Instructional Shifts in Mathematics

This resource is informed by and adapted from the research of *13 Rules That Expire and The Math Pact by Karen S. Karp, Sarah B. Bush, and Barbara J. Dougherty* and *Figuring Out Fluency by Jennifer Bay-Williams and John SanGiovanni*. It identifies common instructional practices that may unintentionally limit student understanding and offers research-based shifts to better support conceptual understanding and procedural fluency.

The document was created by District Content Specialist Wendy Stove, in collaboration with K-6 Academic Instructional Coaches, to guide intentional planning during grade-level meetings and PLCs. It also serves as a reference for administrators during classroom walkthroughs to help align instruction with best practices in mathematics.

We hope this resource supports your work.

Shift Away From	Why	Shift Toward
General Instruction		
Using timed tests to assess fact fluency	Timed tests can trigger or increase math anxiety in students even when they know the answers. Timed tests only assess speed and recall; they do not attend to efficiency and flexibility and give little insight into student thinking. Supportive timed activities (not tests) to practice *mastered* facts can help students become more efficient, but they must be LOW stakes.	Strategy based assessments Fluency interviews Games and tasks Observations during discussions
Telling students to show your steps.	Focuses on procedure over understanding.	Ask students to explain or justify their thinking.
Using I do, we do, you do model of gradual release	Does not position students as doers of mathematics. This model is teacher centered rather than student centered and limits opportunities for productive struggle.	Launch-Explore-Discuss-Close lesson format Promote Productive Struggle
Teaching procedural steps without conceptual understanding.	This can lead to shallow learning, misconceptions, and difficulty applying the math to novel problems.	Incorporate and make connections between conceptual thinking, manipulatives, models, and abstract thinking. This helps

		students develop a deep, flexible understanding of mathematical ideas.
Teaching standard algorithms prior to grade-level expectations. 4th gr: addition/subtraction 5th gr: multiplication 6th gr: long division	Hides place value, discourages flexibility and efficiency and encourages digit-focused procedures. Algorithms can trap students into less sophisticated thinking.	Focus on conceptual understanding, models, strategies that emphasize place value and the relationships between quantities. Follow procedural fluencies outlined in grade level standards. Remember fluency standards are end-of-year expectations.
Requiring students to use the standard algorithm for all problem types once it is learned.	The standard algorithm is not the most efficient strategy for all problem types. Ex: 200-198	Students should be proficient in using algorithms, it should be in addition to, rather than instead of, using flexible and efficient strategy selection.
Using 'cute' language or mnemonics (e.g., Gus the Plus, flip flop property)	Can confuse or distract from math concepts.	Use precise vocabulary and meaningful contexts.
Key words and problem-solving mnemonics (Cubes) for word problems	They remove all need for students to make sense of the problem as it is presented, and the mathematics included in it. Students can be incorrectly influenced by key words, and many problems don't contain them. Falls apart in multi-step problems.	Increase readability of problems. Make the context relevant Employ schema-based instruction. (Problem types)
Teaching tricks, shortcuts, and rules	These are replacements for deep conceptual understanding and procedural fluency. Can also give the message the math is too hard for them to understand so just use this trick to get the answer.	Develop students' procedural fluency built on the foundation of conceptual understanding and understanding the meaning of procedures.
Fluency = speed and memorization	Rote memory limits transfer and understanding.	Fluency = efficiency, flexibility, and accuracy.

Overreliance on worksheets	Lacks engagement and depth. Students need multi-sensory experiences to retain information and make meaning out of math. Use worksheets in moderation.	Use manipulatives, cooperative learning, games, and hands-on tasks.
Using naked numbers (equations) to introduce concepts	Numbers in isolation are abstract. Exposing students to verbal and written contexts helps them make meaning of the mathematics and connects math to the real world.	Use real-world or meaningful contexts to help students make sense of the mathematics.
Saying 'makes' or 'the answer is' for the equal sign. Always showing the equal sign on the right side of the equation (4 +3 = 7)	Implies the equal sign is an action, or an operation rather than a relationship. Undermines the meaning of equivalence.	Use 'is the same as' or equal to. Vary the equation structure.
Using run-on equations (Connecting things that are not equal – ex: 4+3=7+3=10)	Connecting unequal expressions gives them a false understanding that the = sign means the answer is rather than balance. Students should the view the = sign as "the same as" or equal to.	Use arrows or separate lines.
Mathematical Language & Precision		
Reading a multi-digit whole number with "and" ex: One hundred and twenty-three	Inserting the word "and" implies that the number consists of a whole and a part as in a fraction or a decimal	Read as One hundred twenty-three
Saying smaller than or bigger than or the bigger number to make comparisons	Greater and less are better choices because they are more closely aligned with comparisons	Use accurate terms: greater than, less than, equal to, or measurement specific terms like longer, shorter, weighs more, weighs less, etc.
Using the word rounding to mean estimating Using guess to mean estimate	Estimates are not random guesses, and rounding is only one strategy for estimation	Say estimate or estimating Estimating is an educated determination of a rough calculation or given quantity.
Writing only a number to represent a unit of measure.	Influences students' grasp of how to accurately use notation in mathematics.	Ex: Area of 9 square units, not an area of 9

