AzM2 | Arizona's Statewide Achievement Assessment for English Language Arts and Mathematics

Mathematics Item Specifications

GRADE 3

Arizona Department of Education with American Institutes for Research - 2019

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Introduction

The Arizona Statewide Achievement Assessment for English Language Arts and Mathematics (AzM2) is Arizona's statewide achievement test. AzM2 assesses the Arizona English Language Arts Standards and Arizona Mathematics Standards adopted by the Arizona State Board of Education in December 2016. AzM2 will inform students, teachers, and parents about preparedness for college and careers upon graduating from high school. AzM2 tests are computer-based, meaning that they can better assess students' critical thinking skills and provide them with opportunities to demonstrate a deeper understanding of the materials. Computer-based testing also allows for the use of a variety of innovative items types.

During the item-development process, all AzM2 items are written in accordance with the Item Specifications and are reviewed and approved by a committee of Arizona educators to confirm alignment and appropriateness for inclusion in the test. AzM2 items are generally representative of Arizona's geographic regions and culturally diverse population. Items are reviewed for the following kinds of bias: gender, racial, ethnic, linguistic, religious, geographic, and socioeconomic. Item reviews also include consideration of issues related to individuals with disabilities. Arizona community members also have an opportunity to review items for issues of potential concern to members of the community at large. Reviewers are asked to consider the variety of cultural, regional, philosophical, political, and religious backgrounds throughout Arizona, and then to determine whether the subject matter will be acceptable to Arizona students, families, and other members of Arizona communities.

This *AzM2 Item Specifications* is a resource document that defines the content and format of the test and test items for item writers and reviewers. Each *Item Specifications* document indicates the alignment of items with the Arizona Mathematics Standards. It also serves to provide all stakeholders with information about the scope and function of assessment items. This document can also serve to assist educators to understand how assessment items are developed in alignment with the standards for English language arts and math. These item specifications for AzM2 are intended to provide information regarding standards, item formats and response types. The descriptions of blueprints, and depth of knowledge in this document are meant to provide an overview of the test. Item specifications are meant for the purposes of assessment, not instruction. They are not intended to be tools for instruction or the basis for curricula. AzM2 has a test blueprint that was developed by Arizona and is different from any other state or consortium test blueprint.

For the math portion of AzM2, all of the test questions are aligned to the mathematic content standards for these subject areas. Any item specifications that are absent for standards listed in this document may be under development. This document does not endorse the exclusion of the instruction of any grade-level content standards. The test will ask questions that check a student's conceptual understanding of math as well as their procedural skills. These items have been written to be free from bias and sensitivity, and widely vary in their degree of difficulty.

Item Development Process

AzM2 items go through a rigorous review before they are operational. When an item is "operational" it means it is used to determine a student's score on the assessment. This is a description of the process every item must go through before it is operational on AzM2.

Item Development

AIR and ADE generate potential items for review.

Educator Review

Committee of Arizona Teachers review items for content and bias. All approved items are moved forward.

Parent Review Committee

Arizona parents/community members review items for bias and sensitivity. All approved items move forward.

Field Test

Items are field tested to see how they operate.

Data Review

Field Test items are reviewed for data to ensure they are valid.

Operational

Field Test items which have made it through all stages are now potentially Operational.

Sample tests are available online for the math portion of AzM2. For more information view the Guide to the Sample Tests at www.AzM2portal.org.

Test Construction Guidelines

The construction of the AzM2 assessment is guided by the depth and rigor of the Arizona College and Career Ready Standards. Items are created to address key components of the standards and assess a range of important skills. The AzM2 Blueprint provides an overview of the distribution of items on the AzM2 according to the standards. The standards for Math Practices are embedded within all AzM2 items. Further, the AzM2 blueprint outlines the Depth of Knowledge distribution of items.

Blueprint

Grade 3 AzM2 Blueprint 2016 Standards		
Reporting Category	**Min.**	**Max.**
Operations and Algebraic Thinking, and Numbers in Base Ten	**49%**	**53%**
Operations and Algebraic Thinking	38%	42%
Numbers in Base Ten	9%	13%
Numbers and Operations-Fractions	**18%**	**22%**
Measurement, Data, and Geometry	**26%**	**30%**
Measurement & Data and Geometry	26%	28%
Geometry	1%	4%

Depth of Knowledge (DOK)

DOK refers to the level of rigor or sophistication of the task in a given item, designed to reflect the complexity of the Arizona Mathematics Standards. Items at DOK level 1 focus on the recall of information, such as definitions, terms, and simple procedures. Items at DOK 2 require students to make decisions, solve problems, or recognize patterns; in general, they require a greater degree of engagement and cognitive processing than items at DOK 1. Items at DOK 3 feature higher-order cognitive tasks that assess students' capacities to approach abstract or complex problems.

Percentage of Points by Depth of Knowledge (DOK) Level			
Grade 3	DOK Level 1	DOK Level 2	DOK Level 3
	10% - 20%	60% - 70%	12% - 30%

For more information on DOK go to www.azed.gov/AzM2.

Calculators

Arizona Desmos Graphing Calculator is not permitted for the paper-based and computer-based assessment for Grade 3 Math.

Item Formats

The AzM2 Assessments are composed of item formats that include traditional multiple- choice response items and technology-enhanced response items (TEI). TEIs are computer- delivered response items that require students to interact with test content to select, construct, and/or support their responses. TEIs are better able to assess a deeper level of understanding.

Currently, there are nine types of TEIs that may appear on the Math computer-based assessment for AzM2:

- Editing Tasks (ET)
- Editing Task Choice (ETC)
- Equation Editor (EQ)
- Graphic Response Item Display (GRID)
- Hot Text (HT)
    - Selectable Hot Text
    - Drag-and-Drop Hot Text
- Matching Item (MI)
- Multi-Select (MS)
- Open Response
- Table Item (TI)

For paper-based assessments (including those for students with an IEP or 504 plan that specifies a paper-based accommodation), TEIs will be modified so that they can be scanned and scored electronically or hand-scored.

See the table below for a description of each TEI. In addition, for examples of each response item format described, see the AzM2 Training Tests at www.AzM2portal.org.

Item Format	Description
Editing Task (ET)	The student clicks on a highlighted word or phrase that may be incorrect, which reveals a text box. The directions in the text box direct the student to replace the highlighted word or phrase with the correct word or phrase. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Editing Task Choice (ETC)	The student clicks a highlighted word or phrase, which reveals a drop-down menu containing options for correcting an error as well as the highlighted word or phrase as it is shown in the sentence to indicate that no correction is needed. The student then selects the correct word or phrase from the drop-down menu. For paper-based assessments, the item is modified so that it can be scanned and scored electronically. The student fills in a circle to indicate the correct word or phrase.

Item Format	Description
Equation Editor (EQ)	The student is presented with a toolbar that includes a variety of mathematical symbols that can be used to create a response. Responses may be in the form of a number, variable, expression, or equation, as appropriate to the test item. For paper-based assessments, this item type may be replaced with a modified version of the item that can be scanned and scored electronically or replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Graphic Response Item Display (GRID)	The student selects numbers, words, phrases, or images and uses the drag-and-drop feature to place them into a graphic. This item type may also require the student to use the point, line, or arrow tools to create a response on a graph. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Hot Text (HT)	**Selectable Hot Text -** Excerpted sentences from the text are presented in this item type. When the student hovers over certain words, phrases, or sentences, the options highlight. This indicates that the text is selectable ("hot"). The student can then click on an option to select it. For paper- based assessments, a "selectable" hot text item is modified so that it can be scanned and scored electronically. In this version, the student fills in a circle to indicate a selection.
	Drag-and-Drop Hot Text - Certain numbers, words, phrases, or sentences may be designated "draggable" in this item type. When the student hovers over these areas, the text highlights. The student can then click on the option, hold down the mouse button, and drag it to a graphic or other format. For paper-based assessments, drag- and-drop hot text items will be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Matching Item (MI)	The student checks a box to indicate if information from a column header matches information from a row. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Multi-Select (MS)	The student is directed to select all of the correct answers from among a number of options. These items are different from multiple-choice items, which allow the student to select only one correct answer. These items appear in the online and paper-based assessments.

Item Format	Description
Open Response	The student uses the keyboard to enter a response into a text field. These items can usually be answered in a sentence or two. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Table Item (TI)	The student types numeric values into a given table. The student may complete the entire table or portions of the table depending on what is being asked. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.

Arizona Math Standards

Operations and Algebraic Thinking (OA) *Note: Grade 3 expectations in this domain are limited to whole number multiplication through 10 x 10 and whole number division with both quotients and divisors less than or equal to 10.*		
3.OA.A **Represent and solve problems involving whole number multiplication and division.**	**3.OA.A.1**	Interpret products of whole numbers as the total number of objects in equal groups (e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each).
	3.OA.A.2	Interpret whole number quotients of whole numbers (e.g., interpret 56 ÷ 8 as the number of objects in each group when 56 objects are partitioned equally into 8 groups, or as a number of groups when 56 objects are partitioned into equal groups of 8 objects each). *See Table 2.*
	3.OA.A.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities. *See Table 2.*
	3.OA.A.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers *For example, determine the unknown number that makes the equation true in each of the equations 8 x □ = 48, 5 = □ ÷ 3, 6 x 6 = □. See Table 2.*
3.OA.B **Understand properties of multiplication and the relationship between multiplication and division.**	**3.OA.B.5**	Apply properties of operations as strategies to multiply and divide. Properties include commutative and associative properties of multiplication and the distributive property. (Students do not need to use the formal terms for these properties.)
	3.OA.B.6	Understand division as an unknown-factor problem (e.g., find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8).
3.OA.C **Multiply and divide within 100.**	**3.OA.C.7**	Fluently multiply and divide within 100. By the end of Grade 3, know from memory all multiplication products through 10 x 10 and division quotients when both the quotient and divisor are less than or equal to 10.
3.OA.D **Solve problems involving the four operations, and identify and explain patterns in arithmetic.**	**3.OA.D.8**	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Utilize understanding of the Order of Operations when there are no parentheses.
	3.OA.D.9	Identify patterns in the addition table and the multiplication table and explain them using properties of operations (e.g. observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends).
	3.OA.D.10	When solving problems, assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Number and Operations in Base Ten (NBT) *Note: A range of algorithms may be used.*		
3.NBT.A **Use place value understanding and properties of operations to perform multi-digit arithmetic.**	**3.NBT.A.1**	Use place value understanding to round whole numbers to the nearest 10 or 100.
	3.NBT.A.2	Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
	3.NBT.A.3	Multiply one-digit whole numbers by multiples of 10 in the range 10 to 90 using strategies based on place value and the properties of operations (e.g., 9 x 80, 5 x 60).
Number and Operations – Fractions (NF) *Note: Grade 3 expectations are limited to fractions with denominators: 2,3,4,6,8.*		
3.NF.A **Understand fractions as numbers.**	**3.NF.A.1**	Understand a fraction (1/*b*) as the quantity formed by one part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by *a* parts of size 1/*b*.
	3.NF.A.2	Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Understand that each part has size 1/*b* and that the end point of the part based at 0 locates the number 1/*b* on the number line. b. Represent a fraction *a/b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Understand that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line including values greater than 1. c. Understand a fraction 1/*b* as a special type of fraction that can be referred to as a unit fraction (e.g. 1/2, 1/4).
	3.NF.A.3	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent if they have the same relative size compared to 1 whole. b. Recognize and generate simple equivalent fractions. Explain why the fractions are equivalent. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Understand that comparisons are valid only when the two fractions refer to the same whole. Record results of comparisons with the symbols >, =, or <, and justify conclusions.

