

3rd Grade – Summary of Revisions and Planning Guidance - Arizona Mathematics Standards - Adopted in 2016

Additions		Deletions
<ul style="list-style-type: none"> • 3.OA.D.10 –When solving problems, assess the reasonableness of answers using mental computation and estimation strategies including rounding. • 3.MD.A.1b – Solve word problems involving money through \$20.00, using symbols \$, ".", ¢. • 3.NF.A.2c – Understand a fraction $1/b$ as a special type of fraction can be referred to as a unit fraction (e.g. $1/2$, $1/4$). 		<ul style="list-style-type: none"> • No deletions
Parameter Changes/Clarifications		Fluency Expectations
3.OA.A.1	Clarification of the meaning of each of the factors in a multiplication problem is provided in the e.g.	<p>2nd 2.OA.B.2 - Fluently add and subtract within 20. By the end of 2nd grade, know from memory all sums of two one-digit numbers.</p> <p>2.NBT.B.5 - Fluently add and subtract within 100.</p> <hr/> <p>3rd 3.NBT.A.2 - Fluently add and subtract within 1000.</p> <p>3.OA.C.7 - Fluently multiply and divide within 100. By the end of 3rd grade, know from memory all multiplication products through 10 x 10 and division quotients when both the quotient and divisor are less than or equal to 10.</p> <hr/> <p>4th 4.NBT.B.4 - Fluently add and subtract multi-digit whole numbers using a standard algorithm.</p>
3.OA.A.2	The example was removed to allow for multiple instructional approaches. A reference to Table 2 is included in this standard to ensure understanding with the unknown in all positions.	
3.OA.A.3	Specific representations were removed from the e.g. in the standard.	
3.OA.A.4	A reference to Table 2 was added to the standard.	
3.OA.B.5	Removed examples of properties within the standard, these can be found in Table 4 within the Glossary. Property expectations are specifically stated in this standard.	
3.OA.C.7	Removed the use of strategies to multiply and divide, added knowing division quotients from memory. The definition of fluency found in the introduction addresses the use of strategies in achieving fluency.	
3.OA.D.8	Removed "assess the reasonableness of answers using mental computation and estimation strategies including rounding" as this is now found in 3.OA.D.10. The limit of using whole numbers with whole number answers removed as it was redundant for this grade level.	
3.OA.D.10	NEW STANDARD	
3.NF.A.2	This standard now states the inclusion of fractions greater than one. Section c is new and emphasizes the importance of understanding unit fractions.	
3.NF.A.3	Section a. removed the limitation of the same point on number line and added specific comparison to 1 whole which could include many representations. Sections b and c removed the examples from the standards. Part d- instead of recognize valid comparison, it is now understand valid comparisons increasing the cognitive demand.	
3.MD.A.1b	NEW STANDARD	
3.MD.B.3	Instead of draw a graph, the standard requires them to create graphs increasing the cognitive demand. This standard also now includes reference to Table 1 to ensure understanding with the unknown in all positions.	
3.MD.C.7	Instead of recognizing area as additive, students need to understand rectilinear figures can be decomposed and the sum of the area is that of the original figure, increasing the cognitive demand.	
		Fluency Definition
		<p>Fluency standard instruction should begin at the beginning of the year and continue throughout the school year. Wherever the word <i>fluently</i> appears in a content standard, the word includes efficiently, accurately, flexibly, and appropriately. Being fluent means that students are able to choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and are able to explain their approaches, and they are able to produce accurate answers efficiently.</p> <ul style="list-style-type: none"> • Efficiency—carries out easily, keeps track of sub-problems, and makes use of intermediate results to solve the problem. • Accuracy—reliably produces the correct answer. • Flexibility—knows more than one approach, chooses a viable strategy, and uses one method to solve and another method to double-check. • Appropriately—knows when to apply a particular procedure.

Defining Standards, Curriculum and Instruction

Standards – What a student needs to know, understand, and be able to do by the end of each grade. Standards build across grade levels in a progression of increasing understanding and through a range of cognitive demand levels. Standards are adopted at the state level by the State Board of Education.

