, student solve real-world and mathematical problems, including finding the area and perimeter of geometric figures drawn on a coordinate plane.

This standard can be taught in conjunction with **6.G.1** to help students develop the formula for the triangle by using the squares of the coordinate grid. Given a triangle, students can make the corresponding square or rectangle and realize the triangle is . . . .

Students progress from counting the squares to making a rectangle and recognizing the triangle as . to the development of the formula for the area of a triangle.

Examples next page

Example 1:
If the points on the coordinate plane below are the three vertices of a rectangle, what are the coordinates of the fourth vertex? How do you know? What are the length and width of the rectangle? Find the area and the perimeter of the rectangle.



Solution:
To determine the distance along the x-axis between the point (-4, 2) and (2, 2) a student must recognize that -4 is |-4| or 4 units to the left of 0 and 2 is |2| or 2 units to the right of zero, so the two points are total of 6 units apart along the x-axis. Students should represent this on the coordinate grid and numerically with an absolute value expression, |-4| + |2| . The length is 6 and the width is 5.

The fourth vertex would be (2, -3). The area would be 5 x 6 or 30 units2.
The perimeter would be 5 + 5 + 6 + 6 or 22 units.

Example 2:
On a map, the library is located at (-2, 2), the city hall building is located at (0,2), and the high school is located at (0,0). Represent the locations as points on a coordinate grid with a unit of 1 mile.
1. What is the distance from the library to the city hall building? The distance from the city hall building to the high school? How do you know?
2. What shape does connecting the three locations form? The city council is planning to place a city park in this area. How large is the area of the planned park?

Solution:
1. The distance from the library to city hall is 2 miles. The coordinates of these buildings have the same y-coordinate. The distance between the x-coordinates is 2 (from -2 to 0).
2. The three locations form a right triangle. The area is 2 mi^2.

Instructional Strategies:
See G.1

Common Misconceptions:
See G.1

Domain: **Geometry (G)**

Cluster**: Solve real-world and mathematical problems involving area, surface area, and volume.**

Standard: **6.G.4** Represent three-dime surface area of three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Standards for Mathematical Practice (MP):

MP 1 Make sense of problems and persevere in solving them.
MP 2 Reason abstractly and quantitatively.
MP 3 Construct viable arguments and critique the reasoning of others.
MP 4 Model with mathematics.
MP 5 Use appropriate tools strategically.
MP 6 Attend to precision.
MP 7 Look for and make use of structure.

Connections:

See G.1

Explanations and Examples:

A net is a two-dimensional representation of a three-dimensional figure. Students represent three-dimensional figures whose nets are composed of rectangles and triangles. Students recognize that parallel lines on a net are congruent. Using the dimensions of the individual faces, students calculate the area of each rectangle and/or triangle and add these sums together to find the surface area of the figure.

It's very important for students to physically manipulate materials and make connections to the symbolic and more abstract aspects of geometry. See Instructional Strategies G.1 for more information on exploring nets and making connections)

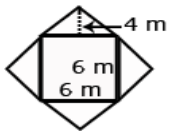
Students construct models and nets of three dimensional figures, and describe them by the number of edges, vertices, and faces. Solids include rectangular and triangular prisms. Students are expected to use the net to calculate the surface area.

Students can create nets of 3D figures with specified dimensions using the Dynamic Paper Tool on NCTM's Illuminations (http://illuminations.nctm.org/ActivityDetail.aspx?ID=205).

Students also describe the types of faces needed to create a three-dimensional figure. Students make and test conjectures by determining what is needed to create a specific three-dimensional figure.

Examples:

- Describe the shapes of the faces needed to construct a rectangular pyramid. Cut out the shapes and create a model. Did your faces work? Why or why not?
- Create the net for a given prism or pyramid, and then use the net to calculate the surface area.