Measurement and Data (MD)		
3.MD.A **Solve problems involving measurement.**	**3.MD.A.1a**	Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes (e.g., representing the problem on a number line diagram).
	3.MD.A.1b	Solve word problems involving money through $20.00, using symbols $, ".", ¢.
	3.MD.A.2	Measure and estimate liquid volumes and masses of objects using metric units. (Excludes compound units such as cm^3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units. Excludes multiplicative comparison problems (problems involving notions of "times as much"). *See Table 2.*
3.MD.B **Represent and interpret data.**	**3.MD.B.3**	Create a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *See Table 1.*
	3.MD.B.4	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch to the nearest quarter-inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.
3.MD.C **Geometric measurement: Understand concepts of area and perimeter.**	**3.MD.C.5**	Understand area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.
	3.MD.C.6	Measure areas by counting unit squares (e.g., square cm, square m, square in, square ft, and improvised units).
	3.MD.C.7	Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. d. Understand that rectilinear figures can be decomposed into non-overlapping rectangles and that the sum of the areas of these rectangles is identical to the area of the original rectilinear figure. Apply this technique to solve problems in real-world contexts.

3.MD.C (cont.)	**3.MD.C.8**	Solve real-world and mathematical problems involving perimeters of plane figures and areas of rectangles, including finding the perimeter given the side lengths, finding an unknown side length. Represent rectangles with the same perimeter and different areas or with the same area and different perimeters.
	Geometry (G)	
3.G.A **Reason with shapes and their attributes.**	**3.G.A.1**	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others)may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples quadrilaterals that do not belong to any of these subcategories.
	3.G.A.2	Partition shapes into *b* parts with equal areas. Express the area of each part as a unit fraction 1/*b* of the whole. (Grade 3 expectations are limited to fractions with denominators *b* = 2,3,4,6,8.)

Grade 3 Item Specifications

Operations and Algebraic Thinking

3.OA.A.1

Content Standards	Interpret products of whole numbers as the total number of objects in equal groups (e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each).
Explanations	Students recognize multiplication as a means to determine the total number of objects when there are a specific number of groups with the same number of objects in each group. Multiplication requires students to think in terms of groups of things rather than individual things. Students learn that the multiplication symbol 'x' means "groups of" and problems such as 5 x 7 refer to 5 groups of 7. To further develop this understanding, students interpret a problem situation requiring multiplication using pictures, objects, words, numbers, and equations. Then, given a multiplication expression (e.g., 5 x 6) students interpret the expression using a multiplication context. (See Table 2) They should begin to use the terms, factor and product, as they describe multiplication.
Content Limits	Products within 100. Whole number factors.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to interpret and/or describe what factor pairs represent in a given arrangement.	• Equation Response • Multiple Choice Response • Multi-Select Response • Proposition Response • Table Response
Students will be required to create a multiplication problem that describes a given arrangement.	
Students will be required to create multiple pairs of factors to create a given arrangement.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify whole number products with visual support.	Interpret whole number products with visual support.
Proficient	**Highly Proficient**
Interpret products of whole numbers as the total number of objects in equal groups (e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each).	Interpret products of whole numbers within 100, representing context using pictures, numbers, and words.

3.OA.A.2

Content Standards	Interpret whole number quotients of whole numbers (e.g., interpret 56 ÷ 8 as the number of objects in each group when 56 objects are partitioned equally into 8 groups, or as a number of groups when 56 objects are partitioned into equal groups of 8 objects each).
Explanations	Students recognize the operation of division in two different types of situations. One situation requires determining how many groups and the other situation requires sharing (determining how many in each group). Students should be exposed to appropriate terminology (quotient, dividend, divisor, and factor). To develop this understanding, students interpret a problem situation requiring division using pictures, objects, words, numbers, and equations. Given a division expression (e.g., 24 ÷ 6) students interpret the expression in contexts that require both interpretations of division.
Content Limits	Dividends up to 100. Whole number dividends. Whole number quotients.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify the quotient for a given problem.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response • Proposition Response
Students will be required to find a number to answer a question based on the interpretation of a quotient within a context.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify whole number quotients with visual support.	Interpret whole number quotients with visual support.
Proficient	**Highly Proficient**
Interpret whole number quotients of whole numbers (e.g., interpret 56 ÷ 8 as the number of objects in each group when 56 objects are partitioned equally into 8 groups, or as a number of groups when 56 objects are partitioned into equal groups of 8 objects each).	Interpret quotients of whole numbers within 100, representing context using pictures, numbers, and words.

3.OA.A.3

Content Standards	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities.
Explanations	Students use a variety of representations for creating and solving one-step word problems, i.e., numbers, words, pictures, physical objects, or equations. They use multiplication and division of whole numbers up to 10 x10. Students explain their thinking, show their work by using at least one representation, and verify that their answer is reasonable.
Content Limits	All numbers must be 100 or less. Use whole numbers only. Give only one unknown per equation. Unknown may be in any position. Do not use letter variables for the unknown in this standard. Instead, use a box or other symbol to represent the unknown. Do not use the words "times as much/many."
Context	Context is required.

Sample Task Demands	**Common Item Formats**
Students will be required to solve a simple word problem involving multiplication or division.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response
Students will be required to create an equation to model a simple situation with multiplication or division.	
Students will be required to model multiplication and division equations by sorting objects into equal groups.	
Students will be required to create an equation to model a complex situation with multiplication or division.	
Students will be required to create a model using a multiplication or division equation that represents a complex situation.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify products and quotients within 100 to solve word problems involving equal groups and arrays when a visual model is given.	Multiply and divide within 100 to solve word problems involving equal groups and arrays when a visual model is given.
Proficient	**Highly Proficient**
Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities.	Multiply and divide within 100 to solve multi-step word problems involving equal groups, arrays, and measurement quantities.

3.OA.A.4

Content Standards	Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 x □ = 48, 5 = □ ÷ 3, 6 x 6 = □.
Explanations	This standard is strongly connected to 3.AO.3 when students solve problems and determine unknowns in equations. Students should also experience creating story problems for given equations. When crafting story problems, they should carefully consider the question(s) to be asked and answered to write an appropriate equation. Students may approach the same story problem differently and write either a multiplication equation or division equation. Students apply their understanding of the meaning of the equal sign as "the same as" to interpret an equation with an unknown. Equations in the form of *a* x *b* = *c* and *c* = *a* x *b* should be used interchangeably, with the unknown in different positions.
Content Limits	Product is less than 100. Whole number factors and quotients. Equation must be given, and not created.
Context	Context is not allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to find the unknown number in a given multiplication or division equation.	• Equation Response • Multiple Choice Response

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Recognize the unknown whole number in a multiplication or division equation, when the unknown number is the solution using visual support/arrays.	Determine the unknown whole number in a multiplication or division equation, when the unknown number is the product or quotient using visual support/arrays.
Proficient	**Highly Proficient**
Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 x □ = 48, 5 = □ ÷ 3, 6 x 6 = □ .	Determine an unknown whole number in a multiplication and division equation. Students will use the given context to generate an equation.

3.OA.B.5

Content Standards	Apply properties of operations as strategies to multiply and divide. Properties include commutative and associative properties of multiplication and the distributive property. (Students do not need to use the formal terms for these properties.)
Explanations	Students represent expressions using various objects, pictures, words and symbols in order to develop their understanding of properties. They multiply by 1 and 0 and divide by 1. They change the order of numbers to determine that the order of numbers does not make a difference in multiplication (but does make a difference in division). Given three factors, they investigate changing the order of how they multiply the numbers to determine that changing the order does not change the product. They also decompose numbers to build fluency with multiplication. Students are introduced to the distributive property of multiplication over addition as a strategy for using products they know to solve products they don't know. To further develop understanding of properties related to multiplication and division, students use different representations and their understanding of the relationship between multiplication and division to determine if the following types of equations
Content Limits	Whole numbers. Product or dividend must be 100 or less. Factors, divisors, and quotients should be 10 or less.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to create an equivalent expression and/or equation based on applying a particular property (i.e., Commutative, Associative, Distributive).	• Equation Response • Graphic Response • Multiple Choice Response • Matching Item Response • Multi-Select Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Apply properties of operations as strategies to multiply and divide. Properties include commutative properties of multiplication. (Students do not need to use the formal terms for these properties.)	Apply properties of operations as strategies to multiply and divide. Properties include commutative and associative properties of multiplication. Students do not need to use the formal terms for these properties.)
Proficient	**Highly Proficient**
Apply properties of operations as strategies to multiply and divide. Properties include commutative and associative properties of multiplication and the distributive property. (Students do not need to use the formal terms for these properties.)	Use multiple strategies of operations to multiply and divide within a word problem.

