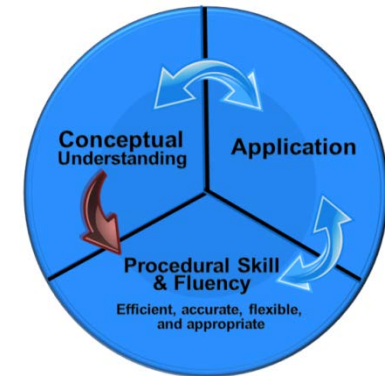# Algebra 2 – Summary of Revisions and Planning Guidance - *Arizona Mathematics Standards - Adopted in 2016*

Additions	Deletions
• **A2.N-Q.A.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data display, include utilizing real-world context. • **A2.N-Q.A.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities utilizing real-world context. • **A2.F-BF.B.4 b** Understand that if a function contains a point (a,b), then the graph of the inverse relation of the function contains the point (b,a). • **A2.F-BF.B.4 c** Interpret the meaning of and relationship between a function and its inverse utilizing real-world context. • **A2.S.ID.C.10** Interpret parameters of exponential models. • **A2.S-CP.B.8** Apply the general Multiplication Rule in a uniform probability model, $P(A$ and $B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model.	• **HS.A.REI.C.6** Remains exclusively in Algebra 1. • **HS.F.IF.A.3** Remains exclusively in Algebra 1. • **HS.F.LE.A.2** Remains exclusively in Algebra 1. • **HS.G.GPE.A.2** Moved to Plus Standards. • **HS.S-CP.A.1** Moved to Algebra 1. • **HS.S-CP.A.2** Moved to Algebra 1. • **HS.S-IC.B.5** Moved to Plus Standards. • **HS.S-IC.B.6** Moved to Plus Standards.

Parameter Changes/Clarifications

Standard	Notes
A2.N-Q.A.2	Added "utilizing real-world context". This is a mathematical modeling standard.
A2.N-CN.A.1 & A.2	Standards were combined into a single standard.
A2.A-SSE.A.2	This standard now includes a defined limit between Algebra 1 and Algebra 2 content. See standard for clarification on limits.
A2.A-SSE.B.3	Added "utilizing real-world context". This is a mathematical modeling standard.
A2.A-APR.B.2	Wording was added to include "Know and apply the Remainder and Factor Theorem" for clarity.
A2. A-APR.B.3	This standard now includes a defined limit between Algebra 1 and Algebra 2 content. See standard for clarification on limits.
A2.A-CED.A.1	Added "utilizing real-world context". This is a mathematical modeling standard. This standard now includes a defined limit between Algebra 1 and Algebra 2 content. See standard for clarification on limits.
A2.A-REI.A.1	This standard now includes a defined limit between Algebra 1 and Algebra 2 content. See standard for clarification on limits.
A2.A-REI.B.4b	This standard now includes a defined limit between Algebra 1 and Algebra 2 content. See standard for clarification on limits to include the addition of the word fluently.
A2.A-REI.D.11	Added "utilizing real-world context". This standard now includes a defined limit between Algebra 1 and Algebra 2 content. See standard for clarification on limits.
A2.F-IF.B.4	Added "utilizing real-world context". This standard now includes a defined limit between Algebra 1 and Algebra 2 content. See standard for clarification on limits.
A2.F-IF.B.6	Added "utilizing real-world context". This standard now includes a defined limit between Algebra 1 and Algebra 2 content. See standard for clarification on limits.
A2.F-IF.C.7	Sub-standards were combined into a single standard. This standard now includes a defined limit between Algebra 1 and Algebra 2 content. See standard for clarification.
A2.F-IF.C.8	Wording was added to including "classify these functions as exponential growth and decay."
A2.F-IF.C.9	This standard now includes a defined limit between Algebra 1 and Algebra 2 content. See standard for clarification on limits.

Vertical Fluency Articulation

Algebra 1	**A1.F-IF.C.7** Graph functions expressed symbolically and show key features of the graph. **A1.A-SSE.A.2** Use structure to identify ways to rewrite numerical and polynomial expressions.
Algebra 2	**A2.A-SSE.A.2** Use the structure of an expression to identify ways to rewrite it. **A2.A-REI.B.4** Fluently solve quadratic equations in one variable **A2.F-BF.B.3** Identify the effect on a graph when changing $f(x)$.