Instructional Strategies:

See G.1

Common Misconceptions:

See G.1

Sixth Grade Mathematics Geometry (G)				
Common Core State Standards		**Essence**	**Extended Common Core**	
Solve real-world and mathematical problems involving area, surface area, and volume.		**Area and perimeter of rectangles.**	**Solve real-world and mathematical problems involving area and perimeter.**	
Cluster	1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. 2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. 3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. 4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.		**Cluster**	1. Determine the perimeter of rectangular figures. 2. Partition rectangular figures into rows and columns of same-size squares without gaps and overlaps and count them to find the area.

EXTENDED 6. G. 1-4					

Domain: Statistics and Probability **(SP)**

Cluster: *Develop understanding of statistical variability.*

Standard: 6.S P.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, "How old am I?" is **not** a statistical question, but "How old are the students in my school?" **is** a statistical question because one anticipates variability in students' ages.

Standards for Mathematical Practice (MP):
MP 1 Make sense of problems and persevere in solving them.
MP 3 Construct viable arguments and critique the reasoning of others.
MP 6 Attend to precision.

Connections:
This cluster is connected to the Grade 6 Critical Area of Focus #4, **Developing Understanding of statistical thinking**.

Measures of center and measures of variability are used to draw informal comparative inferences about two populations in 7.SP.4.

Explanations and Examples:
6.SP.1 Statistics are numerical data relating to a group of individuals; statistics is also the name for the science of collecting, analyzing and interpreting such data. A statistical question anticipates an answer that varies from one individual to the next and is written to account for the variability in the data. Data are the numbers produced in response to a statistical question. Data are frequently collected from surveys or other sources (i.e. documents).

Students differentiate between statistical questions and those that are not. A statistical question is one that collects information that addresses differences in a population. The question is framed so that the responses will allow for the differences. For example, the question, "How tall am I?" is not a statistical question because there is only one response; however, the question, "How tall are the students in my class?" is a statistical question since the responses anticipates variability by providing a variety of possible anticipated responses that have numerical answers. Questions can result in a narrow or wide range of numerical values.

Students might want to know about the fitness of the students at their school. Specifically, they want to know about the exercise habits of the students. So rather than asking "Do you exercise?" they should ask about the amount of exercise the students at their school get per week. A statistical question for this study could be: "How many hours per week on average do students at Jefferson Middle School exercise?"

To collect this information, students might design a survey question that anticipates variability by providing a variety of possible anticipated responses that have numerical answers, such as: 3 hours per week, 4 hours per week, and so on. Be sure that students ask questions that have specific numerical answers.

Instructional Strategies:
Grade 6 is the introduction to the formal study of statistics for students. Students need multiple opportunities to look at data to determine and word statistical questions. Data should be analyzed from many sources, such as organized lists, box-plots, bar graphs and stem-and-leaf plots. This will help students begin to understand that responses to a statistical question will vary, and that this variability is described in terms of spread and overall shape. At the same time, students should begin to relate their informal knowledge of mean, mode and median to understand that data can also be described by single numbers. The single value for each of the measures of center (mean, median or mode) and measures of spread (range, interquartile range, mean absolute deviation) is used to summarize the data. Given measures of center for a set of data, students should use the value to describe the data in words. The important purpose of the number is not the value itself, but the interpretation it provides for the variation of the data. Interpreting different measures of center for the same data develops the understanding of how each measure sheds a different light on the data. The use of a similarity and difference matrix to compare mean, median, mode and range may facilitate understanding the distinctions of purpose between and among the measures of center and spread.

Include activities that require students to match graphs and explanations, or measures of center and explanations prior to interpreting graphs based upon the computation measures of center or spread. The determination of the measures of center and the process for developing graphical representation is the focus of the cluster "Summarize and describe distributions" in the Statistics and Probability domain for Grade 6. Classroom instruction should integrate the two clusters.

Instructional Resources/Tools
Newspaper and magazine graphs for analysis of the spread, shape and variation of data
From the National Council of Teachers of Mathematics, Illuminations: Numerical and Categorical Data. In this unit of three lessons, students formulate and refine questions, and collect, display and analyze data. (ORC # 391, 392, 393)

Data Analysis and Probability Virtual Manipulatives Grades 6-8
#5048 Students can use the appropriate applet from this page of virtual manipulatives to create graphical displays of the data set. This provides an important visual display of the data without requiring students to spend time hand-drawing the display. Classroom time can then be spent discussing the patterns and variability of the data.

Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report, American Statistics Association

Common Misconceptions:
Students may believe all graphical displays are symmetrical. Exposing students to graphs of various shapes will show this to be false.

The value of a measure of center describes the data, rather than a value used to interpret and describe the data.

	6.SP.1				

Domain: Statistics and Probability **(SP)**

Cluster: Develop understanding of statistical variability.

Standard: 6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its **center**, **spread**, and overall **shape**.

Standards for Mathematical Practice (MP):
MP 2 Reason abstractly and quantitatively.
MP 4 Model with mathematics.
MP 5 Use appropriate tools strategically.
MP 6 Attend to precision.
MP 7 Look for and make use of structure.

Connections:
See 6.SP.1

Explanations and Examples:
6.SP.2 The distribution is the arrangement of the values of a data set. Distribution can be described using center (median or mean), and spread. Data collected can be represented on graphs, which will show the shape of the distribution of the data. Students examine the distribution of a data set and discuss the center, spread and overall shape with dot plots, histograms and box plots.

Example 1:
The dot plot shows the writing scores for a group of students on organization. Describe the data.

Solution:
The values range from 0 – 6. There is a peak at 3. The median is 3, which means 50% of the scores are greater than or equal to 3 and 50% are less than or equal to 3. The mean is 3.68. If all students scored the same, the score would be 3.68.

NOTE: Mode as a measure of center and range as a measure of variability are not addressed in the CCSS and as such are not a focus of instruction. These concepts can be introduced during instruction as needed.

Examples continued next page

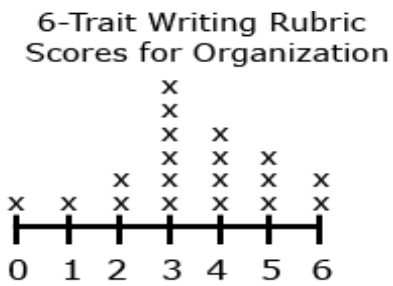
Additional Examples:

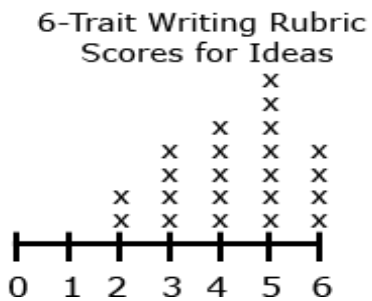
Domain: Statistics and Probability (SP)

Cluster: Develop understanding of statistical variability.

The two dot plots show the 6-trait writing scores for a group of students on two different traits, organization and ideas. The center, spread and overall shape can be used to compare the data sets. Students consider the context in which the data were collected and identify clusters, peaks, gaps, and symmetry.

Showing the two graphs vertically rather than side by side helps students make comparisons. For example, students would be able to see from the display of the two graphs that the ideas scores are generally higher than the organization scores. One observation students might make is that the scores for organization are clustered around a score of 3 whereas the scores for ideas are clustered around a score of 5.





Instructional Strategies & Misconceptions:
See 6.SP.1

Domain: Statistics and Probability (SP)

Cluster: Summarize and describe distributions

Standard: **6.SP.3**
Recognize that a measure of center for a numerical data set summarizes all its values with a single number, while a measure of variation describes how its values vary with a single number.

Standards for Mathematical Practice (MP):
MP 2 Reason abstractly and quantitatively.
MP 4 Model with mathematics.
MP 5 Use appropriate tools strategically.
MP 6 Attend to precision.
MP 7 Look for and make use of structure.

Connections:
See 6.SP.1

Explanations and Examples:
6.SP.3 Data sets contain many numerical values that can be summarized by one number such as a measure of center. The measure of center gives a numerical value to represent the center of the data (ie. midpoint of an ordered list or the balancing point). Another characteristic of a data set is the variability (or spread) of the values.

Measures of variability are used to describe this characteristic.