3.OA.B.6

Content Standards	Understand division as an unknown-factor problem (e.g., find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8).
Explanations	Multiplication and division are inverse operations and that understanding can be used to find the unknown. Fact family triangles demonstrate the inverse operations of multiplication and division by showing the two factors and how those factors relate to the product and/or quotient. Students use their understanding of the meaning of the equal sign as “the same as” to interpret an equation with an unknown. Equations in the form of $a \div b = c$ and $c = a \div b$ need to be used interchangeably, with the unknown in different positions.
Content Limits	Whole numbers. Quotients up to 100.
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to write division problems as equivalent multiplication problems.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify division as unknown factor problems by finding missing number in the second factor position with visual support/ arrays.	Solve division as unknown factor problems by finding missing number in the second factor position with visual support/arrays.
Proficient	**Highly Proficient**
Understand division as an unknown-factor problem (e.g., find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8).	Solve division as unknown factor problems by using the relationship between multiplication and division. Model multiplication and division in a variety of ways.

3.OA.C.7

Content Standards	Fluently multiply and divide within 100. By the end of Grade 3, know from memory all multiplication products through 10 x 10 and division quotients when both the quotient and divisor are less than or equal to 10.
Explanations	By studying patterns and relationships in multiplication facts and relating multiplication and division, students build a foundation for fluency with multiplication and division facts. Students demonstrate fluency with multiplication facts through 10 and the related division facts. Multiplying and dividing fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently. General Note: Students should have exposure to multiplication and division problems presented in both vertical and horizontal forms.
Content Limits	Whole numbers. Multiply and divide within 100. Factors, divisors, and quotients should be 10 or less.
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to find the product or dividend.	• Equation Response • Multiple Choice Response • Multi-Select Response • Table Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Multiply and divide within 100 using visual support/arrays.	Organize expressions to multiply and divide within 100 using visual support/ arrays.
Proficient	**Highly Proficient**
Fluently multiply and divide within 100. By the end of Grade 3, know from memory all multiplication products through 10 x 10 and division quotients when both the quotient and divisor are less than or equal to 10.	Fluently multiply and divide within 100 within range of contexts.

3.OA.D.8

Content Standards	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Utilize understanding of Order of Operations when there are no parentheses.
Explanations	Students should be exposed to multiple problem-solving strategies (using any combination of words, numbers, diagrams, physical objects or symbols) and be able to choose which ones to use. When students solve word problems, they use various estimation skills which include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of solutions.
Content Limits	Dividends up to 100. Whole number dividends. Whole number quotients.
Context	Context is required.

Sample Task Demands	Common Item Formats
Students will be required to determine a solution to a two-step word problem.	• Equation Response • Graphic Response • Multiple Choice Response • Proposition Response
Students will be required to determine whether an answer is reasonable based on estimation and/or rounding.	
Students will be required to construct an equation that models a multi-step word problem.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Solve one-step word problems using the four operations with visual support/arrays. Represent these problems using equations with a letter standing for the unknown quantity. Utilize understanding of the Order of Operations when there are no parentheses.	Solve two-step word problems using the four operations using visual support. Represent these problems using equations with a letter standing for the unknown quantity. Utilize understanding of the Order of Operations when there are no parentheses.
Proficient	**Highly Proficient**
Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Utilize understanding of the Order of Operations when there are no parentheses.	Solve two-step word problems with large whole numbers and using multiple operations.

3.OA.D.9

Content Standards	Identify patterns in the addition table and the multiplication table and explain them using properties of operations (e.g. observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends).
Explanations	Students need ample opportunities to observe and identify important numerical patterns related to operations. They should build on their previous experiences with properties related to addition and subtraction. Students investigate addition and multiplication tables in search of patterns and explain why these patterns make sense mathematically. Students also investigate a hundreds chart in search of addition and subtraction patterns. They record and organize all the different possible sums of a number and explain why the pattern makes sense.
Content Limits	Adding and subtracting whole numbers within 1,000. Multiplying and dividing whole numbers within 100.
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify numbers in a well-known pattern, such as an addition or multiplication table.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response • Proposition Response • Table Response
Students will be required to identify unknown numbers in a pattern.	
Students will be required to identify the pattern in a sequence of numbers.	
Students will be required to determine characteristics or trends across numerical situations such as sum, doubles, and/or multiples.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify addition patterns using visual supports.	Identify multiplication and subtraction patterns using visual supports.
Proficient	**Highly Proficient**
Identify patterns in the addition table and the multiplication table and explain them using properties of operations (e.g. observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends).	Create and extend arithmetic patterns, explain patterns using properties of operations.

3.OA.D.10

Content Standards	When solving problems, assess the reasonableness of answers using mental computation and estimation strategies including rounding.
Explanations	Solve problems involving the four operations, and identify and explain patterns in arithmetic.
Content Limits	Multiplication through 10 x 10 Division with quotients and divisors less than or equal to 10 Addition and subtraction is limited to 1,000. Rounding is to the nearest 10 or the nearest 100. Operations are only with whole numbers.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to determine the best estimation strategy given the context of a situation.	• Equation Response • Graphic Response • Multiple Choice Response • Table Response
Students will be required to determine whether an answer is appropriate in a given context.	
Students will be required to recognize when an estimation strategy is or is not appropriate.	
Students will be required to use estimation strategies to solve a problem.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Recognize whether an answer is reasonable or not when rounding.	Use rounding to determine the reasonableness of answers when using the four operations to solve problems.
Proficient	**Highly Proficient**
When solving problems, assess the reasonableness of answers using mental computation and estimation strategies including rounding.	Recognize the reasonableness of answers using different types of estimation strategies when using the four operations to solve problems. Choose the best estimation strategy for a specific purpose.

3.NBT.A.1

Content Standards	Use place value understanding to round whole numbers to the nearest 10 or 100.
Explanations	Students learn when and why to round numbers. They identify possible answers and halfway points. Then they narrow where the given number falls between the possible answers and halfway points. They also understand that by convention if a number is exactly at the halfway point of the two possible answers, the number is rounded up.
Content Limits	Whole numbers up to 1000. Avoid situations where the place the student rounded to is ambiguous. For example, asking a student to round 697 to the nearest ten is not a good item, as the student would get the exact same answer if he or she mistakenly rounded to the nearest hundred.
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify the value of a given number rounded to the nearest 10 or 100.	• Equation Response • Graphic Response • Matching Item Response • Multi-Select Response • Table Response
Students will be required to identify the numbers that round to a given value.	
Students will be required to plot points to represent values that round to a given value.	
Students will be required to interpret and distinguish between different rounding procedures used in rounding to a number in order to create a number that fits certain parameters.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Use place value understanding to round a two-digit number to the nearest 10.	Use place value understanding to round a three-digit number to the nearest 100.
Proficient	**Highly Proficient**
Use place value understanding to round whole numbers to the nearest 10 or 100.	Use rounding strategies in real-world situations.

3.NBT.A.2

Content Standards	Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
Explanations	Problems should include both vertical and horizontal forms, including opportunities for students to apply the commutative and associative properties. Adding and subtracting fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently. Students explain their thinking and show their work by using strategies and algorithms, and verify that their answer is reasonable. An interactive whiteboard or document camera may be used to show and share student thinking.
Content Limits	Addends and sums are less than or equal to 1000. Minuends, subtrahends, and differences are less than or equal to 1000.
Context	Context is not allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to calculate the sum or difference of two or more numbers.	• Equation Response • Multi-Select Response

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Fluently add and subtract within 1000 using strategies and algorithms based on the relationship between addition and subtraction.	Fluently add and subtract within 1000 using strategies and algorithms based on place value and/or the relationship between addition and subtraction.
Proficient	**Highly Proficient**
Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	Explain the method used in finding the sum or difference; recognize and identify an error and shows the correct answer.

3.NBT.A.3

Content Standards	Multiply one-digit whole numbers by multiples of 10 in the range 10 to 90 using strategies based on place value and the properties of operations (e.g., 9 x 80, 5 x 60).
Explanations	Students use base ten blocks, diagrams, or hundreds charts to multiply one-digit numbers by multiples of 10 from 10-90. They apply their understanding of multiplication and the meaning of the multiples of 10.
Content Limits	Largest product is 810 (9 x 90 = 810)
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to calculate the product of a one-digit number by a multiple of 10 without context.	• Equation Response • Matching Item Response • Multi-Select Response
Students will be required to calculate the product of a one-digit number by a multiple of 10 within the context of a word problem.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Skip count by 10, 20 or 50 to multiply single-digit whole numbers by multiples of 10 in the range 10-90.	Use grouping strategies (associative property) to multiply single-digit whole numbers by multiples of 10 in the range 10-90.
Proficient	**Highly Proficient**
Multiply one-digit whole numbers by multiples of 10 in the range 10 to 90 using strategies based on place value and the properties of operations (e.g., 9 x 80, 5 x 60).	Show product of single-digit whole numbers by multiples of 10 using multiple strategies.

Numbers and Operations – Fractions

3.NF.A.1

Content Standards	Understand a fraction (1/*b*) as the quantity formed by one part when a whole is partitioned into b equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.
Explanations	Students express fractions as fair sharing, parts of a whole, and parts of a set. They use various contexts (candy bars, fruit, and cakes) and a variety of models (circles, squares, rectangles, fraction bars, and number lines) to develop understanding of fractions and represent fractions. Students need many opportunities to solve word problems that require fair sharing. To develop understanding of fair shares, students first participate in situations where the number of objects is greater than the number of children and then progress into situations where the number of objects is less than the number of children.
Content Limits	Denominators limited to 2, 3, 4, 6, and 8. Combining or putting together unit fractions rather than formal addition or subtraction of fractions. Maintain concept of a whole as one entity that can be equally partitioned in various ways when working with unit fractions. Limit usage of the words numerator and denominator in items—focus should not be on assessing vocabulary terms. Fractions *a*/*b* can be improper fractions and students should not be guided to put fractions in lowest terms or to simplify. Focus more on area models since 3.NF.2 uses number lines exclusively.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify a model given a fraction.	• Equation Response • Graphic Response • Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to identify a fraction given a model.	
Students will be required to partition a whole into equal parts and identify that each part is a unit fraction.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify a fraction (1/*b*) as the quantity formed by one part when a whole is partitioned into *b* equal parts given visual support.	Understand a fraction (1/*b*) as the quantity formed by one part when a whole is partitioned into *b* equal parts.
Proficient	**Highly Proficient**
Understand a fraction (1/*b*) as the quantity formed by one part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by a parts of size 1/*b*.	Apply understanding of unit fractions to real world, multi-step problems.