Curriculum – The resources used for teaching and learning the standards. Curricula are adopted at a local level by districts and schools.

Instruction – The methods used by teachers to teach their students. Instructional techniques are employed by individual teachers in response to the needs of the students in their classes to help them progress through the curriculum in order to master the standards.

3rd Grade Content Emphasis

Operations and Algebraic Thinking (OA)

- Represent and solve problems involving whole number multiplication and division.
- Understand properties of multiplication and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Number and Operations in Base Ten (NBT)

- ▲ Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations - Fractions (NF)

- Understand fractions as numbers.

Measurement and Data (MD)

- ▲ Solve problems involving measurement.
- ▲ Represent and interpret data.
- Geometric measurement: Understand concepts of area and perimeter.

Geometry (G)

- ▲ Reason with shapes and their attributes.

● - Major Content ▲ - Supporting Content

Major Content (●) from the content emphasis section should account for approximately 70% of instructional time.

Standards that reference Table 2 in 3rd Grade

3.OA.A.2

3.OA.A.3

3.OA.A.4

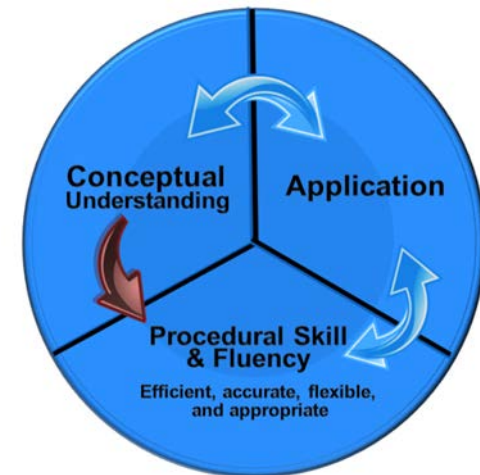
3.MD.A.2

The Standards for Mathematical Practice

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

The Arizona Mathematics Standards now include narratives for each of the 8 Mathematical Practices

Balance of Rigor in the Math Classroom



Changes in Cognitive Demand

There are times in which the standards were changed, resulting in an increase or decrease in cognitive demand expectations within the standards. This is an important aspect of the standard to examine, keeping in mind that cognitive demand refers to the complexity of thinking involved in which students interact with the content; it does **not** refer to difficulty.

Changes in Cognitive Demand in the 3rd Grade Standards

3.NF.A.3

3.MD.B.3

3.MD.C.7

Comparison of Arizona Mathematics Standards – 2010 to 2016

Adopted 2010		Adopted 2016		
Operations and Algebraic Thinking (OA)		Operations & Algebraic Thinking (OA)		
		<i>Note: Grade 3 expectations in this domain are limited to whole number multiplication through 10×10 and whole number division with both quotients and divisors less than or equal to 10.</i>		
3.OA.A	Represent and solve problems involving multiplication and division.			
	3.OA.A.1. Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. <i>For example, describe a context in which a total number of objects can be expressed as 5×7.</i>	3.OA.A Represent and solve problems involving whole number multiplication and division.	3.OA.A.1	Interpret products of whole numbers as the total number of objects in equal groups (e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each).
	3.OA.A.2. Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. <i>For example, describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$.</i>		3.OA.A.2	Interpret whole number quotients of whole numbers (e.g., interpret $56 \div 8$ as the number of objects in each group when 56 objects are partitioned equally into 8 groups, or as a number of groups when 56 objects are partitioned into equal groups of 8 objects each). <i>See Table 2.</i>
	3.OA.A.3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See Table 2.)		3.OA.A.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities. <i>See Table 2.</i>
	3.OA.A.4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. <i>For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = \square \div 3$, $6 \times 6 = ?$.</i>		3.OA.A.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers <i>For example, determine the unknown number that makes the equation true in each of the equations $8 \times \square = 48$, $5 = \square \div 3$, $6 \times 6 = \square$.</i> <i>See Table 2.</i>