Example 1:
Consider the data shown in the dot plot of the six trait scores for organization for a group of students.
• How many students are represented in the data set?
• What are the mean and median of the data set? What do these values mean? How do they compare?
• What is the range of the data? What does this value mean?

Solution:
• 19 students are represented in the data set.
• The mean of the data set is 3.5. The median is 3. The mean indicates that is the values were equally distributed, all students would score a 3.5. The median indicates that 50% of the students scored a 3 or higher; 50% of the students scored a 3 or lower.
• The range of the data is 6, indicating that the values vary 6 points between the lowest and highest scores.

Common Misconceptions:
See 6.SP.1

Standard: 6.SP.4
Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

Standards for Mathematical Practice (MP):
MP 2 Reason abstractly and quantitatively.
MP 4 Model with mathematics.
MP 5 Use appropriate tools strategically.
MP 6 Attend to precision.
MP 7 Look for and make use of structure.

Connections:
This cluster is connected to the Grade 6 Critical Area of Focus #4, **Developing Understanding of statistical thinking**.

Measures of center and measures of variability are used to draw informal comparative inferences about two populations in Grade 7 Statistics and Probability.

Explanations and Examples:
6.SP.4 Students display data set using number lines. Dot plots, histograms and box plots are three graphs to be used. A dot plot is a graph that uses a point (dot) for each piece of data. The plot can be used with data sets that include fractions and decimals.

A histogram shows the distribution of continuous data using intervals on the number line. The of each bar represents the number of data values in that interval.

Box plots are another useful way to display data and are plotted horizontally or vertically on a number line These values give a summary of the shape of a distribution of values in a data set by dividing the set into quartiles. The box plot is constructed from the five-number summary (minimum, lower quartile, median, upper quartile, and maximum). Students understand that the size of the box or whiskers represent the middle 50% of the data. Students can readily compare two sets of data if they are displayed with side by side box plots on the same scale. Box plots display the degree of spread of the data and the skewness of the data.

Dot plots are simple plots on a number line where each dot represents a piece of data in the data set. Dot plots are suitable for small to moderate size data sets and are useful for highlighting the distribution of the data including clusters, gaps, and outliers.

In most real data sets, there is a large amount of data and many numbers will be unique. A graph (such as a dot plot) that shows how many ones, how many twos, etc. would not be meaningful; however, a histogram can be used. Students organize the data into convenient ranges and use these intervals to generate a frequency table and histogram. Note that changing the size of the range changes the appearance of the graph and the conclusions you may draw from it.

In order to display numerical data in dot plots, histograms or box plots, students need to make decisions and perform calculations. Students are expected to display data graphically in a format appropriate for that data set as well as reading data from graphs generated by others students or contained in reference materials. Students can use applets to create data displays.

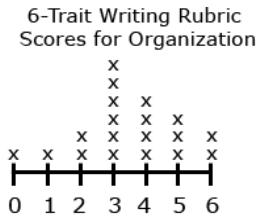
Continued next page
Examples of applets include the Box Plot Tool and Histogram Tool on NCTM's Illuminations.
Box Plot Tool - http://illuminations.nctm.org/ActivityDetail.aspx?ID=77
Histogram Tool -- http://illuminations.nctm.org/ActivityDetail.aspx?ID=78

Examples:
In order to display numerical data in dot plots, histograms or box plots, students need to make decisions and perform calculations. Students are expected to display data graphically in a format appropriate fo that data set as well as reading data from graphs generated by others students or contained in reference materials. Students can use applets to create data displays.

Examples of applets include the Box Plot Tool and Histogram Tool on NCTM's Illuminations.
Box Plot Tool - http://illuminations.nctm.org/ActivityDetail.aspx?ID=77
Histogram Tool -- http://illuminations.nctm.org/ActivityDetail.aspx?ID=78

Examples:

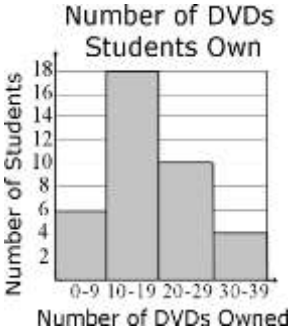
- Nineteen students completed a writing sample that was scored using the six traits rubric. The scores for the trait of organization were 0, 1, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6. Create a data display. What are some observations that can be made from the data display?