3.NF.A.2, 3.NF.A.2a, 3.NF.A.2b, and 3.NF.A.2c

Content Standards	**3.NF.A.2** Understand a fraction as a number on the number line; represent fractions on a number line diagram. **3.NF.A.2a** Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Understand that each part has size 1/*b* and that the end point of the part based at 0 locates the number 1/*b* on the number line. **3.NF.A.2b** Represent a fraction *a*/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Understand that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line including values greater than 1. **3.NF.A.2c** Understand a fraction 1/*b* as a special type of fraction that can be referred to as a unit fraction (e.g. 1/2, 1/4).
Explanations	Students transfer their understanding of parts of a whole to partition a number line into equal parts. New is the inclusion of fractions greater than one, and the emphasis on the importance of understanding unit fractions.
Content Limits	Denominators limited to 2, 3, 4, 6, and 8 Models restricted to number lines starting at 0. Part A: number line interval from 0 to 1. Part B: number lines can extend from 0 to 1+.
Common Item Formats	The Item Formats section on pages 10 through 12 provides a list of item formats that may be used to assess this standard. The common item formats include, but are not limited to, those shown with the sample task demands.
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify and represent unit fractions of 1/*b* on a number line.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response
Students will be required to identify and represent fractions of size *a*/*b* as "*a*" 1/*b* sized segments on the number line starting from 0.	
Students will be required to identify and interpret fractional values on number lines.	
Students will be required to reason and draw conclusions about partitioning wholes and constructing fractional models, and will be required to compare fractions and justify decisions using number line representations.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Identify a unit fraction as being between 0 and 1 on a number line. b. Recognize a partition that creates 1/2 or 1/4 on a number line. c. Recognize that if 1 is in the numerator of a fraction, then it is a unit fraction.	Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Identify 1/2 and 1/3 on a number line. b. Identify the partitions on a number line that would represent common fractions. c. Identify unit fractions.
Proficient	**Highly Proficient**
Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Understand that each part has size 1/*b* and that the end point of the part based at 0 locates the number 1/*b* on the number line. b. Represent a fraction *a*/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Understand that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line including values greater than 1. c. Understand a fraction 1/*b* as a special type of fraction that can be referred to as a unit fraction (e.g. 1/2, 1/4).	Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Create a number line to locate any unit fraction that represents a real world value b. Create a number line to locate fractions greater than 1 that represents a real world value c. Create unit fractions to compare values in multi-step, real world contexts

3.NF.A.3, 3.NF.A.3a, 3.NF.A.3b, 3.NF.A.3c, and 3.NF.A.3d.

Content Standards	**3.NF.A.3** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. **3.NF.A.3a** Understand two fractions as equivalent if they have the same relative size compared to 1 whole. **3.NF.A.3b** Recognize and generate simple equivalent fractions. Explain why the fractions are equivalent. **3.NF.A.3c** Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. **3.NF.A.3d** Compare two fractions with the same numerator or the same denominator by reasoning about their size. Understand that comparisons are valid only when the two fractions refer to the same whole. Record results of comparisons with the symbols >, =, or <, and justify conclusions.
Explanations	An important concept when comparing fractions is to look at the size of the parts and the number of the parts. Students recognize when examining fractions with common denominators, the wholes have been divided into the same number of equal parts. So the fraction with the larger numerator has the larger number of equal parts. To compare fractions that have the same numerator but different denominators, students understand that each fraction has the same number of equal parts but the size of the parts are different. They can infer that the same number of smaller pieces is less
Content Limits	Denominators limited to 2, 3, 4, 6, and 8. Fractions must refer to the same whole unless intent of item is to assess reasoning about wholes. The vocabulary of lowest terms or simplify should not be used. Limit to a maximum of 3 when ordering fractions. Visual models primarily used include number lines and area models (circles, rectangles, regular polygons - see shapes from geometry standards).
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to represent equivalent fractions.	• Graphic Response • Multiple Choice Response • Matching Item Response • Multi-Select Response • Table Response
Students will be required to compare fractions with the same denominator.	
Students will be required to express whole numbers as fractions (over 1) and recognize equivalent fraction forms of whole numbers (n*p/0p).	
Students will be required to represent and explain equivalent fractions by creating fraction models.	
Students will be required to compare fractions with the same numerator and unlike denominators.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand equivalent fractions using denominators of 2, 4 and 8 given visual models. b. Recognize and generate equivalent fractions using denominators of 2, 4 and 8 given visual models. c. Express and recognize fractions that are equivalent to 1. d. Compare two fractions with the same denominator and records results using symbols.	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand equivalent fractions using denominators of 2, 4 and 8. b. Recognize and generate equivalent fractions using denominators of 2, 4 and 8. c. Express and recognize fractions that are equivalent to whole numbers. d. Compare two fractions with the same numerator and records results using symbols.

Proficient	Highly Proficient
Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent if they have the same relative size compared to 1 whole. b. Recognize and generate simple equivalent fractions. Explain why the fractions are equivalent. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Understand that comparisons are valid only when the two fractions refer to the same whole. Record results of comparisons with the symbols >, =, or <, and justify conclusions.	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Identify equivalent fractions by creating fraction models to compare fractions that pertain to the same whole. b. Explain why two fractions are equivalent. Identify equivalent fractions by creating fraction models to compare fractions that pertain to the same whole. c. Express whole numbers as fractions with denominators greater than 1. d. Create and compare two fractions that have the same numerator or same denominator using symbols within a context.

Measurement and Data & Geometry

3.MD.A.1a and 3.MD.A.1b

Content Standards	**3.MD.A.1a** Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes (e.g., representing the problem on a number line diagram). **3.MD.A.1b** Solve word problems involving money through $20.00, using symbols $, ".", ¢.
Explanations	Students in second grade learned to tell time to the nearest five minutes. In third grade, they extend telling time in minute intervals and measure elapsed time using clocks and number lines in an abstract sense or within a larger context. Students in second grade learn to solve problems involving collections of dollar bills, dimes, nickels, and pennies as well as recoring totals using $ and ¢. In third grade, they extend beyond finding totals of money collections to solving a wider variety of problems involving money through $20.00.
Content Limits	Times should be to the nearest minute. Addition and subtraction Problems involving money are limited to $20.00.
Context	Context is allowed.

<table>
<tr><th>Sample Task Demands</th><th>Common Item Formats</th></tr>
<tr><td>Students will be required to recognize and identify a time shown to a single-minute increment on a clock.</td><td rowspan="5">Equation Response
Graphic Response
Multiple Choice Response
Table Response</td></tr>
<tr><td>Students will be required to calculate a change of time.</td></tr>
<tr><td>Students will be required to show change of time on a number line or clock.</td></tr>
<tr><td>Students will be required to construct a schedule by adding and subtracting time intervals.</td></tr>
<tr><td>Students will be required to determine the sum and/or difference of values using symbols $, ".", ¢.</td></tr>
</table>

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Solve problems involving measurement. a. Tell, write, and measure time to the nearest minute. b. Can add money using symbols $, ".", ₵.	Solve problems involving measurement. a. Solve one-step word problems involving addition or subtraction of time intervals in minutes with scaffolding. b. Can add money using symbols $, ".", ₵.
Proficient	**Highly Proficient**
Solve problems involving measurement. a. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes (e.g., representing the problem on a number line diagram). b. Solve word problems involving money through $20.00, using symbols $, ".", ₵.	Solve problems involving measurement. a. Create and solve multi-step time interval problems. b. Solve two-step word problems involving money through $20 using symbols $, ".", ₵.

3.MD. A.2

Content Standards	Measure and estimate liquid volumes and masses of objects using metric units. (Excludes compound units such as cm^3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units. Excludes multiplicative comparison problems (problems involving notions of "times as much").
Explanations	Students need multiple opportunities weighing classroom objects and filling containers to help them develop a basic understanding of the size and weight of a liter, a gram, and a kilogram. Milliliters may also be used to show amounts that are less than a liter.
Content Limits	Excludes compound units such as cm^3 and finding the geometric volume of a container. Excludes multiplicative comparison problems (problems involving notions of "times as much").
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to identify a given measured amount.	• Equation Response • Multiple Choice Response
Students will be required to estimate an unknown quantity by comparing it with a given measurement.	
Students will be required to interpret and calculate a one-step word problem involving measurement.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Using grams, kilograms or liters, measure and estimate liquid volumes and masses of objects using models.	Using grams, kilograms or liters, solve simple one-step measurement word problems using either addition or subtraction.
Proficient	**Highly Proficient**
Measure and estimate liquid volumes and masses of objects using metric units. (Excludes compound units such as cm^3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units. Excludes multiplicative comparison problems (problems involving notions of "times as much").	Using grams, kilograms or liters, estimate and solve multi-step measurement word problems involving any of the four operations.

3.MD.B.3

Content Standards	Create a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs.
Explanations	Students should have opportunities reading and solving problems using scaled graphs before being asked to draw one. The following graphs all use five as the scale interval, but students should experience different intervals to further develop their understanding of scale graphs and number facts.
Content Limits	Categories are five or fewer and use multiplication and division within 100.
Context	Context is required.

