AzM2 | Arizona's Statewide Achievement Assessment for English Language Arts and Mathematics

Mathematics Item Specifications

GRADE 5

Arizona Department of Education with American Institutes for Research - 2019

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Introduction

The Arizona Statewide Achievement Assessment for English Language Arts and Mathematics (AzM2) is Arizona's statewide achievement test. AzM2 assesses the Arizona English Language Arts Standards and Arizona Mathematics Standards adopted by the Arizona State Board of Education in December 2016. AzM2 will inform students, teachers, and parents about preparedness for college and careers upon graduating from high school. AzM2 tests are computer-based, meaning that they can better assess students' critical thinking skills and provide them with opportunities to demonstrate a deeper understanding of the materials. Computer-based testing also allows for the use of a variety of innovative items types.

During the item-development process, all AzM2 items are written in accordance with the Item Specifications and are reviewed and approved by a committee of Arizona educators to confirm alignment and appropriateness for inclusion in the test. AzM2 items are generally representative of Arizona's geographic regions and culturally diverse population. Items are reviewed for the following kinds of bias: gender, racial, ethnic, linguistic, religious, geographic, and socioeconomic. Item reviews also include consideration of issues related to individuals with disabilities. Arizona community members also have an opportunity to review items for issues of potential concern to members of the community at large. Reviewers are asked to consider the variety of cultural, regional, philosophical, political, and religious backgrounds throughout Arizona, and then to determine whether the subject matter will be acceptable to Arizona students, families, and other members of Arizona communities.

This *AzM2 Item Specifications* is a resource document that defines the content and format of the test and test items for item writers and reviewers. Each *Item Specifications* document indicates the alignment of items with the Arizona Mathematics Standards. It also serves to provide all stakeholders with information about the scope and function of assessment items. This document can also serve to assist educators to understand how assessment items are developed in alignment with the standards for English language arts and math. These item specifications for AzM2 are intended to provide information regarding standards, item formats and response types. The descriptions of blueprints, and depth of knowledge in this document are meant to provide an overview of the test. Item specifications are meant for the purposes of assessment, not instruction. They are not intended to be tools for instruction or the basis for curricula. AzM2 has a test blueprint that was developed by Arizona and is different from any other state or consortium test blueprint.

For the math portion of AzM2, all of the test questions are aligned to the mathematic content standards for these subject areas. Any item specifications that are absent for standards listed in this document may be under development. This document does not endorse the exclusion of the instruction of any grade-level content standards. The test will ask questions that check a student's conceptual understanding of math as well as their procedural skills. These items have been written to be free from bias and sensitivity, and widely vary in their degree of difficulty.

Item Development Process

AzM2 items go through a rigorous review before they are operational. When an item is "operational" it means it is used to determine a student's score on the assessment. This is a description of the process every item must go through before it is operational on AzM2.

Sample tests are available online for the math portion of AzM2. For more information view the Guide to the Sample Tests at www.AzM2portal.org.

Test Construction Guidelines

The construction of the AzM2 assessment is guided by the depth and rigor of the Arizona College and Career Ready Standards. Items are created to address key components of the standards and assess a range of important skills. The AzM2 Blueprint provides an overview of the distribution of items on the AzM2 according to the standards. The standards for Math Practices are embedded within all AzM2 items. Further, the AzM2 blueprint outlines the Depth of Knowledge distribution of items.

Blueprint

Grade 5 AzM2 Math Blueprint 2016 Standards		
Reporting Category	**Min.**	**Max.**
Operations & Algebraic Thinking and Numbers & Operations in Base Ten	**38%**	**42%**
Numbers in Base Ten	31%	35%
Algebraic Thinking	4%	8%
Number and Operations-Fractions	**31%**	**35%**
Measurement, Data, and Geometry	**24%**	**28%**
Measurement and Data	18%	20%
Geometry	7%	11%

Depth of Knowledge (DOK)

DOK refers to the level of rigor or sophistication of the task in a given item, designed to reflect the complexity of the Arizona Mathematics Standards. Items at DOK level 1 focus on the recall of information, such as definitions, terms, and simple procedures. Items at DOK 2 require students to make decisions, solve problems, or recognize patterns; in general, they require a greater degree of engagement and cognitive processing than items at DOK 1. Items at DOK 3 feature higher-order cognitive tasks that assess students' capacities to approach abstract or complex problems.

Percentage of Points by Depth of Knowledge (DOK) Level			
Grade 5	DOK Level 1	DOK Level 2	DOK Level 3
	10% - 20%	60% - 70%	12% - 30%

For more information on DOK go to www.azed.gov/AzM2.

Calculators

Arizona Desmos Graphing Calculator is not permitted for the paper-based and computer-based assessment for Grade 5 Math.

Item Formats

The AzM2 Assessments are composed of item formats that include traditional multiple- choice response items and technology-enhanced response items (TEI). TEIs are computer- delivered response items that require students to interact with test content to select, construct, and/or support their responses. TEIs are better able to assess a deeper level of understanding.

Currently, there are nine types of TEIs that may appear on the Math computer-based assessment for AzM2:

- Editing Tasks (ET)
- Editing Task Choice (ETC)
- Equation Editor (EQ)
- Graphic Response Item Display (GRID)
- Hot Text (HT)
    - Selectable Hot Text
    - Drag-and-Drop Hot Text
- Matching Item (MI)
- Multi-Select (MS)
- Open Response
- Table Item (TI)

For paper-based assessments (including those for students with an IEP or 504 plan that specifies a paper based accommodation), TEIs will be modified so that they can be scanned and scored electronically or hand-scored.

See the table below for a description of each TEI. In addition, for examples of each response item format described, see the AzM2 Training Tests at www.AzM2portal.org.

Item Format	Description
Editing Task (ET)	The student clicks on a highlighted word or phrase that may be incorrect, which reveals a text box. The directions in the text box direct the student to replace the highlighted word or phrase with the correct word or phrase. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Editing Task Choice (ETC)	The student clicks a highlighted word or phrase, which reveals a drop-down menu containing options for correcting an error as well as the highlighted word or phrase as it is shown in the sentence to indicate that no correction is needed. The student then selects the correct word or phrase from the drop-down menu. For paper-based assessments, the item is modified so that it can be scanned and scored electronically. The student fills in a circle to indicate the correct word or phrase.