- Grade 6 students were collecting data for a math class project. They decided they would survey the other two grade 6 classes to determine how many DVDs each student owns. A total of 48 students were surveyed. The data are shown in the table below in no specific order. Create a data display. What are some observations that can be made from the data display?

11	21	5	12	10	31	19	13	23	33
10	11	25	14	34	15	14	29	8	5
22	26	23	12	27	4	25	15	7	
2	19	12	39	17	16	15	28	16	

A histogram using 5 ranges (0-9, 10-19, …30-39) to organize the data is displayed below.

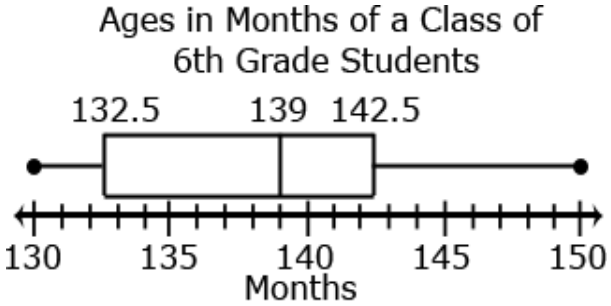


- Ms. Hancock asked each student in her class to write their age in months on a sticky note. The 28 students in the class brought their sticky note to the front of the room and posted them in order

on the white board. The data set is listed below in order from least to greatest. Create a data display. What are some observations that can be made from the data display?

130	130	131	131	132	132	132	133	134	136
137	137	138	139	139	139	140	141	142	142
142	143	143	144	145	147	149	150		

Quartile 3 (Q3) – (142 + 143) ÷ 2 = 142.5 months
Maximum – 150 months



Five number summary
Minimum – 130 months
Quartile 1 (Q1) – (132 + 133) ÷ 2 = 132.5 months
Median (Q2) – 139 months
This box plot shows that

- ¼ of the students in the class are from 130 to 132.5 months old
- ¼ of the students in the class are from 142.5 months to 150 months old
- ½ of the class are from 132.5 to 142.5 months old
- the median class age is 139 months.

Instructional Strategies:

This cluster builds on the understandings developed in the Grade 6 cluster "Develop understanding of statistical variability." Students have analyzed data displayed in various ways to see how data can be described in terms of variability. Additionally, in Grades 3-5 students have created scaled picture and bar graphs, as well as line plots. Now students learn to organize data in appropriate representations such as box plots (box-and-whisker plots), dot plots, and stem-and-leaf plots. Students need to display the same data using different representations. By comparing the different graphs of the same data, students develop understanding of the benefits of each type of representation.

Further interpretation of the variability comes from the range and center-of-measure numbers. Prior to learning the computation procedures for finding mean and median, students will benefit from concrete experiences.

To find the median visually and kinesthetically, students should reorder the data in ascending or descending order, then place a finger on each end of the data and continue to move toward the center by the same increments until the fingers touch. This number is the median.

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The concept of mean (concept of fair shares or "*evening out*") can be demonstrated visually and kinesthetically by using stacks of linking cubes. The blocks are redistributed among the towers so that all towers have the same number of blocks. Students should not only determine the range and centers of measure, but also use these numbers to describe the variation of the data collected from the statistical question asked. The data should be described in terms of its shape, center, spread (range) and interquartile range or mean absolute deviation (the absolute value of each data point from the mean of the data set). Providing activities that require students to sketch a representation based upon given measures of center and spread and a context will help create connections between the measures and real-life situations.

Using graphing calculators to explore box plots (box-and-whisker plots) removes the time intensity from their creation and permits more time to be spent on the meaning. It is important to use the interquartile range in box plots when describing the variation of the data. The mean absolute deviation describes the distance each point is from the mean of that data set. Patterns in the graphical displays should be observed, as should any outliers in the data set.