Sample Task Demands	Common Item Formats
Students will be required to compare two or more data values from a given graph to solve one- and two-step word problems.	• Equation Response • Graphic Response • Table Response
Students will be required to construct a scaled bar or picture graph based on given data.	
Students will be required to create a scale for given data and construct a graph.	
Students will be required to construct a scaled bar or picture graph based on parameters.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Complete a scaled picture graph or bar graph (with a scale factor of 1 or 5) to represent data set with support.	Complete a scaled picture graph or bar graph to represent a data set with support. Solve one-step "how many more" and "how many less" problems using information presented in scaled bar graphs.
Proficient	**Highly Proficient**
Create a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs.	Create own scale and graph based on given data parameters.

3.MD.B.4

Content Standards	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch to the nearest quarter-inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.
Explanations	Students in second grade measured length in whole units using both metric and U.S. customary systems. It's important to review with students how to read and use a standard ruler including details about halves and quarter marks on the ruler. Students should connect their understanding of fractions to measuring to one-half and one-quarter inch. Third graders need many opportunities measuring the length of various objects in their environment. Some important ideas related to measuring with a ruler are: The starting point of where one places a ruler to begin measuring; Measuring is approximate (Items that students measure will not always measure exactly 1/4, 1/2 or one whole inch. Students will need to decide on an appropriate estimate length); Making paper rulers and folding to find the half and quarter marks will help students develop a stronger understanding of measuring length. Students generate data by measuring and create a line plot to display their findings.
Content Limits	Units are limited to whole numbers, halves, or quarters. Standard rulers should not be used - only special rulers that are marked off in halves or quarters. Measurements are limited to inches.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to measure the length of a given object.	• Equation Response • Graphic Response • Matching Item Response • Multi-Select Response
Students will be required to classify and/or sort objects based on their measure.	
Students will be required to construct a line plot for given data.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Generate measurement data by measuring lengths to the nearest half-inch. Show the data by making a line plot, where the horizontal scale is marked by whole numbers or halves with supports.	Generate measurement data by measuring lengths to the nearest quarter-inch. Show the data by making a line plot, where the horizontal scale is marked by whole numbers, halves, or quarters with supports.
Proficient	**Highly Proficient**
Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch to the nearest quarter-inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.	Show the data by making a line plot, where the student decides whether the horizontal scale is marked by whole numbers, halves, or quarters based on the given data.

3.MD.C.5, 3.MD.C.5a, and 3.MD.C.5b

Content Standards	**3.MD.C.5** Understand area as an attribute of plane figures and understand concepts of area measurement. **3.MD.C.5a** A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area. **3.MD.C.5b** A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.
Explanations	Students develop understanding of using square units to measure area by: Using different sized square units, filling in an area with the same sized square units and counting the number of square units. Using different sized graph paper, students can explore the areas measured in square centimeters and square inches.
Content Limits	Plane figures that can be covered by unit squares. Note: Exponential notation is not expected at this grade level (square cm is acceptable, but cm^2 is not)
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify what the area of a figure means and represents. (5a/5b)	• Equation Response • Multiple Choice Response • Multi-Select Response
Students will be required to recognize a square with side length 1 unit as a unit square. (5a)	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Understand area as an attribute of plane figures and understand concepts of area measurement. a. Can identify a square unit. b. Can distinguish area from length and width.	Understand area as an attribute of plane figures and understand concepts of area measurement. a. Understand area is measured using square units. b. Recognize overlapping and gaps in square unit place on a figure would not accurately describe area
Proficient	**Highly Proficient**
Understand area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.	Understand area as an attribute of plane figures and understand concepts of area measurement. a. Can identify and use different unit squares. b. Cover a plane figure with unit squares of different sizes to show that the area of the same figure can be expressed as different numbers in different units.

3.MD.C.6

Content Standards	Measure areas by counting unit squares (e.g., square cm, square m, square in, square ft, and improvised units).
Explanations	Students develop understanding of using square units to measure area by: Using different sized square units, filling in an area with the same sized square units and counting the number of square units. Using different sized graph paper, students can explore the areas measured in square centimeters and square inches.
Content Limits	Plane figures that can be covered by unit squares. Note: Exponential notation is not expected at this grade level (square cm is acceptable, but cm^2 is not)
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to find the area of a rectilinear figure by counting squares.	• Equation Response • Multiple Choice Response • Multi-Select Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Find the area of a rectangle by counting squares covering the rectangle and expresses the area without units.	Find area of a rectangle by counting unit squares.
Proficient	**Highly Proficient**
Measure areas by counting unit squares (e.g., square cm, square m, square in, square ft, and improvised units).	Find the area of 2 plane figures by creating and counting unit squares.

3.MD.C.7, 3.MD.C.7a, 3.MD.C.7b, 3.MD.C.7c, and 3.MD.C.7d

Content Standards	**3.MD.C.7** Relate area to the operations of multiplication and addition. **3.MD.C.7a** Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. **3.MD.C.7b** Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. **3.MD.C.7c** Use tiling to show that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. **3.MD.C.7d** Understand that rectilinear figures can be decomposed into non-overlapping rectangles and that the sum of the areas of these rectangles is identical to the area of the original rectilinear figure. Apply this technique to solve problems in real-world contexts.
Explanations	Students tile areas of rectangles, determine the area, record the length and width of the rectangle, investigate the patterns in the numbers, and discover that the area is the length times the width.
Content Limits	Rectangles and shapes that can be decomposed into rectangles. Whole-number side lengths Multiplication is within 100.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to find the area of a rectangle using various strategies, such as multiplying side lengths and using tiling to demonstrate the distributive property as it relates to area.	• Equation Response • Graphic Response • Matching Item Response • Multi-Select Response
Students will be required to find the area of rectilinear figures by decomposing them into non-overlapping rectangles.	
Students will be required to draw conclusions about unknown side lengths in order to calculate the area of a rectilinear figure.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Relate area to the operations of multiplication and addition. a. Find the area of one rectangles by tiling. b. Multiply side lengths with both sides less than or equal to 5 to find area. c. Determine a missing value in an area model that represents the distributive property where all values are less than of equal to 5. d. Find the area of a rectilinear figure that is composed of two rectangles with side lengths less than or equal to 5 in a mathematical context.	Relate area to the operations of multiplication and addition. a. Show that the area of a rectangle found by tiling is the same as would be found by multiplying the side lengths. b. Multiply side lengths with one side less than or equal to 5 to find area. c. Determines a missing value in an area model that represents the distributive property. d. Find the area of a simple decomposition.

Proficient	Highly Proficient
Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. d. Understand that rectilinear figures can be decomposed into non-overlapping rectangles and that the sum of the areas of these rectangles is identical to the area of the original rectilinear figure. Apply this technique to solve problems in real-world contexts.	Relate area to the operations of multiplication and addition. a. Confirm tiling and multiplication of side lengths in self-created example. b. Compare the area of 2 plane figures by multiplying their side lengths and compares their sizes. c. Create a word problem using the distributive property to find the area of rectangles. d. Design area problems in which decomposition is integral to understanding and solving the problem.

3.MD.C.8

Content Standards	Solve real-world and mathematical problems involving perimeters of plane figures and areas of rectangles, including finding the perimeter given the side lengths, finding an unknown side length. Represent rectangles with the same perimeter and different areas or with the same area and different perimeters.
Explanations	Students develop an understanding of the concept of perimeter by walking around the perimeter of a room, using rubber bands to represent the perimeter of a plane figure on a geoboard. They find the perimeter of objects; use addition to find perimeters; and recognize the patterns that exist when finding the sum of the lengths and widths of rectangles.
Content Limits	Polygons that can be tiled with square units. Whole-number side lengths Multiplication is within 100.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to construct a polygon with a given perimeter or area.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response
Students will be required to find the perimeter of a polygon given the side lengths.	
Students will be required to find an unknown side length of a polygon given the perimeter.	
Students will be required to construct a rectangle with a given perimeter based on area (or a given area based on perimeter).	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Find the perimeter of plane figures (given the side lengths).	Solve mathematical problems involving perimeters of plane figures, understand the difference in area and perimeter.
Proficient	**Highly Proficient**
Solve real-world and mathematical problems involving perimeters of plane figures and areas of rectangles, including finding the perimeter given the side lengths, finding an unknown side length. Represent rectangles with the same perimeter and different areas or with the same area and different perimeters.	Construct rectangles that have the same perimeter but different areas and the reverse.

Measurement and Data & Geometry

3.G.A.1

Content Standards	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples quadrilaterals that do not belong to any of these subcategories.
Explanations	In third grade, students identify and draw triangles, quadrilaterals, pentagons, and hexagons. Third graders build on this experience and further investigate quadrilaterals (technology may be used during this exploration). Students recognize shapes that are and are not quadrilaterals by examining the properties of the geometric figures. They conceptualize that a quadrilateral must be a closed figure with four straight sides and begin to notice characteristics of the angles and the relationship between opposite sides. Students should be encouraged to provide details and use proper vocabulary when describing the properties of quadrilaterals. They sort geometric figures and identify squares, rectangles, and rhombuses as quadrilaterals.
Content Limits	Shapes include two dimensional shapes and the following quadrilaterals in particular: rhombus, rectangle, and square. Attributes include: number of sides, number of angles, whether the shape has a right angle or not, whether sides are the same length or not, whether the sides are straight lines or not. Do not include references to parallel or perpendicular sides - this is assessed in grade 4.
Context	Context is not allowed.

<table>
<tr><th>Sample Task Demands</th><th>Common Item Formats</th></tr>
<tr><td>Students will be required to select attributes that are shared by a set of shapes.</td><td rowspan="6">• Graphic Response
• Multiple Choice Response
• Matching Item Response
• Multi-Select Response
• Open Response</td></tr>
<tr><td>Students will be required to select shapes that belong to the same sub-categories.</td></tr>
<tr><td>Students will be required to use a set of attributes to construct a shape.</td></tr>
<tr><td>Students will be required to use a set of attributes to name a shape.</td></tr>
<tr><td>Students will be required to classify shapes based on attributes.</td></tr>
<tr><td>Students will be required to explain why the shapes were classified in a given way, given a set of shapes in two groups.</td></tr>
</table>

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify properties of squares.	Understand the properties of quadrilaterals and the subcategories of quadrilaterals.
Proficient	**Highly Proficient**
Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples quadrilaterals that do not belong to any of these subcategories.	Recognize and sort examples of quadrilaterals that have shared attributes and that the shared attributes can define a larger category; draw examples and non-examples of quadrilaterals that are not rhombuses, rectangles, or squares.