Item Format	Description
Equation Editor (EQ)	The student is presented with a toolbar that includes a variety of mathematical symbols that can be used to create a response. Responses may be in the form of a number, variable, expression, or equation, as appropriate to the test item. For paper-based assessments, this item type may be replaced with a modified version of the item that can be scanned and scored electronically or replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Graphic Response Item Display (GRID)	The student selects numbers, words, phrases, or images and uses the drag-and-drop feature to place them into a graphic. This item type may also require the student to use the point, line, or arrow tools to create a response on a graph. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Hot Text (HT)	**Selectable Hot Text -** Excerpted sentences from the text are presented in this item type. When the student hovers over certain words, phrases, or sentences, the options highlight. This indicates that the text is selectable ("hot"). The student can then click on an option to select it. For paper- based assessments, a "selectable" hot text item is modified so that it can be scanned and scored electronically. In this version, the student fills in a circle to indicate a selection.
	Drag-and-Drop Hot Text - Certain numbers, words, phrases, or sentences may be designated "draggable" in this item type. When the student hovers over these areas, the text highlights. The student can then click on the option, hold down the mouse button, and drag it to a graphic or other format. For paper-based assessments, drag-and-drop hot text items will be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Matching Item (MI)	The student checks a box to indicate if information from a column header matches information from a row. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Multi-Select (MS)	The student is directed to select all of the correct answers from among a number of options. These items are different from multiple-choice items, which allow the student to select only one correct answer. These items appear in the online and paper-based assessments.
Open Response	The student uses the keyboard to enter a response into a text field. These items can usually be answered in a sentence or two. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.

Item Format	Description
Table Item (TI)	The student types numeric values into a given table. The student may complete the entire table or portions of the table depending on what is being asked. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.

Arizona Math Standards

Operations and Algebraic Thinking (OA)		
5.OA.A **Write and interpret numerical expressions.**	**5.OA.A.1**	Use parentheses and brackets in numerical expressions, and evaluate expressions with these symbols (Order of Operations).
	5.OA.A.2	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them (e.g., express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x (18,932 + 921) is three times as large as 18,932 + 921, without having to calculate the indicated sum or product).
5.OA.B **Analyze patterns and relationships.**	**5.OA.B.3**	Generate two numerical patterns using two given rules (e.g., generate terms in the resulting sequences). Identify and explain the apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane (e.g., given the rule "add 3" and the starting number 0, and given the rule "add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence).
	5.OA.B.4	Understand primes have only two factors and decompose numbers into prime factors.
Number and Operations in Base Ten (NBT)		
5.NBT.A **Understand the place value system.**	**5.NBT.A.1**	Apply concepts of place value, multiplication, and division to understand that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
	5.NBT.A.2	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10.
	5.NBT.A.3	Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form. b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
	5.NBT.A.4	Use place value understanding to round decimals to any place.
5.NBT.B **Perform operations with multi-digit whole numbers and with decimals to hundredths.**	**5.NBT.B.5**	Fluently multiply multi-digit whole numbers using a standard algorithm.
	5.NBT.B.6	Apply and extend understanding of division to find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors.
	5.NBT.B.7	Add, subtract, multiply, and divide decimals to hundredths, connecting objects or drawings to strategies based on place value, properties of operations, and/or the relationship between operations. Relate the strategy to a written form.

Number and Operations – Fractions (NF)		
5.NF.A **Use equivalent fractions to add and subtract fractions.**	**5.NF.A.1**	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators (e.g., 2/3 + 5/4 = 8/12 + 15/12 = 23/12).
	5.NF.A.2	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by using a variety of representations, equations, and visual models to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers (e.g. recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2).
5.NF.B **Use previous understandings of multiplication and division to multiply and divide fractions.**	**5.NF.B.3**	Interpret a fraction as the number that results from dividing the whole number numerator by the whole number denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people, each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*
	5.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number and a fraction by a fraction. a. Interpret the product (*a/b*) x *q* as *a* parts of a partition of *q* into *b* equal parts. *For example, use a visual fraction model to show (2/3) x 4 = 8/3, and create a story context for this equation.* b. Interpret the product of a fraction multiplied by a fraction (*a/b*) x (*c/d*). Use a visual fraction model and create a story context for this equation. *For example, use a visual fraction model to show (2/3) x (4/5) = 8/15, and create a story context for this equation.* In general, (*a/b*) x (*c/d*) = *ac/bd*. c. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
	5.NF.B.5	Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number; explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $\frac{a}{b} = \frac{n \times a}{n \times b}$ to the effect of multiplying $\frac{a}{b}$ by 1.
5.NF.B (cont.)	**5.NF.B.6**	Solve problems in real-world contexts involving multiplication of fractions, including mixed numbers, by using a variety of representations including equations and models.
	5.NF.B.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. Use the relationship between multiplication and division to justify conclusions. b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient.* Use the relationship between multiplication and division to justify conclusions (e.g., 4 ÷ (1/5) = 20 because 20 x (1/5) = 4). c. Solve problems in real-world context involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, using a variety of representations.
Measurement and Data (MD)		
5.MD.A **Convert like measurement units within a given measurement system.**	**5.MD.A.1**	Convert among different-sized standard measurement units within a given measurement system, and use these conversions in solving multi-step, real-world problems.
5.MD.B **Represent and interpret data.**	**5.MD.B.2**	Make a line plot to display a data set of measurements in fractions of a unit (1/8, 1/2, 3/4). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*
5.MD.C **Geometric measurement: Understand concepts of volume and relate volume to multiplication and to addition.**	**5.MD.C.3**	Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.
	5.MD.C.4	Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

5.MD.C (cont.)	**5.MD.C.5**	Relate volume to the operations of multiplication and addition and solve mathematical problems and problems in real-world contexts involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes (e.g., to represent the associative property of multiplication). b. Understand and use the formulas $V = l \times w \times h$ and $V = B \times h$, where in this case B is the area of the base ($B = l \times w$), for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths to solve mathematical problems and problems in real-world contexts. c. Understand volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms, applying this technique to solve mathematical problems and problems in real-world contexts.
Geometry (G)		
5.G.A **Graph points on the coordinate plane to solve mathematical problems as well as problems in real-world context.**	**5.G.A.1**	Understand and describe a coordinate system as perpendicular number lines, called axes, that intersect at the origin (0 , 0). Identify a given point in the first quadrant of the coordinate plane using an ordered pair of numbers, called coordinates. Understand that the first number (x) indicates the distance traveled on the horizontal axis, and the second number (y) indicates the distance traveled on the vertical axis.
	5.G.A.2	Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.
5.G.B **Classify two-dimensional figures into categories based on their properties.**	**5.G.B.3**	Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.
	5.G.B.4	Classify two-dimensional figures in a hierarchy based on properties.

Grade 5 Item Specifications

Measurement and Data & Geometry

5.G.A.1

Content Standards	Understand and describe a coordinate system as perpendicular number lines, called axes, that intersect at the origin (0, 0). Identify a given point in the first quadrant of the coordinate plane using an ordered pair of numbers, called coordinates. Understand that the first number (*x*) indicates the distance traveled on the horizontal axis, and the second number (*y*) indicates the distance traveled on the vertical axis.
Explanations	Graph points on the coordinate plane to solve real-world and mathematical problems.
Content Limits	Whole numbers Use only points located in the first quadrant of the coordinate plane. Plotting points given the ordered pair is aligned to 5.G.2
Context	Context is not allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to find the coordinates of a point based on its distance from the origin in the direction of the axes.	• Graphic Response • Multiple Choice Response • Multi-Select Response
Students will be required to plot a point based on its distance from the origin in the direction of the axes.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify the axes and the origin (0 , 0) of a coordinate system. Identify the *x*- and *y*-coordinates of an ordered pair.	Describe a coordinate system as having 2 axes that intersect at the origin (0 , 0). Identify an ordered pair and the x- and y-coordinates of an ordered pair.
Proficient	**Highly Proficient**
Understand and describe a coordinate system as perpendicular number lines, called axes, that intersect at the origin (0 , 0). Identify a given point in the first quadrant of the coordinate plane using an ordered pair of numbers, called coordinates. Understand that the first number (*x*) indicates the distance traveled on the horizontal axis, and the second number (*y*) indicates the distance traveled on the vertical axis.	Understand and describe a coordinate system. Identify points in the coordinate plane using coordinates. Explain that the x-coordinate indicates the distance traveled on the horizontal axis, and the y-coordinate indicates the distance traveled on the vertical axis.