Fluency Definition

Wherever the word *fluently* appears in a content standard, the word includes *efficiently, accurately, flexibly,* and *appropriately*. Being fluent means that students are able to choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and are able to explain their approaches, and they are able to produce accurate answers efficiently.

• **Efficiency**—carries out easily, keeps track of sub-problems, and makes use of intermediate results to solve the problem.
• **Accuracy**—reliably produces the correct answer.
• **Flexibility**—knows more than one approach, chooses a viable strategy, and uses one method to solve and another method to double-check.
• **Appropriately**—knows when to apply a particular procedure.

A2.F-BF.A.1	Added "utilizing real-world context". This is a mathematical modeling standard. This standard now includes a defined limit between Algebra 1 and Algebra 2 content. See standard for clarification.
A2.F-BF.B.3	This standard now includes a defined limit between Algebra 1 and Algebra 2 content. See standard for clarification on limits.
A2.F-BF.B.4a	See standard for revision.
A2.F-LE.B.5	Added "utilizing real-world context". This is a mathematical modeling standard. This standard now includes a defined limit between Algebra 1 and Algebra 2 content. See standard for clarification.
A2.F-TF.B.5	Wording change; "Choose" changed to "Create and interpret".
A2.F-TF.C.8	Wording change influences cognitive demand. "Prove" changed to "Use" the Pythagorean Identity.
A2.S-ID.A.4	Wording change from normal distribution to normal curve.
A2.S-ID.B.6	This standard now includes a defined limit between Algebra 1 and Algebra 2 content. See standard for clarification on limits. Wording of standard was clarified.
A2.S-IC.A.2	Cognitive demand changed. Decide changed to explain.
A2.S-IC.B	Cluster heading wording was changed – "sample surveys" was removed.
A2.S-IC.B.3	See standards for revision. Randomization piece removed.
A2.S-IC.B.4	See standard for revision.
A2.S-CP.B.5	Added "utilizing real-world context". This is a mathematical modeling standard.
A2.S-CP.B.6	Wording change; addition of Bayes Rule for clarification.

Algebra 2-Content Emphasis

● Major Content   ▲ Supporting Content

Major Content (●) from the content emphasis section should account for approximately 70% of instructional time.

The Real Number System (N-RN)	Creating Equations (A-CED)	Trigonometric Functions (F-TF)
● Extend the properties of exponents to rational exponents.	● Create equations that describe numbers or relationships.	▲ Extend the domain of trigonometric functions using the unit circle.
Quantities (N-Q)	**Arithmetic with Polynomials and Rational Expressions (A-APR)**	● Model periodic phenomena with trigonometric functions.
▲ Reason quantitatively and use units to solve problems.	● Understand the relationship between zeros and factors of polynomials.	▲ Apply trigonometric identities.
The Complex Number System (N-CN)	▲ Use polynomial identities to solve problems.	**Interpreting Categorical and Quantitative Data (S-ID)**
▲ Perform arithmetic operations with complex numbers.	▲ Rewrite rational expressions.	▲ Summarize, represent, and interpret data on a single count or measurement variable.
● Use complex numbers in polynomial identities and equations.	**Interpreting Functions (F-IF)**	▲ Summarize, represent, and interpret data on two categorical and quantitative variables.
Seeing Structure in Expressions (A-SSE)	● Interpret functions that arise in applications in terms of the context.	● Interpret models.
▲ Interpret the structure of expressions.	● Analyze functions using different representations.	**Making Inferences and Justifying Conclusions (S-IC)**
▲ Write expressions in equivalent forms to solve problems.	**Building Functions (F-BF)**	▲ Understand and evaluate random processes underlying statistical experiments.
Reasoning with Equations and Inequalities (A-REI)	● Build a function that models a relationship between two quantities.	▲ Make inferences and justify conclusions from experiments, and observational studies.
● Understand solving equations as a process of reasoning and explain the reasoning.	▲ Build new functions from existing functions.	**Conditional Probability and the Rules of Probability (S-CP)**
● Solve equations and inequalities in one variable.	**Linear, Quadratic, and Exponential Models (F-LE)**	▲ Understand independence and conditional probability and use them to interpret data.
▲ Solve systems of equations.	● Construct and compare linear, quadratic, and exponential models and solve problems.	● Use the rules of probability to compute probabilities of compound events in a uniform probability model.
▲ Represent and solve equations and inequalities graphically.	● Interpret expressions for functions in terms of the situation they model.	