3.G.A.2

Content Standards	Partition shapes into b parts with equal areas. Express the area of each part as a unit fraction 1/*b* of the whole. (Grade 3 expectations are limited to fractions with denominators *b* = 2,3,4,6,8.)
Explanations	Given a shape, students partition it into equal parts, recognizing that these parts all have the same area. They identify the fractional name of each part and are able to partition a shape into parts with equal areas in several different ways.
Content Limits	Fractions can have denominators of 2, 3, 4, 6, and 8 (per 3.NF). The fractions must be unit fractions. Shapes include quadrilateral (rhombus, rectangle, square, isosceles trapezoid), isosceles triangle, regular hexagon, circle (these are all the shapes covered in geometry standards K-3). The shape used and the number of partitions should be suitable for this grade. For example, having a student partition a hexagon into 6 parts is acceptable, but 8 is not.
Context	Context is not allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to recognize the fraction an area of a shape represents.	• Equation Response • Graphic Response • Multi-Select Response • Table Response
Students will be required to identify the shapes that are divided into equal parts.	
Students will be required to partition a shape into equal areas.	
Students will be required to shade a fraction of shape.	
Students will be required to match given partitions with the fraction each represents.	
Students will be required to construct a complete shape given only one of the partitioned areas of the whole shape.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Partition shapes into b parts with equal areas. Express the area of each part as a unit fraction 1/*b* of the whole. (limited to halves and quarters).	Partition shapes into b parts with equal areas. Express the area of each part as a unit fraction 1/*b* of the whole. (limited to halves, quarters, and eighths).
Proficient	**Highly Proficient**
Partition shapes into b parts with equal areas. Express the area of each part as a unit fraction 1/*b* of the whole. (Grade 3 expectations are limited to fractions with denominators *b* = 2,3,4,6,8.)	Partition shapes into parts with equal areas and expresses the area as a unit fraction of the whole to answer questions presented in a context.

AzM2 | Arizona's Statewide Achievement Assessment for English Language Arts and Mathematics

Mathematics Item Specifications

GRADE 4

Arizona Department of Education with American Institutes for Research - 2019

Table of Contents

Introduction

The Arizona Statewide Achievement Assessment for English Language Arts and Mathematics (AzM2) is Arizona's statewide achievement test. AzM2 assesses the Arizona English Language Arts Standards and Arizona Mathematics Standards adopted by the Arizona State Board of Education in December 2016. AzM2 will inform students, teachers, and parents about preparedness for college and careers upon graduating from high school. AzM2 tests are computer-based, meaning that they can better assess students' critical thinking skills and provide them with opportunities to demonstrate a deeper understanding of the materials. Computer-based testing also allows for the use of a variety of innovative items types.

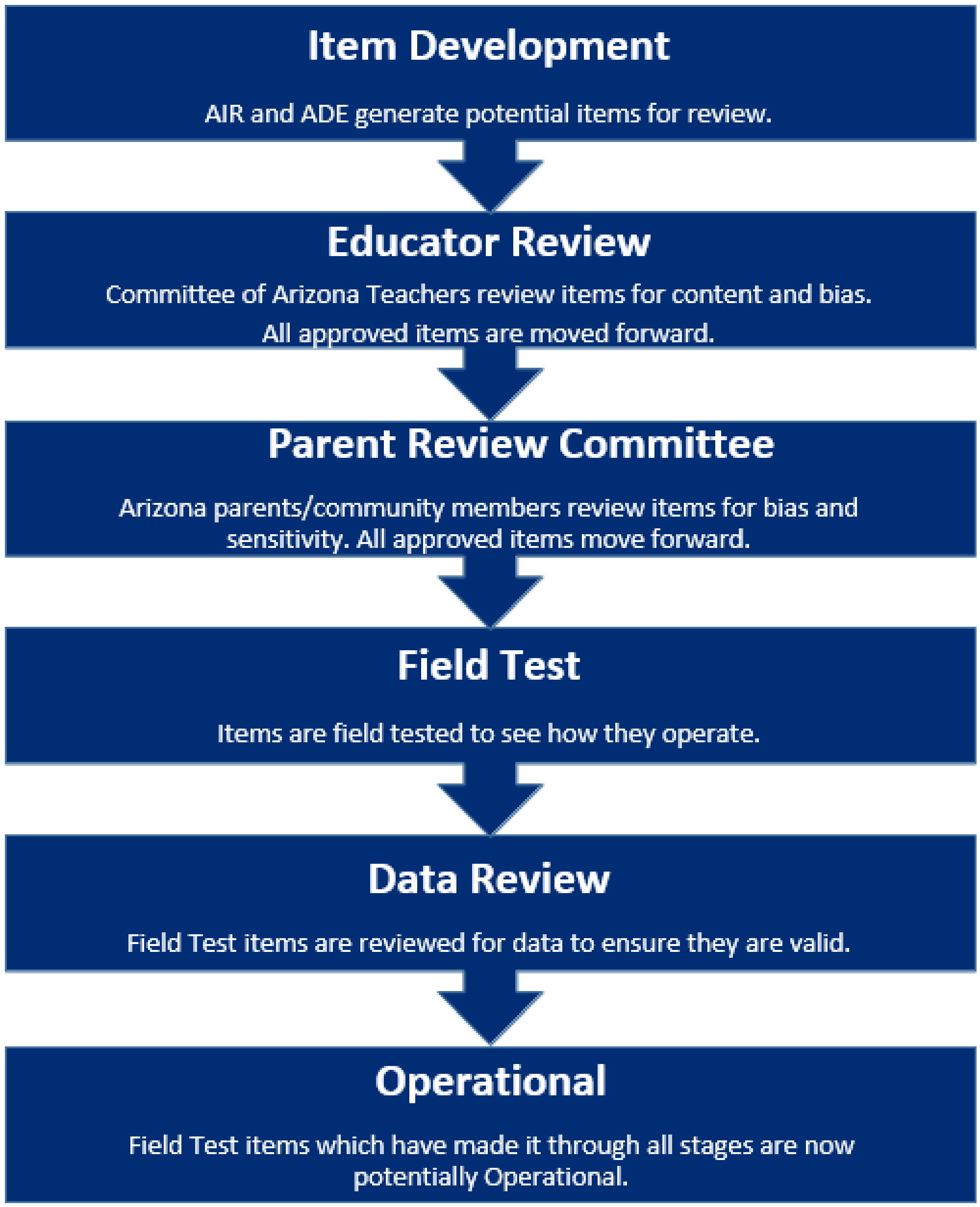
During the item-development process, all AzM2 items are written in accordance with the Item Specifications and are reviewed and approved by a committee of Arizona educators to confirm alignment and appropriateness for inclusion in the test. AzM2 items are generally representative of Arizona's geographic regions and culturally diverse population. Items are reviewed for the following kinds of bias: gender, racial, ethnic, linguistic, religious, geographic, and socioeconomic. Item reviews also include consideration of issues related to individuals with disabilities. Arizona community members also have an opportunity to review items for issues of potential concern to members of the community at large. Reviewers are asked to consider the variety of cultural, regional, philosophical, political, and religious backgrounds throughout Arizona, and then to determine whether the subject matter will be acceptable to Arizona students, families, and other members of Arizona communities.

This *AzM2 Item Specifications* is a resource document that defines the content and format of the test and test items for item writers and reviewers. Each *Item Specifications* document indicates the alignment of items with the Arizona Mathematics Standards. It also serves to provide all stakeholders with information about the scope and function of assessment items. This document can also serve to assist educators to understand how assessment items are developed in alignment with the standards for English language arts and math. These item specifications for AzM2 are intended to provide information regarding standards, item formats and response types. The descriptions of blueprints, and depth of knowledge in this document are meant to provide an overview of the test. Item specifications are meant for the purposes of assessment, not instruction. They are not intended to be tools for instruction or the basis for curricula. AzM2 has a test blueprint that was developed by Arizona and is different from any other state or consortium test blueprint.

For the math portion of AzM2, all of the test questions are aligned to the mathematic content standards for these subject areas. Any item specifications that are absent for standards listed in this document may be under development. This document does not endorse the exclusion of the instruction of any grade-level content standards. The test will ask questions that check a student's conceptual understanding of math as well as their procedural skills. These items have been written to be free from bias and sensitivity, and widely vary in their degree of difficulty.

Item Development Process

AzM2 items go through a rigorous review before they are operational. When an item is "operational" it means it is used to determine a student's score on the assessment. This is a description of the process every item must go through before it is operational on AzM2.



Sample tests are available online for the math portion of AzM2. For more information view the Guide to the Sample Tests at [www.AzM2portal.org.](http://www.AzM2portal.org)

Test Construction Guidelines

The construction of the AzM2 assessment is guided by the depth and rigor of the Arizona College and Career Ready Standards. Items are created to address key components of the standards and assess a range of important skills. The AzM2 Blueprint provides an overview of the distribution of items on the AzM2 according to the standards. The standards for Math Practices are embedded within all AzM2 items. Further, the AzM2 blueprint outlines the Depth of Knowledge distribution of items.

Blueprint

Grade 4 AzM2 Math Blueprint 2016 Standards		
Reporting Category	**Min.**	**Max.**
Operations and Algebraic Thinking and Numbers & Operations in Base Ten	**46%**	**54%**
Operations & Algebraic Thinking	22%	26%
Numbers in Base Ten	24%	28%
Numbers and Operations - Fractions	**29%**	**33%**
Measurement, Data, and Geometry	**15%**	**19%**
Measurement and Data	9%	13%
Geometry	4%	7%

Depth of Knowledge (DOK)

DOK refers to the level of rigor or sophistication of the task in a given item, designed to reflect the complexity of the Arizona Mathematics Standards. Items at DOK level 1 focus on the recall of information, such as definitions, terms, and simple procedures. Items at DOK 2 require students to make decisions, solve problems, or recognize patterns; in general, they require a greater degree of engagement and cognitive processing than items at DOK 1. Items at DOK 3 feature higher-order cognitive tasks that assess students' capacities to approach abstract or complex problems.