5.G.A.2

Content Standards	Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.
Explanations	Graph points on the coordinate plane to solve real-world and mathematical problems.
Content Limits	Whole numbers Use only points located in the first quadrant of the coordinate plane. Mathematical and real-world problems must have axes scaled to whole numbers (not letters).
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to find the coordinates of a point based on a graphed point in a coordinate plane.	• Graphic Response • Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to plot points based on given coordinates.	
Students will be required to plot points based on the relationship between their locations on the coordinate plane.	
Students will be required to identify how many units and which direction one point is from another point.	
Students will be required to interpret meaning of coordinate values within a context (axes indicate specific units).	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify points graphed in the first quadrant of the coordinate plane.	Graph points in the first quadrant of the coordinate plane, and identify the coordinate values of points in the context of the situation.
Proficient	**Highly Proficient**
Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	Create real-world and mathematical problems that can be solved by graphing points in the first quadrant of the coordinate plane. Explain the meaning of the coordinate values of points in the context of the situation.

5.G.B.3

Content Standards	Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.
Explanations	Geometric properties include properties of sides (parallel, perpendicular, congruent), properties of angles (type, measurement, congruent), and properties of symmetry (point and line). Properties of figure may include: Properties of sides - parallel, perpendicular, congruent, number of sides - or properties of angles - types of angles, congruent.
Content Limits	Focus should be on quadrilaterals, although other polygons can be included as well. There are two competing definitions for trapezoids - one that requires exactly one pair or parallel sides, and another that requires at least one pair of parallel sides (using this definition, parallelograms are trapezoids). Some students are taught one definition, others, the other. Thus, items that require the student to choose a definition in order to arrive at the correct answer should be avoided. Do not use Venn diagrams to represent hierarchy.
Context	Context is not allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to select shapes based on the attributes of a specific category.	• Graphic Response • Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to select attributes that categories share.	
Students will be required to select shapes that can be treated the same way as shapes in an upper category.	
Students will be required to show a hierarchy of shapes categorized by their attributes.	
Students will be required to select the categories a shape belongs.	
Students will be required to select shapes belonging to a particular subcategory.	
Students will be required to support/refute statements about categorizing shapes.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify attributes belonging to a category of two-dimensional figures.	Recognize that attributes belonging to a category of two-dimensional figures also belong to a subcategory of that category.
Proficient	**Highly Proficient**
Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.	Explain why attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.

5.G.B.4

Content Standards	Classify two-dimensional figures in a hierarchy based on properties.
Explanations	Geometric properties include properties of sides (parallel, perpendicular, congruent), properties of angles (type, measurement, congruent), and properties of symmetry (point and line). Properties of figure may include: Properties of sides - parallel, perpendicular, congruent, number of sides - or properties of angles - types of angles, congruent
Content Limits	Focus should be on quadrilaterals, although other polygons can be included as well. There are two competing definitions for trapezoids - one that requires exactly one pair or parallel sides, and another that requires at least one pair of parallel sides (using this definition, parallelograms are trapezoids). Some students are taught one definition, others, the other. Thus, items that require the student to choose a definition in order to arrive at the correct answer should be avoided. Do not use Venn diagrams to represent hierarchy.
Context	Context is not allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to select shapes based on the attributes of a specific category.	• Graphic Response • Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to select attributes that categories share.	
Students will be required to select shapes that can be treated the same way as shapes in an upper category.	
Students will be required to show a hierarchy of shapes categorized by their attributes.	
Students will be required to select the categories a shape belongs.	
Students will be required to select shapes belonging to a particular subcategory.	
Students will be required to support/refute statements about categorizing shapes.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify two-dimensional figures based on properties limited to sides and angles.	Classify two-dimensional figures based on properties limited to sides and angles.
Proficient	**Highly Proficient**
Classify two-dimensional figures in a hierarchy based on properties.	Draw or construct two-dimensional figures based on properties or classifications.

5.MD.A.1

Content Standards	Convert among different-sized standard measurement units within a given measurement system, and use these conversions in solving multi-step, real-world problems.
Explanations	In fifth grade, students build on their prior knowledge of related measurement units to determine equivalent measurements. Prior to making actual conversions, they examine the units to be converted, determine if the converted amount will be more or less units than the original unit, and explain their reasoning. They use several strategies to convert measurements. When converting metric measurement, students apply their understanding of place value and decimals.
Content Limits	Measurement values can be whole, decimal, and/or fractional values. Conversion is within the same system. Units of measurement include: kilometer, meter, centimeter, millimeter, liter, milliliter, kilogram, gram, milligram, mile, yard, foot, inch, gallon, quart, pint, cup, ton, pound, and ounce.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to calculate a measurement conversion within a problem with no context.	• Equation Response • Multiple Choice Response • Multi-Select Response • Table Response
Students will be required to solve a real world problem involving measurement conversions.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify equivalent, different-sized standard measurement units within a given measurement system, and use these conversions in solving one-step, real-world problems.	Convert among different-sized standard measurement units within a given measurement system, and use these conversions in solving two-step, real-world problems.
Proficient	**Highly Proficient**
Convert among different-sized standard measurement units within a given measurement system, and use these conversions in solving multi-step, real-world problems.	Create multi-step, real-world problems that require converting among different-sized standard measurement units within a given measurement system.

5.MD.B.2

Content Standards	Make a line plot to display a data set of measurements in fractions of a unit (1/8, 1/2, 3/4). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*
Explanations	Students apply their understanding of operations with fractions. They use either addition and/or multiplication to determine the total number of liters in the beakers. Then the sum of the liters is shared evenly among the ten beakers.
Content Limits	Measurement units are limited to halves, quarters, and eighths. Division is limited to a whole number divided by a unit fraction or a unit fraction divided by a whole number.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to construct a line plot based on given data comprised of fractions.	• Equation Response • Graphic Response • Multiple Choice Response • Table Response
Students will be required to interpret data in a line plot to solve problems involving addition, subtraction, multiplication, and division of fractions.	
Students will be required to interpret data in a line plot to solve problems involving addition, subtraction, multiplication, and division of unit fractions, where information is not fully provided.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify a line plot that displays a data set of measurements in fractions of a unit (1/2, 1/4). Use operations on fractions for this grade to identify solutions to one-step problems involving information presented in line plots.	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4). Use operations on fractions for this grade to solve one- or two-step problems involving information presented in line plots.
Proficient	**Highly Proficient**
Make a line plot to display a data set of measurements in fractions of a unit (1/8, 1/2, 3/4). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*	Make a line plot to display a data set of measurements in fractions of a unit. Use operations on fractions for this grade to solve multi-step problems involving information presented in line plots.