3.OA.B	Understand properties of multiplication and the relationship between multiplication and division.			
	3.OA.B.5. Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) <i>Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property of multiplication.)</i>	3.OA.B Understand properties of multiplication and the relationship between multiplication and division.	3.OA.B.5	Apply properties of operations as strategies to multiply and divide. Properties include commutative and associative properties of multiplication and the distributive property. (Students do not need to use the formal terms for these properties.)
	3.OA.B.6. Understand division as an unknown-factor problem. For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.		3.OA.B.6	Understand division as an unknown-factor problem (e.g., find $32 \div 8$ by finding the number that makes 32 when multiplied by 8).
3.OA.C	Multiply and divide within 100.			
	3.OA.C.7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.	3.OA.C Multiply and divide within 100.	3.OA.C.7	Fluently multiply and divide within 100. By the end of Grade 3, know from memory all multiplication products through 10×10 and division quotients when both the quotient and divisor are less than or equal to 10.
3.OA.D	Solve problems involving the four operations, and identify and explain patterns in arithmetic			
	3.OA.D.8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).)	3.OA.D Solve problems involving the four operations, and identify and explain patterns in arithmetic.	3.OA.D.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Utilize understanding of the Order of Operations when there are no parentheses.

	3.OA.D.9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. <i>For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.</i>	3.OA.D (cont.)	3.OA.D.9	Identify patterns in the addition table and the multiplication table and explain them using properties of operations (e.g. observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends).
			3.OA.D.10	When solving problems, assess the reasonableness of answers using mental computation and estimation strategies including rounding.
Number and Operations in Base Ten (NBT)		Number and Operations in Base Ten (NBT) <i>Note: A range of algorithms may be used.</i>		
3.NBT.A	Use place value understanding and properties of operations to perform multi-digit arithmetic. (A range of algorithms may be used)			
	3.NBT.A.1. Use place value understanding to round whole numbers to the nearest 10 or 100.	3.NBT.A Use place value understanding and properties of operations to perform multi-digit arithmetic.	3.NBT.A.1	Use place value understanding to round whole numbers to the nearest 10 or 100.
	3.NBT.A.2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.		3.NBT.A.2	Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
	3.NBT.A.3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80 , 5×60) using strategies based on place value and properties of operations.		3.NBT.A.3	Multiply one-digit whole numbers by multiples of 10 in the range 10 to 90 using strategies based on place value and the properties of operations (e.g., 9×80 , 5×60).
Number and Operations—Fractions (NF) (Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8)		Number and Operations-Fractions (NF) <i>Note: Grade 3 expectations are limited to fractions with denominators: 2,3,4,6,8.</i>		
3.NF.A	Develop understanding of fractions as numbers.			
	3.NF.A.1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.	3.NF.A Understand fractions as numbers.	3.NF.A.1	Understand a fraction ($1/b$) as the quantity formed by one part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

	<p>3.NF.A.2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.</p> <p>a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.</p> <p>b. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.</p>	<p>3.NF.A (cont.)</p>	<p>3.NF.A.2</p>	<p>Understand a fraction as a number on the number line; represent fractions on a number line diagram.</p> <p>a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Understand that each part has size $1/b$ and that the end point of the part based at 0 locates the number $1/b$ on the number line.</p> <p>b. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Understand that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line including values greater than 1.</p> <p>c. Understand a fraction $1/b$ as a special type of fraction can be referred to as a unit fraction (e.g. $1/2$, $1/4$).</p>
	<p>3.NF.A.3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.</p> <p>a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.</p> <p>b. Recognize and generate simple equivalent fractions, e.g., $1/2 = 2/4$, $4/6 = 2/3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.</p> <p>c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form $3 = 3/1$; recognize that $6/1 = 6$; locate $4/4$ and 1 at the same point of a number line diagram.</p> <p>d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.</p>			<p>3.NF.A.3</p>
<p>Measurement and Data (MD)</p>		<p>Measurement and Data (MD)</p>		