Balance of Rigor in the Math Classroom

"Tasks that ask students to perform a memorized procedure in a routine manner lead to one type of opportunity for student thinking; tasks that require students to think conceptually and that stimulate students to make connections lead to a different set of opportunities for student thinking."
(Stein & Smith, 1998)

Defining Standards, Curriculum and Instruction

Standards – What a student needs to know, understand, and be able to do by the end of each grade. Standards build across grade levels in a progression of increasing understanding and through a range of cognitive demand levels. Standards are adopted at the state level by the State Board of Education.

Curriculum – The resources used for teaching and learning the standards. Curricula are adopted at a local level by districts and schools.

Instruction – The methods used by teachers to teach their students. Instructional techniques are employed by individual teachers in response to the needs of the students in their classes to help them progress through the curriculum in order to master the standards.

The Standards for Mathematical Practice

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

The Arizona Mathematics Standards has included narratives for each of the 8 Mathematical Practices.

Comparison of Arizona Mathematics Standards – 2010 to 2016

Algebra 2				
Number and Quantity – N				
The Real Number System (N-RN)				
Adopted 2010		**Adopted 2016**		
HS.N-RN.A Extend the properties of exponents to rational exponents.	**HS.N-RN.A.1.** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*	**A2.N-RN.A Extend the properties of exponents to rational exponents.**	**A2.N-RN.A.1**	Explain how the definition of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
	HS.N-RN.A.2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.		**A2.N-RN.A.2**	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
Number and Quantity – N				
Quantities (N-Q)				
HS.N-Q.A Reason quantitatively and use units to solve problems.		**A2.N-Q.A Reason quantitatively and use units to solve problems.**	**A2.N-Q.A.1**	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays, include utilizing real-world context.
	HS.N-Q.A.2. Define appropriate quantities for the purpose of descriptive modeling.		**A2.N-Q.A.2**	Define appropriate quantities for the purpose of descriptive modeling. Include problem-solving opportunities utilizing real-world context.
			A2.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities utilizing real-world context.

Number and Quantity – N				
The Complex Number System (N-CN)				
HS.N-CN.A Perform arithmetic operations with complex numbers.	**HS.N-CN.A.1.** Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	**A2.N-CN.A Perform arithmetic operations with complex numbers.**	**A2.N-CN.A.1**	Apply the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. Write complex numbers in the form $(a+bi)$ with a and b real.
HS.N-CN.C Use complex numbers in polynomial identities and equations.	**HS.N-CN.C.7.** Solve quadratic equations with real coefficients that have complex solutions.	**A2.N-CN.C Use complex numbers in polynomial identities and equations.**	**A2.N-CN.C.7**	Solve quadratic equations with real coefficients that have complex solutions.
Algebra – A				
Seeing Structure in Expressions (A-SSE)				
HS.A-SSE.A Interpret the structure of expressions.	**HS.A-SSE.A.2.** Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*	**A2.A-SSE.A Interpret the structure of expressions.**	**A2.A-SSE.A.2**	Use structure to identify ways to rewrite polynomial and rational expressions. Focus on polynomial operations and factoring patterns.
HS-A-SSE.B Write expressions in equivalent forms to solve problems.	**HS.A-SSE.B.3.** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%*	**A2.A-SSE.B Write expressions in equivalent forms to solve problems.**	**A2.A-SSE.B.3**	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Include problem-solving opportunities utilizing real-world context and focus on expressions with rational exponents. c. Use the properties of exponents to transform expressions for exponential functions.