5.MD.C.3, 5.MD.C.3a, and 5.MD.C.3b

Content Standards	**5.MD.C.3** Recognize volume as an attribute of solid figures and understand concepts of volume measurement. **5.MD.C.3a** A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. **5.MD.C.3b** A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of *n* cubic units.
Explanations	Students' prior experiences with volume were restricted to liquid volume. As students develop their understanding volume they understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. This cube has a length of 1 unit, a width of 1 unit and a height of 1 unit and is called a cubic unit. This cubic unit is written with an exponent of 3 (e.g., in3, m3). Students connect this notation to their understanding of powers of 10 in our place value system. Models of cubic inches, centimeters, cubic feet, etc., are helpful in developing an image of a cubic unit. Student's estimate how many cubic yards would be needed to fill the classroom or how many cubic centimeters would be needed to fill a pencil box.
Content Limits	Right rectangular prisms with whole-number side lengths Graphics include unit cube Labels can include cubic units (i.e. cubic centimeters, cubic feet, etc) or exponential units (i.e. cm3, ft3, etc.)
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to recognize volume as an attribute of solid figures. (3a, 3b)	• Equation Response • Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to identify a unit cube as 1 cubic unit of volume. (3a)	
Students will be required to recognize the use of n unit cubes packed in a solid figure to find the volume of that figure in n cubic units. (3b)	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. Identify a “unit cube,” and know that it can be used to measure volume. b. Match the number of unit cubes it takes to pack a solid figure without gaps or overlaps to the volume of the figure.	Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. Define a “unit cube” and "one cubic unit." b. Identify that a solid figure which can be packed without gaps or overlaps using n unit cubes, and thus has a volume of n cubic units.
Proficient	**Highly Proficient**
Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.	Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. Explain why a cube with side length 1 unit, called a “unit cube,” and why it is said to have “one cubic unit” of volume. b. Explain why a solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

5.MD.C.4

Content Standards	Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
Explanations	Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.
Content Limits	Right rectangular prisms with whole-number side lengths Graphics include unit cube Labels can include cubic units (i.e. cubic centimeters, cubic feet, etc) or exponential units (i.e. cm3, ft3, etc.)
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to identify the volume of a rectangular prism by counting unit cubes, and compare volumes of multiple prisms.	• Equation Response • Multiple Choice Response • Matching Item Response • Multi-Select Response

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify volumes by counting unit cubes.	Measure volumes by counting unit cubes.
Proficient	**Highly Proficient**
Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	Look for patterns in measuring volumes of prisms by counting unit cubes. Fluently use cubic cm, cubic in, cubic ft, and improvised units.

5.MD.C.5, 5.MD.C.5a, 5.MD.C.5b, and 5.MD.C.5c

Content Standards	**5.MD.C.5** Relate volume to the operations of multiplication and addition and solve mathematical problems and problems in real-world contexts involving volume. **5.MD.C.5a** Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes (e.g., to represent the associative property of multiplication). **5.MD.C.5b** Understand and use the formulas $V = l \times w \times h$ and $V = B \times h$, where in this case B is the area of the base ($B = l \times w$), for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths to solve mathematical problems and problems in real-world contexts. **5.MD.C.5c** Understand volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms, applying this technique to solve mathematical problems and problems in real-world contexts.
Explanations	Students need multiple opportunities to measure volume by filling rectangular prisms with cubes and looking at the relationship between the total volume and the area of the base. They derive the volume formula (volume equals the area of the base times the height) and explore how this idea would apply to other prisms. Students use the associative property of multiplication and decomposition of numbers using factors to investigate rectangular prisms with a given number of cubic units.
Content Limits	Whole number side lengths Right rectangular prisms No more than two non-overlapping prisms - non-overlapping means that two prisms may share a face, but they do not share the same volume
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to calculate the volume of a right rectangular prism when given the formula.	• Equation Response • Graphic Response • Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to determine the volume of a right rectangular prism without the formula given.	
Students will be required to determine the dimensions of a right rectangular prism given the volume.	
Students will be required to compare volumes of rectangular prisms using the formula for volume.	
Students will be required to show how to determine the volume of a solid composed of 2 non-overlapping rectangular prisms (e.g. by writing an expression with an unknown.)	
Students will be required to calculate the volume of a solid figure that is composed of 2 non-overlapping rectangular prisms.	
Students will be required to identify an additional volume needed to complete a larger volume.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Relate volume to the operations of multiplication and addition and solve mathematical problems and problems in real-world contexts involving volume. a. Identify the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, or by multiplying the edge lengths. b. Understand and use the formula $V = l \times w \times h$ for rectangular prisms to identify volumes of right rectangular prisms with whole-number edge lengths. c. Understand volume as additive. Identify volumes of solid figures composed of two non-overlapping right rectangular prisms.	Relate volume to the operations of multiplication and addition and solve mathematical problems and problems in real-world contexts involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, or by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. b. Understand and use the formulas $V = l \times w \times h$ and $V = B \times h$, where in this case B is the area of the base ($B = l \times w$), for rectangular prisms to identify volumes of right rectangular prisms with whole-number edge lengths to solve mathematical problems. c. Understand volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms, applying this technique to solve mathematical problems.
Proficient	**Highly Proficient**
Relate volume to the operations of multiplication and addition and solve mathematical problems and problems in real-world contexts involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes (e.g., to represent the associative property of multiplication). b. Understand and use the formulas $V = l \times w \times h$ and $V = B \times h$, where in this case B is the area of the base ($B = l \times w$), for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths to solve mathematical problems and problems in real-world contexts. c. Understand volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms, applying this technique to solve mathematical problems and problems in real-world contexts.	Relate volume to the operations of multiplication and addition and solve mathematical problems and problems in real-world contexts involving volume. a. Explain why the volume of a right rectangular prism can be calculated by multiplying the edge lengths, and explain why this is equivalent to multiplying the height by the area of the base. Represent threefold whole-number products as volumes (e.g., to represent the associative property of multiplication). b. Create problems in real-world contexts that require understanding and using the formulas $V = l \times w \times h$ and $V = B \times h$. c. Understand volume as additive. Find volumes of solid figures composed of more than two non-overlapping right rectangular prisms, applying this technique to solve mathematical problems and problems in real-world contexts.