Percentage of Points by Depth of Knowledge (DOK) Level			
Grade 4	DOK Level 1	DOK Level 2	DOK Level 3
	10% - 20%	60% - 70%	12% - 30%

For more information on DOK go to www.azed.gov/AzM2.

Calculators

Arizona Desmos Graphing Calculator is not permitted for the paper-based and computer-based assessment for Grade 4 Math.

Item Formats

The AzM2 Assessments are composed of item formats that include traditional multiple- choice response items and technology-enhanced response items (TEI). TEIs are computer- delivered response items that require students to interact with test content to select, construct, and/or support their responses. TEIs are better able to assess a deeper level of understanding.

Currently, there are nine types of TEIs that may appear on the Math computer-based assessment for AzM2:

- Editing Tasks (ET)
- Editing Task Choice (ETC)
- Equation Editor (EQ)
- Graphic Response Item Display (GRID)
- Hot Text (HT)
  - Selectable Hot Text
  - Drag-and-Drop Hot Text
- Matching Item (MI)
- Multi-Select (MS)
- Open Response
- Table Item (TI)

For paper-based assessments (including those for students with an IEP or 504 plan that specifies a paper based accommodation), TEIs will be modified so that they can be scanned and scored electronically or hand-scored.

See the table below for a description of each TEI. In addition, for examples of each response item format described, see the AzM2 Training Tests at www.AzM2portal.org.

Item Format	Description
Editing Task (ET)	The student clicks on a highlighted word or phrase that may be incorrect, which reveals a text box. The directions in the text box direct the student to replace the highlighted word or phrase with the correct word or phrase. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Editing Task Choice (ETC)	The student clicks a highlighted word or phrase, which reveals a drop-down menu containing options for correcting an error as well as the highlighted word or phrase as it is shown in the sentence to indicate that no correction is needed. The student then selects the correct word or phrase from the drop-down menu. For paper-based assessments, the item is modified so that it can be scanned and scored electronically. The student fills in a circle to indicate the correct word or phrase.

Equation Editor (EQ)	The student is presented with a toolbar that includes a variety of mathematical symbols that can be used to create a response. Responses may be in the form of a number, variable, expression, or equation, as appropriate to the test item. For paper-based assessments, this item type may be replaced with a modified version of the item that can be scanned and scored electronically or replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Graphic Response Item Display (GRID)	The student selects numbers, words, phrases, or images and uses the drag-and-drop feature to place them into a graphic. This item type may also require the student to use the point, line, or arrow tools to create a response on a graph. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Hot Text (HT)	**Selectable Hot Text -** Excerpted sentences from the text are presented in this item type. When the student hovers over certain words, phrases, or sentences, the options highlight. This indicates that the text is selectable ("hot"). The student can then click on an option to select it. For paper- based assessments, a "selectable" hot text item is modified so that it can be scanned and scored electronically. In this version, the student fills in a circle to indicate a selection.
	Drag-and-Drop Hot Text - Certain numbers, words, phrases, or sentences may be designated "draggable" in this item type. When the student hovers over these areas, the text highlights. The student can then click on the option, hold down the mouse button, and drag it to a graphic or other format. For paper-based assessments, drag- and-drop hot text items will be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Matching Item (MI)	The student checks a box to indicate if information from a column header matches information from a row. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Multi-Select (MS)	The student is directed to select all of the correct answers from among a number of options. These items are different from multiple-choice items, which allow the student to select only one correct answer. These items appear in the online and paper-based assessments.

Open Response	The student uses the keyboard to enter a response into a text field. These items can usually be answered in a sentence or two. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.

Operations and Algebraic Thinking (OA)		
4.OA.A Use the four operations with whole numbers to solve problems.	**4.OA.A.1**	Represent verbal statements of multiplicative comparisons as multiplication equations. Interpret a multiplication equation as a comparison (e.g., 35 is the number of objects in 5 groups, each containing 7 objects, and is also the number of objects in 7 groups, each containing 5 objects).
	4.OA.A.2	Multiply or divide within 1000 to solve word problems involving multiplicative comparison (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison). *See Table 2.*
	4.OA.A.3	Solve multistep word problems using the four operations, including problems in which remainders must be interpreted. Understand how the remainder is a fraction of the divisor. Represent these problems using equations with a letter standing for the unknown quantity.
4.OA.B Gain familiarity with factors and multiples.	**4.OA.B.4**	Find all factor pairs for a whole number in the range 1 to 100 and understand that a whole number is a multiple of each of its factors.
4.OA.C Generate and analyze patterns.	**4.OA.C.5**	Generate a number pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself and explain the pattern informally (e.g., given the rule “add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers).
	4.OA.C.6	When solving problems, assess the reasonableness of answers using mental computation and estimation strategies including rounding.
Number and Operations in Base Ten (NBT) *Note: Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.*		
4.NBT.A Generalize place value understanding for multi-digit whole numbers.	**4.NBT.A.1**	Apply concepts of place value, multiplication, and division to understand that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.
	4.NBT.A.2	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
	4.NBT.A.3	Use place value understanding to round multi-digit whole numbers to any place.

4.NBT.B **Use place value understanding and properties of operations to perform multi-digit arithmetic.**	**4.NBT.B.4**	Fluently add and subtract multi-digit whole numbers using a standard algorithm.
	4.NBT.B.5	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
	4.NBT.B.6	Demonstrate understanding of division by finding whole-number quotients and remainders with up to four-digit dividends and one-digit divisors.
Number and Operations – Fractions (NF) *Note: Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.*		
4.NF.A **Extend understanding of fraction equivalence and ordering.**	**4.NF.A.1**	Explain why a fraction *a/b* is equivalent to a fraction (*n x a*)/(*n x b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to understand and generate equivalent fractions.
	4.NF.A.2	Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators and by comparing to a benchmark fraction). a. Understand that comparisons are valid only when the two fractions refer to the same size whole. b. Record the results of comparisons with symbols >, =, or <, and justify the conclusions.
4.NF.B **Build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers.**	**4.NF.B.3**	Understand a fraction *a/b* with *a* > 1 as a sum of unit fractions (1/*b*). a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way (e.g., 3/8 = 1/8 + 1/8+1/8; 3/8 = 2/8 + 1/8; 2 1/8 = 1 + 1 + 1/8 + or 2 1/8 = 8/8 + 8/8 + 1/8). c. Add and subtract mixed numbers with like denominators (e.g., by using properties of operations and the relationship between addition and subtraction and/or by replacing each mixed number with an equivalent fraction). d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.
	4.NF.B.4	Build fractions from unit fractions. a. Understand a fraction $\frac{a}{b}$ as a multiple of a unit fraction $\frac{1}{b}$. In general, $\frac{a}{b} = a \times \frac{1}{b}$. b. Understand a multiple of $\frac{a}{b}$ as a multiple of a unit fraction $\frac{1}{b}$, and use this understanding to multiply a whole number by a fraction. In general, $n \times \frac{a}{b} = \frac{n \times a}{b}$. c. Solve word problems involving multiplication of a whole number by a fraction. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

4.NF.C Understand decimal notation for fractions, and compare decimal fractions.	**4.NF.C.5**	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 (tenths) and 100 (hundredths). *For example, express 3/10 as 30/100, and and 3/10 + 4/100 = 34/100.* (Note: Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators, in general, is not a requirement at this grade.)
	4.NF.C.6	Use decimal notation for fractions with denominators 10 (tenths) or 100 (hundredths), and locate these decimals on a number line.
	4.NF.C.7	Compare two decimals to hundredths by reasoning about their size. Understand that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <.
Measurement and Data (MD)		
4.MD.A Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.	**4.MD.A.1**	Know relative sizes of measurement units within one system of units which could include km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit and in a smaller unit in terms of a larger unit. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1,12), 2,24), (3,36).*
	4.MD.A.2	Use the four operations to solve word problems and problems in real-world context involving distances, intervals of time (hr, min, sec), liquid volumes, masses of objects, and money, including decimals and problems involving fractions with like denominators, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using a variety of representations, including number lines that feature a measurement scale.
	4.MD.A.3	Apply the area and perimeter formulas for rectangles in mathematical problems and problems in real-world contexts including problems with unknown side lengths. See *Table 2.*
4.MD.B Represent and interpret data.	**4.MD.B.4**	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots.
4.MD.C Geometric measurement: Understand concepts of angle and measure angles.	**4.MD.C.5**	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.
	4.MD.C.6	Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
4.MD.C (cont.)	**4.MD.C.7**	Understand angle measures as additive. (When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts.) Solve addition and subtraction problems to find unknown angles on a diagram within mathematical problems as well as problems in real-world contexts.
Geometry (G)		
4.G.A Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	**4.G.A.1**	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
	4.G.A.2	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size (e.g., understand right triangles as a category, and identify right triangles).
	4.G.A.3	Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Grade 4 Item Specifications

Measurement and Data & Geometry

4.G.A.1

Content Standards	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
Explanations	Examples of points, line segments, lines, angles, parallelism, and perpendicularity can be seen daily. Students do not easily identify lines and rays because they are more abstract
Content Limits	All objects (point, line, line segment, angles) and properties (right, acute, obtuse, perpendicular, parallel) noted in the standard, as individual objects or within two-dimensional figures.
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify geometric objects and properties, either as individual objects or as part of a more complex figure.	• Graphic Response • Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to construct a geometric figure based on given constraints/properties.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify points, lines, line segments, rays, angles, and lines in two-dimensional figures.	Identify and draw points, lines, line segments, rays, angles, and perpendicular and parallel lines in two-dimensional figures.
Proficient	**Highly Proficient**
Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	Explain characteristics that define points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines.