	HS.A-SSE.B.4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*		**A2.A-SSE.B.4**	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*
center	*Algebra – A*				
center	**Arithmetic with Polynomials and Rational Expressions (A-APR)**				
HS.A-APR.B Understand the relationship between zeros and factors of polynomials.	**HS.A-APR.B.2.** Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	**A2.A-APR.B Understand the relationship between zeros and factors of polynomials.**	**A2.A-APR.B.2**	Know and apply the Remainder and Factor Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $(x - a)$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
	HS.A-APR.B.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.		**A2.A-APR.B.3**	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. Focus on quadratic, cubic, and quartic polynomials including polynomials for which factors are not provided
HS.A-APR.C Use polynomial identities to solve problems.	**HS.A-APR.C.4.** Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2+y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.*	**A2.A-APR.C Use polynomial identities to solve problems.**	**A2.A-APR.C.4**	Prove polynomial identities and use them to describe numerical relationships.
	HS.A-APR.D.6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection,	**A2.A-APR.D Rewrite rational expressions.**	**A2.A-APR.D.6**	Rewrite rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x), b(x), q(x),$ and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or for the more complicated examples, a computer algebra system.

	long division, or, for the more complicated examples, a computer algebra system.			

Algebra – A				
Creating Equations (A-CED)				
HS.A-CED.A Create equations that describe numbers or relationships.	**HS.A-CED.A.1.** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	**A2.A-CED.A Create equations that describe numbers or relationships.**	**A2.A-CED.A.1**	Create equations and inequalities in one variable and use them to solve problems. Include problem-solving opportunities utilizing real-world context. Focus on equations and inequalities arising from linear, quadratic, rational, and exponential functions.

Algebra – A				
Reasoning with Equations and Inequalities (A-REI)				
HS.A-REI.A Understand solving equations as a process of reasoning and explain the reasoning.	**HS.A-REI.A.1.** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	**A2.A-REI.A Understand solving equations as a process of reasoning and explain the reasoning.**	**A2.A-REI.A.1**	Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Extend from quadratic equations to rational and radical equations.
	HS.A-REI.A.2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.		**A2.A-REI.A.2**	Solve rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
HS.A-REI.B Solve equations and inequalities in one variable.	**HS.A-REI.B.4.** Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and	**A2.A-REI.B Solve equations and inequalities in one variable.**	**A2.A-REI.B.4**	Fluently solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

	factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.			
HS.A-REI.C Solve systems of equations.	**HS.A-REI.C.7.** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line y = −3x and the circle $x^2 + y^2 = 3$.*	**A2.A-REI.C Solve systems of equations.**	**A2.A-REI.C.7**	Solve a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line y = -3x and the circle $x^2 + y^2 = 3$.*
HS.A-REI.D Represent and solve equations and inequalities graphically.	**HS.A-REI.D.11.** Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	**A2.A-REI.D Represent and solve equations and inequalities graphically.**	**A2.A-REI.D.11**	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) =g(x)$; find the solutions approximately (e.g., using technology to graph the functions, make tables of values, or find successive approximations). Include problems utilizing real-world context. Extend from linear, quadratic, and exponential functions to cases where $f(x)$ and/or $g(x)$ are polynomial, rational, exponential, and logarithmic functions.

Functions - F
Interpreting Functions (F-IF)
HS.F-IF.B Interpret functions that arise in applications in terms of the context.	**HS.F-IF.B.4.** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*	**A2.F-IF.B Interpret functions that arise in applications in terms of the context.**	**A2.F-IF.B.4**	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Include problem-solving opportunities utilizing real-world context. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.
	HS.F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.		**A2.F-IF.B.6**	Calculate and interpret the average rate of change of a continuous function (presented symbolically or as a table) on a closed interval. Estimate the rate of change from a graph. Include problem-solving opportunities utilizing real-world context. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.
HS.F-IF.C Analyze functions using different representations.	**HS.F-IF.C.7.** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. e. Graph exponential and	**A2.F-IF.C Analyze functions using different representations.**	**A2.F-IF.C.7**	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.

	logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.			
	HS.F-IF.C.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify* percent rate of change *in functions such as y = (1.02)^t, y = (0.97)^t, y = (1.01)^{12t}, y = (1.2)$^{t/10}$, and classify them as representing exponential growth or decay.*		**A2.F-IF.C.8**	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Use the properties of exponents to interpret expressions for exponential functions and classify those functions as exponential growth or decay.
	HS.F-IF.C.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*		**A2.F-IF.C.9**	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions.). Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.