Numbers and Operations – Fractions

5.NF.A.1

Content Standards	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators (e.g., 2/3 + 5/4 = 8/12 + 15/12 = 23/12).
Explanations	Students should apply their understanding of equivalent fractions developed in fourth grade and their ability to rewrite fractions in an equivalent form to find common denominators. They should know that multiplying the denominators will always give a common denominator but may not result in the smallest denominator.
Content Limits	Improper fractions and mixed numbers included. Least common denominator is not necessary to calculate sums of fractions. Do not use the terms "simplify" or "lowest terms". Denominators should be one-digit or two-digit.
Context	Context is not allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to calculate the sum or difference of two or more fractions with unlike denominators.	• Equation Response • Multiple Choice Response

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify the sum or difference of fractions with unlike denominators.	Identify the sum or difference of fractions with unlike denominators (including mixed numbers).
Proficient	**Highly Proficient**
Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators (e.g., 2/3 + 5/4 = 8/12 + 15/12 = 23/12).	Explain how to find the sum or difference of fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.

5.NF.A.2

Content Standards	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by using a variety of representations, equations, and visual models to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers (e.g. recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2).
Explanations	Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies for calculations with fractions extend from students' work with whole number operations and can be supported through the use of physical models.
Content Limits	Improper fractions and mixed numbers included. Least common denominator is not necessary to calculate sums of fractions. Do not use the terms "simplify" or "lowest terms".
Context	Context is required.

Sample Task Demands	**Common Item Formats**
Students will be required to calculate the sum or difference of two or more fractions with like and/or unlike denominators in a given word problem.	• Equation Response • Multiple Choice Response
Students will be required to determine a missing numerator or denominator in the addend, subtrahend, or minuend of an addition or subtraction problem with fractions in a given word problem.	
Students will be required to use benchmark fractions to explain why an assertion is or is not reasonable.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the solution to word problems involving addition and subtraction of fractions referring to the same whole, by using visual models to represent the problem. Use benchmark fractions and number sense of fractions to identify an estimate.	Identify the solution to word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by using a variety of representations, equations, and visual models to represent the problem. Use benchmark fractions and number sense of fractions to identify an estimate and assess the reasonableness of answers.
Proficient	**Highly Proficient**
Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by using a variety of representations, equations, and visual models to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers (e.g., recognize an incorrect result $2/5 + 1/2 = 3/7$, by observing that $3/7 < 1/2$).	Create word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators. Explain how to estimate mentally and assess the reasonableness of answers.

5.NF.B.3

Content Standards	Interpret a fraction as the number that results from dividing the whole number numerator by the whole number denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people, each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*
Explanations	Students are expected to demonstrate their understanding using concrete materials, drawing models, and explaining their thinking when working with fractions in multiple contexts. They read 3/5 as "three fifths" and after many experiences with sharing problems, learn that 3/5 can also be interpreted as "3 divided by 5."
Content Limits	Quotients in division problems should not be equivalent to a whole number. Only use whole numbers for the divisor and dividend of a fraction.
Context	Context is required.

Sample Task Demands	**Common Item Formats**
Students will be required to express a given division problem as a fraction.	• Equation Response • Multiple Choice Response • Table Response
Students will be required to identify a given fraction as a division problem.	
Students will be required to find the solution to a division word problem and express the quotient as a fraction.	
Students will be required to with or without context, determine the two consecutive whole numbers between which the answer lies in a given division problem.	
Students will be required to identify an area model or number line model that shows the solution to a division word problem.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify a fraction that results from dividing the whole number numerator by the whole number denominator. Identify the solution to word problems involving division of whole numbers leading to answers in the form of fractions.	Determine the fraction that results from dividing the whole number numerator by the whole number denominator. Identify the solution to word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers.
Proficient	**Highly Proficient**
Interpret a fraction as the number that results from dividing the whole number numerator by the whole number denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people, each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*	Explain the meaning of a fraction as the number that results from dividing the whole number numerator by the whole number denominator, and why multiplying a fraction by the denominator results in the numerator. Create word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers.

5.NF.B.4, 5.NF.B.4a, 5.NF.B.4b, and 5.NF.B.4c

Content Standards	**5.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number and a fraction by a fraction. **5.NF.B.4a** Interpret the product (a/b) x q as a parts of a partition of q into b equal parts. *For example, use a visual fraction model to show (2/3) x 4 = 8/3, and create a story context for this equation.* **5.NF.B.4b** Interpret the product of a fraction multiplied by a fraction (a/b) x (c/d). Use a visual fraction model and create a story context for this equation. For example, use a visual fraction model to show (2/3) x (4/5) = 8/15, and create a story context for this equation. In general, (a/b) x (c/d) = ac/bd. **5.NF.B.4c** Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
Explanations	Students are expected to multiply fractions including proper fractions, improper fractions, and mixed numbers. They multiply fractions efficiently and accurately as well as solve problems in both contextual and non-contextual situations.
Content Limits	Multiply whole numbers by fractions or fractions by fractions Visual models Part a - any appropriate fraction model. (e.g. circles, tape, polygons, etc…) Part b - rectangle models only, tile with unit squares For tiling, the dimensions of the tile should be unit fractions with the same denominator as the given rectangular shape (see p. 13 of the progression document for demonstration) Problems do not require simplifying or lowest form.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to multiply a fraction by a whole number or a fraction.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response
Students will be required to relate a scenario that describes "a parts of a partition of q into b equal parts" to an expression of the form (a/b) x q and/or a x q ÷ b.	
Students will be required to tile a rectangular shape to find the area, either given the dimensions of the tile, or the dimensions of the shape.	
Students will be required to multiply length and width to find the area of a rectangular shape with fractional side lengths.	
Students will be required to identify rectangular shape(s) with a given area, where the shapes have given side lengths or are tiled with tiles of given dimensions.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Apply and extend previous understandings of multiplication to multiply a fraction by a whole number and a fraction by a fraction. a. Identify the product $(a/b) \times q$ as a parts of a partition of q into b equal parts using a visual fraction model. b. Identify the product of a fraction multiplied by a fraction $(a/b) \times (c/d)$ as (ac/bd) using a visual fraction model. c. Identify the area of a rectangle with fractional side lengths that has been tiled with unit squares of the appropriate unit fraction side lengths. Identify the product of fractional side lengths to find areas of rectangles.	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number and a fraction by a fraction. a. Identify the product $(a/b) \times q$ as a parts of a partition of q into b equal parts. b. Identify the product of a fraction multiplied by a fraction $(a/b) \times (c/d)$ as ac/bd. Identify the correct story context for a given equation in the form $(a/b) \times (c/d) = ac/bd$. c. Find the area of a rectangle with fractional side lengths that has been tiled with unit squares of the appropriate unit fraction side lengths. Identify the product of fractional side lengths to find areas of rectangles. Recognize that fraction products are rectangular areas.
Proficient	**Highly Proficient**
Apply and extend previous understandings of multiplication to multiply a fraction by a whole number and a fraction by a fraction. a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts. *For example, use a visual fraction model to show (2/3) x 4 = 8/3, and create a story context for this equation.* b. Interpret the product of a fraction multiplied by a fraction $(a/b) \times (c/d)$. Use a visual fraction model and create a story context for this equation*. For example, use a visual fraction model to show (2/3) x (4/5) = 8/15, and create a story context for this equation.* In general, $(a/b) \times (c/d) = ac/bd$. c. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number and a fraction by a fraction. a. Explain why the product $(a/b) \times q$ is a parts of a partition of q into b equal parts, and create a word problem for an equation given in the form $(a/b) \times q$. b. Explain why the product of a fraction multiplied by a fraction $(a/b) \times (c/d)$ is the product of the numerators divided by the product of the denominators ac/bd. Create a story context for a given equation in the form $(a/b) \times (c/d) = ac/bd$. c. Given a rectangle with fractional side lengths, explain how tiling the rectangle with unit squares of the appropriate fractional side lengths and calculating the sum of area of those tiles is the same as multiplying the side lengths of the rectangle. Explain the connection between the product of two fractions and the area of a rectangle with side lengths equal to those fractions.