Students should identify the attributes of the data and know the appropriate use of the attributes when describing the data. Pairing contextual situations with data and its box-and-whisker plot is essential.

Continue to have students connect contextual situations to data to describe the data set in words prior to computation. Therefore, determining the measures of spread and measures of center mathematically need to follow the development of the conceptual understanding.

Students should experience data which reveals both different and identical values for each of the measures. Students need opportunities to explore how changing a part of the data may change the measures of center and measure of spread. Also, by discussing their findings, students will solidify understanding of the meanings of the measures of center and measures of variability, what each of the measures do and do not tell about a set of data, all leading to a better understanding of their usage.

Common Misconceptions:

Students often use words to help them recall how to determine the measures of center. However, student's lack of understanding of what the measures of center actually represent tends to confuse them. Median is the number in the middle, but that middle number can only be determined after the data entries are arranged in ascending or descending order. Mode is remembered as the "most," and often students think this means the largest value, not the "most frequent" entry in the set. Vocabulary is important in mathematics, but conceptual understanding is equally as important. Usually the mean, mode, or median have different values, but sometimes those values are the same.

Arizona, Ohio & NC DOE

Domain: **Statistics and Probability (SP)**

Cluster**:** Summarize and describe distributions.

Standard: **6.SP.5**

Summarize numerical data sets in relation to their context, such as by:

a. Reporting the number of observations.
b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement
c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Standards for Mathematical Practice (MP):

MP 2 Reason abstractly and quantitatively.
MP 3 Construct viable arguments and critique the reasoning of others.
MP 4 Model with mathematics.
MP 5 Use appropriate tools strategically.
MP 6 Attend to precision.
MP 7 Look for and make use of structure.

Connections:

See SP.4

Explanations and Examples:

6.SP.5 Students record the number of observations. Using histograms, students determine the number of values between specified intervals. Given a box plot and the total number of data values, students identify the number of data points that are represented by the box. Reporting of the number of observations must consider the attribute of the data sets, including units (when applicable). Consideration may need to be given to how the data was collected (ie. random sampling)

Given a set of data values, students summarize the measure of center with the median or mean. The median is the value in the middle of a ordered list of data. This value means that 50% of the data is greater than or equal to it and that 50% of the data is less than or equal to it.

The mean is the arithmetic average or balance point of a distribution. The mean is the sum of the values in a data set divided by how many values there are in the data set. The mean represents the value if all pieces of the data set had the same value. As a balancing point, the mean is the value where the data values above and the data values below have the same value.

Measures of variation can be described using the interquartile range or the Mean Absolute Deviation. The interquartile range describes the variability between the middle 50% of a data set. It is found by subtracting the lower quartile from the upper quartile. It represents the length of the box in a box plot and is not affected by outliers.

The Mean Absolute Deviation describes the variability of the data set by determining the absolute value deviation (the distance) of each data piece from the mean and then finding the average of these deviations.

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Both the interquartile range and the Mean Absolute Deviation are represented by a single numerical value. Higher values represent a greater variability in the data.
Students understand how the measures of center and measures of variability are represented by the graphical display.

Students describe the context of the data, using the shape of the data and are able to use this information to determine an appropriate measure of center and measure of variability.

Students summarize numerical data by providing background information about the attribute being measured, methods and unit of measurement, the context of data collection activities, the number of observations, and summary statistics. Summary statistics include quantitative measures of center, spread, and variability including extreme values (minimum and maximum), mean, median, mode, range, quartiles, interquartile ranges, and mean absolute deviation.

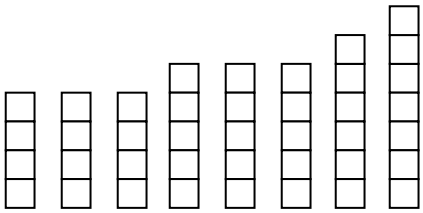
The measure of center that a student chooses to describe a data set will depend upon the shape of the data distribution and context of data collection. The mode is the value in the data set that occurs most frequently. The mode is the least frequently used as a measure of center because data sets may not have a mode, may have more than one mode, or the mode may not be descriptive of the data set. The mean is a very common measure of center computed by adding all the numbers in the set and dividing by the number of values. The mean can be affected greatly by a few data points that are very low or very high. In this case, the median or middle value of the data set might be more descriptive. In data sets that are symmetrically distributed, the mean and median will be very close to the same. In data sets that are skewed, the mean and median will be different, with the median frequently providing a better overall description of the data set.