Functions - F				
Building Functions (F-BF)				
HS.F-BF.A **Build a function that models a relationship between two quantities.**	**HS.F-BF.A.1.** Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*	**A2.F-BF.A** **Build a function that models a relationship between two quantities.**	**A2.F-BF.A.1**	Write a function that describes a relationship between two quantities. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions. Include problem-solving opportunities utilizing real-world context. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine function types using arithmetic operations and function composition.
	HS.F-BF.A.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.		**A2.F-BF.A.2**	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
HS.F-BF.B **Build new functions from existing functions.**	**HS.F-BF.B.3.** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions*	**A2.F-BF.B** **Build new functions from existing functions.**	**A2.F-BF.B.3**	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.

	for them.			
	HS.F-BF.B.4 Find inverse functions. a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) =2 x^3$ or $f(x) = (x+1)/(x-1)$ for $x ≠ 1$.*		**A2.F-BF.B.4**	Find inverse functions. a. Understand that an inverse function can be obtained by expressing the dependent variable of one function as the independent variable of another, recognizing that functions f and g are inverse functions if and only if $f(x) = y$ and $g(y) = x$ for all values of x in the domain of f and all values of y in the domain of g. b. Understand that if a function contains a point (a,b), then the graph of the inverse relation of the function contains the point (b,a). c. Interpret the meaning of and relationship between a function and its inverse utilizing real-world context.

Functions - F

Linear, Quadratic, and Exponential Models (F-LE)

HS.F-LE.A **Construct and compare linear, quadratic, and exponential models and solve problems.**	**HS.F-LE.A.4.** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	**A2.F-LE.A** **Construct and compare linear, quadratic, and exponential models and solve problems.**	**A2.F-LE.A.4**	For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithms that are not readily found by hand or observation using technology.
	HS.F-LE.B.5. Interpret the parameters in a linear or exponential function in terms of a context.	**A2.F-LE.B** **Interpret expressions for functions in terms of the situation they model.**	**A2.F-LE.B.5**	Interpret the parameters in an exponential function with rational exponents utilizing real-world context.

Functions - F				
Trigonometric Functions (F-TF)				
HS.F-TF.A **Extend the domain of trigonometric functions using the unit circle.**	**HS.F-TF.A.1.** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	**A2.F-TF.A** **Extend the domain of trigonometric functions using the unit circle.**	**A2.F-TF.A.1**	Understand radian measure of an angle as the length of the arc on any circle subtended by the angle, measured in units of the circle's radius.
	HS.F-TF.A.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.		**A2.F-TF.A.2**	Explain how the unit circle in the coordinate plane enables the extension of sine and cosine functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
HS.F-TF.B **Model periodic phenomena with trigonometric functions.**	**HS.F-TF.B.5.** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	**A2.F-TF.B** **Model periodic phenomena with trigonometric functions.**	**A2.F-TF.B.5**	Create and interpret sine, cosine and tangent functions that model periodic phenomena with specified amplitude, frequency, and midline.
A2.F-TF.C **Prove and apply trigonometric identities.**	**HS.F-TF.C.8.** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.	**A2.F-TF.C** **Apply trigonometric identities.**	**A2.F-TF.C.8**	Use the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

Statistics and Probability - S				
Interpreting Categorical and Quantitative Data (S-ID)				
HS.S-ID.A Summarize, represent, and interpret data on a single count or measurement variable.	**HS.S-ID.A.4.** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	**A2.S-ID.A Summarize, represent, and interpret data on a single count or measurement variable.**	**A2.S-ID.A.4**	Use the mean and standard deviation of a data set to fit it to a normal curve, and use properties of the normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, or tables to estimate areas under the normal curve.
HS.S-ID.B Summarize, represent, and interpret data on two categorical and quantitative variables.	**HS.S-ID.B.6.** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or chooses a function suggested by the context. Emphasize linear, quadratic, and exponential models.*	**A2.S-ID.B Summarize, represent, and interpret data on two categorical and quantitative variables.**	**A2.S-ID.B.6**	Represent data of two quantitative variables on a scatter plot, and describe how the quantities are related. Extend to polynomial and exponential models. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Uses given functions, or choose a function, suggested by the context.
		A2.S-ID.C Interpret models.	**A2.S-ID.C.10**	Interpret parameters of exponential models.