	Numbers without units are without meaning.	
Using PEMDAS	Students incorrectly believe Parentheses are the only form of grouping and that the order is rigid, meaning multiplication before division, addition before subtractions	In addition to sense-making use GEMS or GEMA instead G – Groupings E – Exponents M – Multiplicative A – Additive
Saying undo for inverse operations	Undo oversimplifies the process and can limit to procedural/surface level understanding	Say the inverse operation encourages reasoning-based thinking and re-emphasizes relationships
	Addition & Subtraction	
Saying addition always makes numbers bigger, and subtraction makes them smaller	False with integers and advanced concepts.	Refer to context and operational meaning.
Saying you always start with the larger number when you subtract.	Expires when students begin working with integers. Does not focus on meaning.	Help students make sense of context problems before they write them symbolically. Ex: Kris gave 3 pencils to her brother. She had 7 to begin with, so how many does she have now? Ask students to act out the situation until they realize that she started with 7 and gave away 3, which should be recorded 7 – 3. If they write 3 – 7, ask them, "Your expression shows that Kris started with 3 pencils and gave away 7. Is that what happened?"
Using the term "take away" as the generic way to read subtraction	Not all subtraction problems are take-away problems	Read the subtraction symbol as minus

Saying 'borrow' and 'carry'	Borrowing & carrying focus on the algorithm and not the actual trading of equal amounts	Use bundling/unbundling, regrouping or trading to indicate the action of exchanging or trading one place value unit for another
Multiplication and Division		
Skip counting as the primary strategy for multiplication	Keeps students in additive thinking. Multiplicative reasoning is key to students' continued development in mathematics.	Encourage multiplicative strategies (doubling, compensation, distributive property) Multiplicative Strategies help students think about groups of groups (thinking in bigger chunks)
Multiplication is repeated addition	This rule expires when students begin using exponents and/or multiplying with fractions or decimals Repeated addition is only one meaning of multiplication and it is the least sophisticated.	Move students from additive thinking to multiplicative thinking. Multiplicative Strategies help students think about groups of groups (thinking in bigger chunks)
Say – When you multiply, the product is always bigger/larger than the factors.	This expires when students begin operating with fractions, decimals, and positive and negative integers.	Focus making sense of the question/problem Use estimation and sense making to assess reasonableness of answers
When multiplying by 10, just add a zero	Students should see the pattern in zeros and connect to place value principles.	Model with manipulatives (base 10 blocks) Use multiplicative comparison language like 10 times larger 100 times larger etc.

	This "rule" is not generalizable to other types of numbers – i.e. decimals	
Saying the quotient is always smaller than the dividend	This expires in fraction and decimal operations	Focus making sense of the question/problem Use estimation and sense making to assess reasonableness of answers
Saying divide the larger number by the smaller number	This does not apply when students learn to divide fractions, decimals, and negative numbers. In fifth grade, students learn that fractions represent division. ¾ is three divided by four.	Promote sense making – what is the problem asking. Use mathematical language such dividend and divisor. Use estimation to check reasonableness.
The Big 7 for partial quotients	Makes this a procedure, steps follow rather than understanding the mathematics	Call it partial quotients
Forcing students to decompose numbers strictly by place value (e.g., 30 as 3 tens) when using the area/box method	This approach can limit flexibility and reinforce a narrow understanding of number structure. It may also make the model feel procedural rather than conceptual	Encourage flexible decomposition of numbers based on relationships and strategy (e.g., decomposing 36 as 40 – 4 or 25 + 11), depending on the problem context and student reasoning.
Using equal-sized boxes in area models regardless of the actual value of the numbers	Equal boxes misrepresent the relative size (magnitude) of the partial products, making it harder for students to grasp the underlying concept of area and value.	Construct area models that reflect the magnitude of each quantity visually, helping students connect the mathematics to the structure and size of the model.
Don't use cover and fill interchangeably when teaching area or volume		Use cover with area Fill with volume
Fractions		
Top number and bottom number	A fraction is a single number that expresses a relationship between two quantities. The numerator tells how	Use precise vocabulary: Numerator and denominator