5.NF.B.5, 5.NF.B.5a, and 5.NF.B.5b

Content Standards	**5.NF.B.5** Interpret multiplication as scaling (resizing), by: **5.NF.B.5a** Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. **5.NF.B.5b** Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number; explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.
Explanations	Use previous understandings of multiplication and division to multiply and divide fractions.
Content Limits	Fractions greater than or equal to 0 Base numbers should be large and unwieldy to discourage students from calculating products Scaling is explored or demonstrated only in terms of quantity. Scaling geometric figures should not be assessed at this standard. Scaling quantities of any kind in 2 dimensions is strictly beyond the scope of this standard.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to identify a statement comparing the value of a given multiplication expression to one of its factors.	• Multiple Choice Response • Multi-Select Response
Students will be required to identify an expression that represents a given statement comparing a product to one of its factors.	
Students will be required to identify expressions that have a value less than or greater than a given number, where the expressions are that number multiplied by another number.	
Students will be required to identify a possible factor of a given expression, given one factor and a comparison of the value of the product to that factor.	
Students will be required to identify an expression that is equivalent to multiplying a given number by 1.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Interpret multiplication as scaling (resizing), by: a. Identifying how the size of the product relates to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication, given a visual model. b. Identifying that multiplying a given number by a fraction greater than 1 results in a product greater than the given number; identifying that multiplying a given number by a fraction less than 1 results in a product smaller than the given number.	Interpret multiplication as scaling (resizing), by: a. Identifying how the size of the product relates to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication, given a visual model. b. Identifying that multiplying a given number by a fraction greater than 1 results in a product greater than the given number; identifying that multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and identifying that multiplying a given fraction by a fraction equal to 1 results in an equivalent fraction.
Proficient	**Highly Proficient**
Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number; explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.	Interpret multiplication as scaling (resizing), by: a. Explaining how the size of a product compares to the size of one factor on the basis of the size of the other factor. b. Demonstrating how multiplying a given number by a fraction greater than 1 results in a product greater than the given number; demonstrating how multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and demonstrating how the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ relates to the effect of multiplying a/b by 1.

5.NF.B.6

Content Standards	Solve problems in real-world contexts involving multiplication of fractions, including mixed numbers, by using a variety of representations including equations and models.
Explanations	Use previous understandings of multiplication and division to multiply and divide fractions.
Content Limits	Items should require student to interpret the context to determine operations.
Context	Context is required.

Sample Task Demands	Common Item Formats
Students will be required to solve simple word problems involving multiplication of fractions .(i.e., multiplying two given values)	• Equation Response • Multiple Choice Response
Students will be required to solve complex word problems involving multiplication of fractions (e.g., multiplying three numbers, involving other operations, finding an unknown. (numerator, denominator, etc.)	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the solutions to problems in real-world contexts involving multiplication of fractions, by using visual models.	Identify the solutions to problems in real-world contexts involving multiplication of fractions, by using a variety of representations including equations and models.
Proficient	**Highly Proficient**
Solve problems in real-world contexts involving multiplication of fractions, including mixed numbers, by using a variety of representations including equations and models.	Create problems in real-world contexts involving multiplication of fractions, including mixed numbers, given a representation such as an equation or a model.

5.NF.B.7, 5.NF.B.7a, 5.NF.B.7b, and 5.NF.B.7c

Content Standards	**5.NF.B.7** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. **5.NF.B.7a** Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. Use the relationship between multiplication and division to justify conclusions. **5.NF.B.7b** Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient.* Use the relationship between multiplication and division to justify conclusions (e.g., 4 ÷ (1/5) = 20 because 20 x (1/5) = 4). **5.NF.B.7c** Solve problems in real-world context involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, using a variety of representations.
Explanations	In fifth grade, students experience division problems with whole number divisors and unit fraction dividends (fractions with a numerator of 1) or with unit fraction divisors and whole number dividends. Students extend their understanding of the meaning of fractions, how many unit fractions are in a whole, and their understanding of multiplication and division as involving equal groups or shares and the number of objects in each group/share. In sixth grade, they will use this foundational understanding to divide into and by more complex fractions and develop abstract methods of dividing by fractions.
Content Limits	All problems should have either: A) Division of unit fractions by a non-zero whole number, or B) Division of a non-zero whole number by a unit fraction
Context	Context is required.

Sample Task Demands	Common Item Formats
Students will be required to divide a fraction by a whole number.	• Equation Response • Graphic Response • Multiple Choice Response
Students will be required to divide a whole number by a fraction.	
Students will be required to select a division expression that represents the scenario of a given division problem.	
Students will be required to select an equivalent multiplication equation for a given division equation.	
Students will be required to solve a simple word problem that involves division, a whole number, and a fraction.	
Students will be required to solve a simple word problem that involves division, and justify the solution using an equation or number line.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. a. Identify the quotient of a unit fraction by a non-zero whole number. b. Identify the quotient of a whole number by a unit fraction. c. Identify the solutions to problems in real-world context involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, using visual models.	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. a. Compute the quotient of a unit fraction by a non-zero whole number. b. Compute the quotient of a whole number by a unit fraction. c. Identify the solutions to problems in real-world context involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, using a variety of representations.
Proficient	**Highly Proficient**
Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. Use the relationship between multiplication and division to justify conclusions. b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient.* Use the relationship between multiplication and division to justify conclusions (e.g., 4 ÷ (1/5) = 20 because 20 x (1/5) = 4). c. Solve problems in real-world context involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, using a variety of representations.	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. a. Use the relationship between multiplication and division to explain how to divide a unit fraction by a non-zero whole number. b. Use the relationship between multiplication and division to explain how to divide a whole number by a unit fraction. c. Create problems in real-world context involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions.