3.MD.A	Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.			
	3.MD.A.1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.	3.MD.A Solve problems involving measurement.	3.MD.A.1a	Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes (e.g., representing the problem on a number line diagram).
			3.MD.A.1b	Solve word problems involving money through \$20.00, using symbols \$, ".", ¢.
	3.MD.A.2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Excludes compound units such as cm ³ and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. Excludes multiplicative comparison problems (problems involving notions of “times as much”; see Table 2).		3.MD.A.2	Measure and estimate liquid volumes and masses of objects using metric units. (Excludes compound units such as cm ³ and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units. Excludes multiplicative comparison problems (problems involving notions of “times as much”). <i>See Table 2.</i>
3.MD.B	Represent and interpret data.			
	3.MD.B.3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. <i>For example, draw a bar graph in which each square in the bar graph might represent 5 pets.</i>	3.MD.B Represent and interpret data.	3.MD.B.3	Create a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. <i>See Table 1.</i>
	3.MD.B.4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.		3.MD.B (cont.)	3.MD.B.4

3.MD.C	Geometric measurement: understand concepts of area and relate area to multiplication and to addition.			
	<p>3.MD.C.5. Recognize area as an attribute of plane figures and understand concepts of area measurement.</p> <p>a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.</p> <p>b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.</p>	<p>3.MD.C Geometric measurement: Understand concepts of area and perimeter.</p>	3.MD.C.5	<p>Understand area as an attribute of plane figures and understand concepts of area measurement.</p> <p>a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.</p> <p>b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.</p>
	<p>3.MD.C.6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).</p>		3.MD.C.6	<p>Measure areas by counting unit squares (e.g., square cm, square m, square in, square ft, and improvised units).</p>

<p>3.MD.C.7. Relate area to the operations of multiplication and addition.</p> <p>a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.</p> <p>b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.</p> <p>c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.</p> <p>d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.</p>		<p>3.MD.C (cont.)</p>	<p>3.MD.C.7</p>	<p>Relate area to the operations of multiplication and addition.</p> <p>a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.</p> <p>b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.</p> <p>c. Use tiling to show that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.</p> <p>d. Understand that rectilinear figures can be decomposed into non-overlapping rectangles and that the sum of the areas of these rectangles is identical to the area of the original rectilinear figure. Apply this technique to solve problems in real-world contexts.</p>
			<p>3.MD.C.8</p>	<p>Solve real-world and mathematical problems involving perimeters of plane figures and areas of rectangles, including finding the perimeter given the side lengths, finding an unknown side length. Represent rectangles with the same perimeter and different areas or with the same area and different perimeters.</p>

3.MD.D	Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.			
	3.MD.D.8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.			
Geometry (G)		Geometry (G)		
3.G.A	Reason with shapes and their attributes.			
	3.G.A.1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.	3.G.A Reason with shapes and their attributes.	3.G.A.1	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
	3.G.A.2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as $\frac{1}{4}$ of the area of the shape.		3.G.A.2	Partition shapes into b parts with equal areas. Express the area of each part as a unit fraction $\frac{1}{b}$ of the whole. (Grade 3 expectations are limited to fractions with denominators $b = 2,3,4,6,8$.)
Standards for Mathematical Practice				
3.MP.1 Make sense of problems and persevere in solving them.				
Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, "Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.				
3.MP.2 Reason abstractly and quantitatively.				
Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.				

3.MP.3 Construct viable arguments, and critique the reasoning of others.

Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming or questioning the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.

3.MP.4 Model with mathematics.

Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

3.MP.5 Use appropriate tools strategically.

Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.

3.MP.6 Attend to precision.

Mathematically proficient students clearly communicate to others using appropriate mathematical terminology, and craft explanations that convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.

3.MP.7 Look for and make use of structure.

Mathematically proficient students use structure and patterns to assist in making connections among mathematical ideas or concepts when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.

3.MP.8 Look for and express regularity in repeated reasoning.

Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.