Understanding the Mean

The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed **equally or "evened out"**, and also in the sense that it is a balance point. Students develop understanding of what the mean represents by redistributing data sets to be level or fair. The leveling process can be connected to and used to develop understanding of the computation of the mean.

For example, students could generate a data set by measuring the number of jumping jacks they can perform in 5 seconds, the length of their feet to the nearest inch, or the number of letters in their names. It is best if the data generated for this activity are 5 to 10 data points which are whole numbers between 1 and 10 that are easy to model with counters or stacking cubes.

Students generate a data set by drawing eight student names at random from the popsicle stick cup. The number of letters in each of the names is used to create the data set. If the names drawn were Carol, Mike, Maria, Luis, Monique, Sierra, John, and Karen there would be 3 names with 4 letters each, 3 names with 5 letters each, 1 name with 6 letters and 1 name with 7 letters.
This data set could be represented with stacking cubes.



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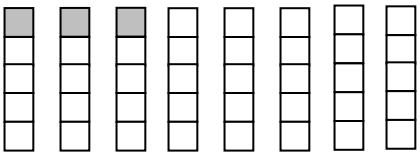
Students can model the mean by "leveling" the stacks or distributing the blocks so the stacks are "fair". Students are seeking to answer the question "If all of the students had the same number of letters in their name, how many letters would each person have?"

One block from the stack of six and two blocks from the stack of 7 can be moved down to the stacks of 4 and then all the stacks have five blocks. If all students had the same number of letters in their name, they would have five letters. The mean number of letters in a name in this data set is 5.

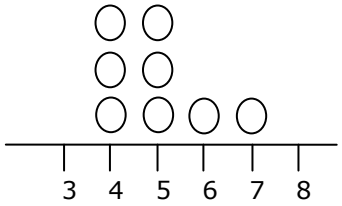


If it was not possible to make the stacks exactly even, students could begin to consider what part of the extra blocks each stack would have.

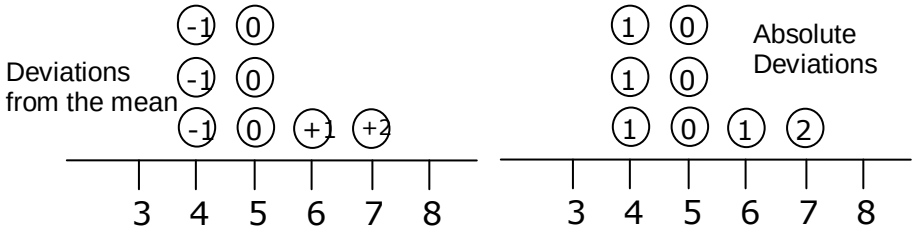
<u>Understanding Mean Absolute Deviation</u>

The use of mean absolute deviation in 6th grade is mainly exploratory. The intent is to build a deeper understanding of variability. Students would understand the mean distance between the pieces of data and the mean of the data set expresses the spread of the data set. Students can see that the larger the mean distance, the greater the variability. Comparisons can be made between different data sets.

In the previous data set, the names drawn were Carol, Mike, Maria, Luis, Monique, Sierra, John, and Karen. There were 3 names with 4 letters each, 3 names with 5 letters each, 1 name with 6 letters and 1 name with 7 letters. This data can be represented on a dot plot. The mean of the data set is 5.



To find the mean absolute deviation, students examine each of the data points and its difference from the mean. This analysis can be represented on the dot plot itself or in a table. Each of the names with 4 letters has one fewer letter than the mean, each of the names with 5 letters has zero difference in letters as compared to the mean, each of the names with 6 letters has one more letter than the mean, and each of the names with 7 letters has two more letters than the mean. The absolute deviations are the absolute value of each difference.



