Mathematics

Table 1 & Table 2

Aligned to the Arizona Mathematics Standards, Adopted 2016

ARIZONA DEPARTMENT OF EDUCATION
HIGH ACADEMIC STANDARDS FOR STUDENTS

Table 1 and Table 2 represent the various types of word problems and situations in which students will engage. Table 1 is explicitly referenced in grades K-3. Table 2 is explicitly referenced in grades 3 and 4. However it would be beneficial for the learner if these problem types were used when teaching fraction and decimal operations as well.

Utilizing the numeracy and operational expectations from specific grade levels, instruction on each of the problem types will support student success in solving problems with the unknown in different positions, as well as help them gain a deeper understanding of the equal sign.

Students in grades K-5 should regularly be exposed to the various word problem types as referenced in their grade level standards. The exposure to different problem types will help students strengthen procedural, conceptual and factual understanding.

Table 1: Addition and Subtraction Situations

Table 1 provides support to clarify the varied problem structures necessary to build student conceptual understanding of addition and subtraction, focusing on developing student flexibility. In order to fully implement the standards, students must solve problems from all problem subcategories relevant to the grade level. All problem types should not be mastered at all grades in Kindergarten through fifth grade. Guidance on what problem types could be mastered at each grade level is available in the Progressions for Operations and Algebraic Thinking document.[1]

Table 2: Multiplication and Division Situations

Table 2 provides support to clarify the varied problem structures necessary to build student conceptual understanding of multiplication and division, focusing on developing student flexibility. In order to fully implement the standards, students must solve problems from all problem subcategories relevant to the grade level.

[1] University of Arizona Institute for Mathematics and Education. (2011). *Progression on Counting and Cardinality and Operations and Algebraic Thinking.*

Table 1. Common Addition and Subtraction Problem Types/Situations.[1]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[2]**
Put Together / Take Apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean ***makes*** or ***results*** in but always does mean ***is the same quantity as***.

[3]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

Table 2. Common Multiplication and Division Problem Types/Situations.[1]

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	3 x 6 = ?	3 x ? = 18 and 18 ÷ 3 = ?	? x 6 = 18 and 18 ÷ 6 = ?
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? ***Measurement example.*** You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? ***Measurement example.*** You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? ***Measurement example.*** You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[2] Area[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? ***Area example.*** What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? ***Area example.*** A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? ***Area example.*** A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A straw hat costs $6. A baseball hat costs 3 times as much as the straw hat. How much does the baseball hat cost? ***Measurement example.*** A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A baseball hat costs $18 and that is 3 times as much as a straw hat costs. How much does a straw hat cost? ***Measurement example.*** A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A baseball hat costs $18 and a straw hat costs $6. How many times as much does the baseball hat cost as the straw hat? ***Measurement example.*** A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[1]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.
[2]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[3]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.