Operations and Algebraic Thinking & Numbers in Base Ten

5.NBT.A.1

Content Standards	Apply concepts of place value, multiplication, and division to understand that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
Explanations	In fourth grade, students examined the relationships of the digits in numbers for whole numbers only. This standard extends this understanding to the relationship of decimal fractions. Students use base ten blocks, pictures of base ten blocks, and interactive images of base ten blocks to manipulate and investigate the place value relationships. They use their understanding of unit fractions to compare decimal places and fractional language to describe those comparisons. Before considering the relationship of decimal fractions, students express their understanding that in multi-digit whole numbers, a digit in one place represents 10 times what it represents in the place to its right and 1/10 of what it represents in the place to its left.
Content Limits	Whole numbers to any place value and decimals to thousandths
Context	Context is not allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to identify the factor by which one number is greater or less than another.	• Equation Response • Multiple Choice Response
Students will be required to compare the value of a digit in different place values of two given numbers and identify the power of 10 by which one number is greater than another.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify which place value in a multi-digit whole number represents 10 times the value of a given place value, or identify which place value in a multi-digit whole number represents 1/10 the value of a given place value.	Given two multi-digit numbers, with a digit in different place values in each number, identify how many times the value of the digit is in one number compared to the other number (e.g. the value of the 4 in 29,143 is 1/100 times the value of the 4 in 74,851).
Proficient	**Highly Proficient**
Apply concepts of place value, multiplication, and division to understand that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	Apply concepts of place value, multiplication, and division to explain why a digit in one place represents ten times what it represents in the place to its right and 1/10 of what it represents in the place to its left.

5.NBT.A.2

Content Standards	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10.
Explanations	Understand the place value system.
Content Limits	Whole number exponents with a base of 10. Decimals to thousandths.
Context	Context is not allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to calculate a power of 10.	• Equation Response • Multiple Choice Response • Multi-Select Response
Students will be required to multiply or divide a decimal by a power of ten.	
Students will be required to find a missing exponent when multiplying or dividing a decimal by a power of ten.	
Students will be required to identify patterns when multiplying or dividing by a power of 10.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify patterns in the number of zeros of the product when multiplying a number by powers of 10, and identify patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10.	Find patterns in the number of zeros of the product when multiplying a number by powers of 10, and find patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10.
Proficient	**Highly Proficient**
Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10.	Given a pattern in the number of zeros of the product when multiplying a number by powers of 10, or a pattern in the placement of the decimal point when multiplying or dividing a number by a power of 10, create a possible equation that represents the pattern and explain why there are multiple correct equations.

5.NBT.A.3, 5.NBT.A.3a, and 5.NBT.A.3b

Content Standards	**5.NBT.A.3** Read, write, and compare decimals to thousandths. **5.NBT.A.3a** Read and write decimals to thousandths using base-ten numerals, number names, and expanded form. **5.NBT.A.3b** Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
Explanations	Students build on the understanding they developed in fourth grade to read, write, and compare decimals to thousandths. They connect their prior experiences with using decimal notation for fractions and addition of fractions with denominators of 10 and 100. They use concrete models and number lines to extend this understanding to decimals to the thousandths. Models may include base ten blocks, place value charts, grids, pictures, drawings, manipulatives, technology-based, etc. They read decimals using fractional language and write decimals in fractional form, as well as in expanded notation as show in the standard 3a. This investigation leads them to understanding equivalence of decimals (0.8 = 0.80 = 0.800). Students need to understand the size of decimal numbers and relate them to common benchmarks such as 0, 0.5 (0.50 and 0.500), and 1. Comparing tenths to tenths, hundredths to hundredths, and thousandths to thousandths is simplified if students use their understanding of fractions to compare decimals.
Content Limits	Decimals to thousandths
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to write a number with a given name in numeric form.	• Equation Response • Graphic Response • Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to identify the name of a given number.	
Students will be required to write a number given in traditional expanded form in numeric form or vice versa.	
Students will be required to compare two decimals.	
Students will be required to order more than two decimals in numeric form.	
Students will be required to identify numbers in non-traditional expanded form (e.g., 47.389 = 9 × (1/1000) + 7 × 1 + 3 × (1/10) + 4 × 10 + 8 x (1/100)).	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Read and write, decimals to tenths. a. Identify decimals to tenths using base-ten numerals and number names. b. Compare two decimals to tenths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	Read, write, and compare decimals to hundredths. a. Identify decimals to hundredths using base-ten numerals, number names, and expanded form. b. Compare two decimals to hundredths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
Proficient	**Highly Proficient**
Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form. b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	Read, write, and compare decimals to thousandths. a. Order multiple decimals to thousandths using base-ten numerals, number names, and expanded form. b. Compare more than two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

5.NBT.A.4

Content Standards	Use place value understanding to round decimals to any place.
Explanations	When rounding a decimal to a given place, students may identify the two possible answers, and use their understanding of place value to compare the given number to the possible answers.
Content Limits	Decimals to thousandths
Context	Context is not allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to identify the value of a decimal number rounded to a place value.	• Equation Response • Multiple Choice Response • Matching Item Response • Multi-Select Response • Table Response
Students will be required to identify the decimal numbers that round to a given value.	
Students will be required to distinguish between different rounding procedures used in order to create a number that fits certain parameters.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Use place value understanding to round decimals to the tenths place.	Use place value understanding to round decimals to the hundredths place.
Proficient	**Highly Proficient**
Use place value understanding to round decimals to any place.	Explain how to use place value understanding to round decimals to any place.

5.NBT.B.5

Content Standards	Fluently multiply multi-digit whole numbers using a standard algorithm.
Explanations	In prior grades, students used various strategies to multiply. Students can continue to use these different strategies as long as they are efficient, but must also understand and be able to use the standard algorithm. In applying the standard algorithm, students recognize the importance of place value.
Content Limits	Multiplication should not exceed 5 digits by 2 digits.
Context	Context is not allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to calculate the product of two numbers.	• Equation Response
Students will be required to identify a missing factor or digit in a multiplication problem.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify the product of two multi-digit whole numbers.	Calculate the product of two multi-digit whole numbers.
Proficient	**Highly Proficient**
Fluently multiply multi-digit whole numbers using a standard algorithm.	Explain how to use a standard algorithm to multiply multi-digit whole numbers.

5.NBT.B.6

Content Standards	Apply and extend understanding of division to find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors.
Explanations	In fourth grade, students' experiences with division were limited to dividing by one-digit divisors. This standard extends students' prior experiences with strategies, illustrations, and explanations. When the two-digit divisor is a "familiar" number, a student might decompose the dividend using place value.
Content Limits	Only 3-digit or 4-digit dividend and 2-digit divisor
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to calculate the quotient of 2 numbers.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response
Students will be required to select expressions that are equivalent to a given quotient.	
Students will be required to illustrate and explain quotients of 2 numbers using equations, rectangular arrays, or area models.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Apply understanding of division to identify whole-number quotients of whole numbers with up to three-digit dividends and two-digit divisors.	Apply understanding of division to identify whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors.
Proficient	**Highly Proficient**
Apply and extend understanding of division to find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors.	Apply and extend understanding of division to find whole-number quotients of whole numbers with more than four-digit dividends and two-digit divisors.