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Name	Number of letters in a name	Deviation from the Mean	Absolute Deviation from the Mean
John	4	-1	1
Luis	4	-1	1
Mike	4	-1	1
Carol	5	0	0
Maria	5	0	0
Karen	5	0	0
Sierra	6	+1	1
Monique	7	+2	2
Total	40	0	6

The mean of the absolute deviations is found by summing the absolute deviations and dividing by the number of data points. In this case, the mean absolute deviation would be 6 ÷ 8 or ¾ or 0.75. The mean absolute deviation is a small number, indicating that there is little variability in the data set.

Consider a different data set also containing 8 names. If the names were Sue, Joe, Jim, Amy, Sabrina, Monique, Timothy, and Anita. Summarize the data set and its variability. How does this compare to the first data set?

The mean of this data set is still 5. $\frac{(3+3+3+3+7+7+7)}{8} = \frac{40}{8} = 5$

Name	Number of letters in a name	Deviation from the Mean	Absolute Deviation from the Mean
Sue	3	-2	2
Joe	3	-2	2
Jim	3	-2	2
Amy	3	-2	2
Sabrina	7	+2	2
Timothy	7	+2	2
Anita	7	+2	2
Monique	7	+2	2
Total	40	0	16

The mean deviation of this data set is 16 ÷ 8 or 2. Although the mean is the same, there is much more variability in this data set.

Understanding Medians and Quartiles

Students can also summarize and describe the center and variability in data sets using the median and a five number summary consisting of the minimum, quartiles, and maximum as seen in the box plot example in **6.SP.4**. The median is the middle number of the data set with half the number below the median and half the numbers above the median. The quartiles partition the data set into four parts by dividing each of the halves of the data set into half again. Quartile 1 (Q1 or the lower quartile) is the middle value of the lower half of the data set and quartile 3 (Q3 or the upper quartile) is the middle value of the upper half of the data set. The median can also be referred to as quartile 2 (Q2). The range of the data is the difference between the minimum and maximum values. The interquartile range of the data is the difference between the lower and upper quartiles (Q3 – Q1). The interquartile range is a measure of the dispersion or spread of the data set: a small value indicates values that are clustered near the median whereas a larger value indicates values that are more distributed.

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Consider the first data set again. Recall that the names drawn were Carol, Mike, Maria, Luis, Monique, Sierra, John, and Karen. The data set can be represented in a numerical list. To find the median and quartile, the values are placed in order from least to greatest.

5 4 5 4 7 6 4 5 $\longrightarrow$ 4 4 4 5 5 5 6 7

The middle value in the ordered data set is the median. If there are an even number of values, the median is the mean of the middle two values. In this case, the median would be 5 because 5 is the average of the 4th and 5th values which are both 5. Students find quartile 1 (Q1) by examining the lower half of the data. Again there are 4 values which is an even number of values. Q1 would be the average of the 2nd and 3rd value in the data set or 4. Students find quartile 3 (Q3) by examining the upper half of the data. Q3 would be the average of the 6th and 7th value in the data set or 5.5. The mean of the data set was 5 and the median is also 5, showing that the values are probably clustered close to the mean. The interquartile range is 1.5 (5.5 – 4). The interquartile range is small, showing little variability in the data.

4 4 4 5 5 5 6 7

Q1 = 4 Q3 = 5.5

Median = 5

<u>Instructional Strategies:</u>

See SP.4

Common Misconceptions:

See SP.4

Arizona, Ohio & NC DOE

Sixth Grade Mathematics Statistics and Probability (SP) continued				
Common Core State Standards		**Essence**	**Extended Common Core**	
Summarize and describe distributions.		**Summarize distributions**	**Summarize distributions on picture graphs, line plots, and bar graphs.**	
Cluster	4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots. 5. Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.		Cluster	2. Display numerical data. 3. Summarize numerical data sets in relation to their context by reporting the number of observations.

EXTENDED 6.SP. 4-5					