	many parts are being considered, and the denominator tells how many equal parts make up one whole. Referring to these as "top" and "bottom" numbers reinforces the misconception that fractions are two numbers.	Understand the meaning of numerator and denominator
Only referring to $\frac{3}{4}$" is 'three out of four parts'" or "three shaded out of four"	This is one definition of fractions that expires by 4th grade. Limits understanding.	$\frac{3}{4}$ is not just "three shaded out of four" parts, but three copies of one-fourth put together. This is the key conceptual idea behind non-unit fractions
Using only labeled models or circles	Limits flexibility and understanding.	Use a variety of fraction models.
Requiring students to simplify fractions	The AZ State Standards do not include simplifying fractions. However, students will be asked to find equivalent fractions of a given number in multiple forms.	Look for equivalent fractions Write equivalent fractions in various forms.
Use the phrase "reducing fractions."	This gives the incorrect impression that the fraction is getting smaller or being reduced in size.	Reinforce fraction understandings (size of the parts) and equivalency.
Using the term "improper fraction" as the only way to name fractions greater that one.	Improper fraction implies something is wrong with the fraction. It has no mathematical meaning.	Teach as fractions greater than one, call attention to these are often referred to as improper fractions.
Always make improper fraction to mixed numbers	This is not a rule and is not required in the AZ State Standards. This also confuses students when they move to middle school, and they need to leave fractions in "fraction form".	Mixed numbers and fractions greater than one (improper fractions) are equivalent fractions. Students should leave their answer in the form required by the context of the problem.
'Keep, Change, Flip' for dividing fractions No standard algorithms for dividing with fractions in K-5	It does not promote conceptual understanding or procedural fluency. Students overgeneralize this "rule" to other fraction operations.	Connect division with fractions to whole number division. Develop conceptual understanding through the use of physical models and through explaining the use of partitioning and sharing, or

		the use of other methods (common denominator strategy)
Teaching butterfly method / cross multiplying for comparing and operations.	This focuses on following steps rather than developing the understanding students need to reason with fractions as numbers. It does not make explicit connections between comparing and visual models. Students also use this inappropriately (when it will not result in the correct answer.)	Use a variety of manipulatives, models and drawings. Use strategies that support reasoning about size.
Decimals		
Moving the decimal to multiply/divide	The decimal point does not move. It always separates the whole units from the fractional parts. This also keeps students from reasoning about magnitude.	Students should develop a deep understanding of how place value shifts when multiplying or dividing by powers of ten.
Use of the words "dot" or "point" when reading decimal numbers.	This takes away from the understanding of place value.	Read with precise place value. Note "dot and point" are widely used to read decimal numbers in the world but use these terms sparingly after students understand place value.
Just line the decimal points up when you add and subtract.	This advice focuses on procedure rather than understanding. Students may overgeneralize this to other operations and does can be confusion when one of the numbers is a whole number.	Emphasize place value, make connections to whole number addition and subtraction. Use models and visuals.
Add a zero for tenths, hundredths	The "zero" in this instance does not change the value of the number the way it does with whole numbers (like thinking 0.7 is less than 0.70 because "70 is bigger than 7"). Students	Instead of teaching them we are expressing equivalent forms of numbers to match their place value structure, so

	might not grasp that 0.7 and 0.70 are *the same number*, just represented differently. They might mistakenly think the number *changed* when a zero was "added," rather than seeing it as an equivalent form.	we can accurately add or subtract corresponding units.
Algebra/Integers		
Teaching a variable as "just a letter that stands for a number"	This often leads students to see variables as meaningless symbols and promotes guessing or plugging in, rather than reasoning.	Teach that a variable represents an unknown or a changing quantity. Use real contexts to connect variables to meaningful values (e.g., "Let *n* represent the number of apples").
Two negatives make a positive	This breaks down when adding or subtracting negative numbers and fails to explain why this happens and works.	Instead of focusing on the rule, consider using patterns of products to develop generalizations about the relationships.
Only representing equations or inequalities with the variable (e.g., *x*) on the left side of the symbol	Students begin to believe that variables must always come first. This limits flexibility and can confuse students when they encounter equivalent but differently structured expressions (e.g., 12 = x + 3 or 5 > x).	Vary the position of the variable to build structural flexibility. Show that $x = 5$, $5 = x$, and $x + 3 = 12$ are all valid and meaningful. Use number balance models or real-world contexts to reinforce the relational meaning.