5.NBT.B.7

Content Standards	Add, subtract, multiply, and divide decimals to hundredths, connecting objects or drawings to strategies based on place value, properties of operations, and/or the relationship between operations. Relate the strategy to a written form.
Explanations	This standard requires students to extend the models and strategies they developed for whole numbers in grades 1-4 to decimal values. Before students are asked to give exact answers, they should estimate answers based on their understanding of operations and the value of the numbers. Students should be able to express that when they add decimals they add tenths to tenths and hundredths to hundredths. So, when they are adding in a vertical format (numbers beneath each other), it is important that they write numbers with the same place value beneath each other. This understanding can be reinforced by connecting addition of decimals to their understanding of addition of fractions. Adding fractions with denominators of 10 and 100 is a standard in fourth grade.
Content Limits	Decimals within hundredths place in all numbers involved (divisors, dividends, quotients and likewise for other operations)
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to perform a calculation involving decimals.	• Equation Response • Multi-Select Response
Students will be required to solve a problem involving decimals and the four operations given a scenario.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Add and subtract decimals (without regrouping) to hundredths, connecting objects or drawings to strategies based on place value, properties of operations, and/or the relationship between operations.	Add, subtract, and multiply decimals to hundredths, connecting objects or drawings to strategies based on place value, properties of operations, and/or the relationship between operations.
Proficient	**Highly Proficient**
Add, subtract, multiply, and divide decimals to hundredths, connecting objects or drawings to strategies based on place value, properties of operations, and/or the relationship between operations. Relate the strategy to a written form.	Add, subtract, multiply, and divide decimals to hundredths. Relate the strategy to a written form. Apply this to real-world context.

5.OA.A.1

Content Standards	Use parentheses and brackets in numerical expressions, and evaluate expressions with these symbols (Order of Operations).
Explanations	This standard builds on the expectations of third grade where students are expected to start learning the conventional order. Students need experiences with multiple expressions that use grouping symbols throughout the year to develop understanding of when and how to use parentheses, brackets, and braces. First, students use these symbols with whole numbers. Then the symbols can be used as students add, subtract, multiply and divide decimals and fractions. To further develop students' understanding of grouping symbols and facility with operations, students place grouping symbols in equations to make the equations true or they compare expressions that are grouped differently.
Content Limits	Whole numbers & simple fraction expressions (single digit denominators, fraction multiplied by a whole number) Do not use nested grouping symbols (based on the progressions document for K-5 operations and algebraic thinking) Expressions should not be more complex than those used in associative or distributive property situations
Context	Context is not allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to evaluate a numerical expression with parentheses.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response
Students will be required to rewrite a given numerical expression with parentheses, brackets and/or braces (by inserting these grouping symbols) such that the expression evaluates to a given answer.	
Students will be required to identify a calculation error when evaluating a numerical expression.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Use parenthesis in numerical expressions and evaluate numeric expressions.	Evaluate numerical expressions with parentheses and brackets.
Proficient	**Highly Proficient**
Use parentheses and brackets in numerical expressions, and evaluate expressions with these symbols (Order of Operations).	Use parentheses and brackets to create multiple numerical expressions equivalent to a given value.

5.OA.A.2

Content Standards	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them (e.g., express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x (18,932 + 921) is three times as large as 18,932 + 921, without having to calculate the indicated sum or product).
Explanations	Students use their understanding of operations and grouping symbols to write expressions and interpret the meaning of a numerical expression.
Content Limits	Whole numbers Simple fraction expressions Do not use nested parentheses Use numeric expressions only. Multiplication cross symbol is the only acceptable symbol for multiplication. Do not use the c-dot. When grouping symbols are part of the expression, the associative property or distributive property should be found in that expression.
Context	Context is not allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to construct a numeric expression given a written statement of numerical values.	• Equation Response • Multiple Choice Response • Proposition Response
Students will be required to interpret the meaning of a written numerical statement without evaluating it.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify simple expressions that record calculations with numbers, and identify numerical expressions without evaluating them.	Write simple expressions that record calculations with numbers, and identify numerical expressions without evaluating them.
Proficient	**Highly Proficient**
Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them (e.g., express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x (18,932 + 921) is three times as large as 18,932 + 921, without having to calculate the indicated sum or product).	Write simple expressions that record multi-step calculations with numbers, and interpret multi-step numerical expressions without evaluating them.

5.OA.B.3

Content Standards	Generate two numerical patterns using two given rules (e.g., generate terms in the resulting sequences). Identify and explain the apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane (e.g., given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence).
Explanations	Analyze patterns and relationships.
Content Limits	Whole numbers & fractions with denominators less than 10 Quadrant I on coordinate plane Acceptable operations: addition, subtraction, multiplication, and division The rule should be no more complex than one finds in an application of the associative or distributive property. Examples should not contain nested grouping symbols.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to find terms of two numerical patterns given rules, including forming ordered pairs determined by the pattern.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response • Table Response
Students will be required to identify specific relationships between terms of two numerical patterns (term when the sequences are equal, where one is twice the other, etc.)	
Students will be required to graph ordered pairs corresponding to terms in two numerical patterns in a coordinate plane.	
Students will be required to identify relationships between two numerical patterns.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify two numerical patterns using two given rules (e.g., identify terms in the resulting sequences). Identify the apparent relationships between corresponding terms. Identify ordered pairs consisting of corresponding terms from the two patterns.	Determine the missing values in two numerical patterns using two given rules (e.g., determine the missing terms in the resulting sequences). Identify the apparent relationships between corresponding terms. Identify ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.
Proficient	**Highly Proficient**
Generate two numerical patterns using two given rules (e.g., generate terms in the resulting sequences). Identify and explain the apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane (e.g., given the rule "add 3" and the starting number 0, and given the rule "add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence).	Explain how the rules for two numerical patterns relate to the relationships between the corresponding terms in those patterns (e.g., given the rule "add 3" and the starting number 0, and given the rule "add 6" and the starting number 0, observe that the terms in one sequence are twice the corresponding terms in the other sequence, and recognize that "add 3" is twice "add 6").

5.OA.B.4

Content Standards	Understand primes have only two factors and decompose numbers into prime factors.
Explanations	A prime number is a whole number greater than 1 that has only 2 factors, 1 and itself. Composite numbers have more than 2 factors. Students investigate whether numbers are prime or composite by building rectangles (arrays) and finding which rectangles can be built using more than one equal row and one equal column. These rectangles represent composite numbers. Rectangles that cannot be built with more than one equal row and one equal column (e.g., 7 can be only be shown as a 1-by-7 or 7-by-1 array) represent prime numbers.
Content Limits	Vocabulary includes “prime,” “composite,” “factor,” and “multiple.”
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will determine whether a whole number is prime or composite.	• Equation Response • Graphic Response • Multiple Choice Response • Table Response
Students will be able to decompose whole numbers into prime factors.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify prime numbers.	Understand prime numbers have only two factors and identify the prime factorization of numbers.
Proficient	**Highly Proficient**
Understand primes have only two factors and decompose numbers into prime factors.	Explain how to decompose numbers into prime factors.