Statistics and Probability - S				
Making Inferences and Justifying Conclusions (S-IC)				
HS.S-IC.A **Understand and evaluate random processes underlying statistical experiments.**	**HS.S-IC.A.1.** Understand statistics as a process for making inferences to be made about population parameters based on a random sample from that population.	**A2.S-IC.A** **Understand and evaluate random processes underlying statistical experiments.**	**A2.S-IC.A.1**	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
	HS.S-IC.A.2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin will fall heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*		**A2.S-IC.A.2**	Explain whether a specified model is consistent with results from a given data-generating process.
HS.S-IC.B **Make inferences and justify conclusions from sample surveys, experiments, and observational studies.**	**HS.S-IC.B.3.** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	**A2.S-IC.B** **Make inferences and justify conclusions from experiments, and observational studies.**	**A2.S-IC.B.3**	Recognize the purposes of and differences between designed experiments, sample surveys and observational studies.
	HS.S-IC.B.4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.		**A2.S-IC.B.4**	Use data from a sample survey to estimate a population mean or proportion; recognize that estimates are unlikely to be correct and the estimates will be more precise with larger sample sizes.

	HS.S-IC.B.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.			
	HS.S-IC.B.6 Evaluate reports based on data.			

		Statistics and Probability - S		
		Conditional Probability and the Rules of Probability (S-CP)		
HS.S-CP.A Understand independence and conditional probability and use them to interpret data.	**HS.S-CP.A.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	**A2.S-CP.A Understand independence and conditional probability and use them to interpret data.**		
	HS.S-CP.A.2 Use the Multiplication Rule for independent events to understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.			
	HS.S-CP.A.3. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.		**A2.S-CP.A.3**	Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of A, and the conditional probability of *B* given *A* is the same as the probability of *B*.

	HS.S-CP.A.4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities*. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*		**A2.S-CP.A.4**	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
	HS.S-CP.A.5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*		**A2.S-CP.A.5**	Recognize and explain the concepts of conditional probability and independence utilizing real-world context.
HS.S-CP.B **Use the rules of probability to compute probabilities of compound events in a uniform probability model.**	**HS.S-CP.B.6.** Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.	**A2.S-CP.B** **Use the rules of probability to compute probabilities of compound events in a uniform probability model.**	**A2.S-CP.B.6**	Use Bayes Rule to find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.

	HS.S-CP.B.7. Apply the Addition Rule, $P(A$ or $B) = P(A) + P(B) − P(A$ and $B)$, and interpret the answer in terms of the model.	**A2.S-CP.B.7**	Apply the Addition Rule, $P(A$ or $B) = P(A) + P(B) − P(A$ and $B)$, and interpret the answer in terms of the model.
		A2.S-CP.B.8	Apply the general Multiplication Rule in a uniform probability model, $P(A$ and $B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model.

Standards for Mathematical Practice

A2.MP.1 Make sense of problems and persevere in solving them.
Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, "Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.

A2.MP.2 Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.

A2.MP.3 Construct viable arguments and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming or questioning the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.

A2.MP.4 Model with mathematics.
Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

A2.MP.5 Use appropriate tools strategically.
Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.

A2.MP.6 Attend to precision.
Mathematically proficient students clearly communicate to others using appropriate mathematical terminology, and craft explanations that convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.

A2.MP.7 Look for and make use of structure.
Mathematically proficient students use structure and patterns to assist in making connections among mathematical ideas or concepts when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.

A2.MP.8 Look for and express regularity in repeated reasoning.
Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.