4.G.A.2

Content Standards	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size (e.g., understand right triangles as a category, and identify right triangles).
Explanations	Two-dimensional figures may be classified using different characteristics such as, parallel or perpendicular lines or by angle measurement. Students should become familiar with the concept of parallel and perpendicular lines. Two lines are parallel if they never intersect and are always equidistant. Two lines are perpendicular if they intersect in right angles (90°). Students may use transparencies with lines to arrange two lines in different ways to determine that the 2 lines might intersect in one point or may never intersect. This expectation is closely connected to 4.MD.5, 4.MD.6, and 4.G.1. Students' experiences with drawing and identifying right, acute, and obtuse angles support them in classifying two-dimensional figures based on specified angle measurements. They use the Right triangles can be a category for classification. A right triangle has one right angle. There are different types of right triangles. An isosceles right triangle has two or more congruent sides and a scalene right triangle has no congruent sides.
Content Limits	For this standard, classifications should focus on parallel/perpendicular lines and the size of angles rather than their side lengths. Triangles: Right triangles, acute triangles, obtuse triangles, scalene triangles, isosceles triangles, and equilateral triangles Quadrilaterals: parallelograms, rectangles, squares, rhombi, trapezoids. Other polygons may be included where appropriate. There are two competing definitions for trapezoids - one that requires exactly one pair or parallel sides, and another that requires at least one pair of parallel sides (using this definition, parallelograms are trapezoids). Thus, items that require the student to choose a definition in order to arrive at the correct answer should be avoided.
Context	Context is not allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to identify types of triangles.	• Graphic Response • Multiple Choice Response • Matching Item Response • Multi-Select Response • Proposition Response
Students will be required to construct a shape based on the shape name.	
Students will be required to given a set of shapes in two groups, explain why the shapes were classified this way.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify two-dimensional figures based on the presence or absence of parallel or perpendicular lines.	Identify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size.
Proficient	**Highly Proficient**
Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size (e.g., understand right triangles as a category, and identify right triangles).	Classify two-dimensional figures into more than one category based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size (e.g., understand right triangles as a category, and identify right triangles).

4.G.A.3

Content Standards	Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.
Explanations	Students need experiences with figures which are symmetrical and non-symmetrical. Figures include both regular and non-regular polygons. Folding cut-out figures will help students determine whether a figure has one or more lines of symmetry.
Content Limits	Be mindful of the graphic response answer space the students work with when considering the number of lines of symmetry of a shape. Avoid a busy figure with many of lines of symmetry that young students would find hard to work with. Items that require constructing a shape based on the number of lines of symmetry should specify the shape category with regards to the number of sides (quadrilateral, triangle, pentagon…).
Context	Context is not allowed.

<table>
<tr><th>Sample Task Demands</th><th>Common Item Formats</th></tr>
<tr><td>Students will be required to identify symmetric figures.</td><td rowspan="6">• Equation Response
• Graphic Response
• Multiple Choice Response
• Matching Item Response
• Multi-Select Response</td></tr>
<tr><td>Students will be required to identify whether a line drawn on a figure represents a line of symmetry of the figure.</td></tr>
<tr><td>Students will be required to determine the number of lines of symmetry a given figure has.</td></tr>
<tr><td>Students will be required to construct lines of symmetry for a given shape.</td></tr>
<tr><td>Students will be required to construct a complete figure based on half of the figure and its line of symmetry.</td></tr>
<tr><td>Students will be required to construct a figure based on two attributes (e.g., the number of lines of symmetry and type of shape, or the lines of symmetry, already drawn, and type of shape).</td></tr>
</table>

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify a line of symmetry for a two-dimensional figure.	Identify line-symmetric figures and draw lines of symmetry.
Proficient	**Highly Proficient**
Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	Explain that a line of symmetry for a two-dimensional figure is a line across the figure such that the figure can be folded along the line into matching parts. Draw line-symmetric figures.

4.MD.A.1

Content Standards	Know relative sizes of measurement units within one system of units which could include km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit and in a smaller unit in terms of a larger unit. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36).*
Explanations	The units of measure that have not been addressed in prior years are pounds, ounces, kilometers, milliliters, and seconds. Students' prior experiences were limited to measuring length, mass, liquid volume, and elapsed time. Students did not convert measurements. Students need ample opportunities to become familiar with these new units of measure.
Content Limits	Measurement units are within a single system. Measurement conversions are from larger units to smaller units. Multiplication is limited to 4-digit numbers by 1-digit numbers and two 2-digit numbers. (4.NBT.B.5) Units of measurement include: kilometer, meter, centimeter, millimeter, liter, milliliter, kilogram, gram, milligram, mile, yard, foot, inch, gallon, quart, pint, cup, ton, pound, and ounce.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify the relative size of a measurement unit.	• Equation Response • Graphic Response • Multiple Choice Response • Matching Item Response • Multi-Select Response • Table Response
Students will be required to calculate measurement conversions.	
Students will be required to order measurements given in different units within the same measurement system.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the relative sizes of measurement units within one system of units which could include km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, identify measurements in a larger unit in terms of a smaller unit.	Identify the relative sizes of measurement units within one system of units which could include km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, identify measurements in a larger unit in terms of a smaller unit and in a smaller unit in terms of a larger unit.
Proficient	**Highly Proficient**
Know relative sizes of measurement units within one system of units which could include km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit and in a smaller unit in terms of a larger unit. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1,12), (2,24), (3,36).*	Explain how different sizes of measurement units within one system of units relate to each other. Within a single system of measurement, explain how to convert measurements from a larger unit to a smaller unit and from a smaller unit to a larger unit. Generate a conversion table for measurements within one system of units.

4.MD.A.2

Content Standards	Use the four operations to solve word problems and problems in real-world context involving distances, intervals of time (hr, min, sec), liquid volumes, masses of objects, and money, including decimals and problems involving fractions with like denominators, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using a variety of representations, including number lines that feature a measurement scale.
Explanations	Number line diagrams that feature a measurement scale can represent measurement quantities. Examples include: ruler, diagram marking off distance along a road with cities at various points, a timetable showing hours throughout the day, or a volume measure on the side of a container.
Content Limits	Measurement conversions are from larger units to smaller units. Calculations are limited to simple fractions or decimals. Operations include addition, subtraction, multiplication, and division. Calculations involving fractions and decimals are limited to addition or subtraction.
Context	Context is required

Sample Task Demands	Common Item Formats
Students will be required to solve a word problem involving specified measurements.	• Equation Response • Graphic Response
Students will be required to represent/model a problem involving specified measurements.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Use the four operations to identify solutions to word problems and problems in real-world context involving distances, intervals of time (hr, min, sec), liquid volumes, masses of objects, and money, including decimals. Represent measurement quantities using number lines that feature a measurement scale.	Use the four operations to identify solutions to word problems and problems in real-world context involving distances, intervals of time (hr, min, sec), liquid volumes, masses of objects, and money, including decimals and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using a variety of representations, including number lines that feature a measurement scale.
Proficient	**Highly Proficient**
Use the four operations to solve word problems and problems in real-world context involving distances, intervals of time (hr, min, sec), liquid volumes, masses of objects, and money, including decimals and problems involving fractions with like denominators, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using a variety of representations, including number lines that feature a measurement scale.	Explain how to use the four operations to solve word problems and problems in real-world context involving distances, intervals of time (hr, min, sec), liquid volumes, masses of objects, and money, including decimals and problems involving fractions with like denominators, and problems that require expressing measurements given in a smaller unit in terms of a larger unit. Represent measurement quantities using a variety of representations, including number lines that feature a measurement scale.

4.MD.A.3

Content Standards	Apply the area and perimeter formulas for rectangles in mathematical problems and problems in real-world contexts including problems with unknown side lengths.
Explanations	Students developed understanding of area and perimeter in 3rd grade by using visual models. While students are expected to use formulas to calculate area and perimeter of rectangles, they need to understand and be able to communicate their understanding of why the formulas work.
Content Limits	Figures are limited to rectangles. Fractions are limited to like denominators. Products of factor pairs are limited to the range 1-100. Multiplication and division is limited to 2-digit by 1-digit, or 2-digit by 2-digit, where one number is a multiple of 10. Addition and subtraction within 1000. When constructing rectangles, the minimum grid size is 20 pixels, and in the context of a situation, one grid must be labeled with the appropriate dimension. That dimension should be "1 ____", as items at this standard should not assess scale.
Context	Context is allowed.

<table>
<tr><th>Sample Task Demands</th><th>Common Item Formats</th></tr>
<tr><td>Students will be required to construct a rectangle with a given perimeter and/or area.</td><td rowspan="5">Equation Response
Graphic Response
Multi-Select Response</td></tr>
<tr><td>Students will be required to calculate perimeter and/or area of a rectangle.</td></tr>
<tr><td>Students will be required to calculate an unknown side length given an area or perimeter.</td></tr>
<tr><td>Students will be required to model with an expression or equation the area or perimeter of a rectangle with an unknown side length.</td></tr>
<tr><td>Students will be required to construct a rectangle based on given parameters (i.e. ranges of possible areas and/or perimeters.)</td></tr>
</table>

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the area and perimeter for rectangles in mathematical problems.	Identify the area and perimeter for rectangles in mathematical problems and problems in real-world contexts.
Proficient	**Highly Proficient**
Apply the area and perimeter formulas for rectangles in mathematical problems and problems in real-world contexts including problems with unknown side lengths.	Explain the difference between the area and perimeter formulas for rectangles. Use the area and perimeter formulas to determine unknown side lengths of a rectangle.

4.MD.B.4

Content Standards	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots.
Explanations	Represent and interpret data.
Content Limits	Measurement units are limited to halves, quarters, and eighths. Addition and subtraction of fractions is limited to fractions with the same denominators. Multiplication and division is limited to 2-digit by 1-digit, or 2-digit by 2-digit, where one number is a multiple of 10. Addition and subtraction within 1000.
Context	Context is allowed

Sample Task Demands	Common Item Formats
Students will be required to construct a line plot based on given data.	• Equation Response • Graphic Response
Students will be required to interpret data in a line plot to solve problems involving addition and subtraction.	
Students will be required to complete a line plot based on the information about the sum or difference of the data.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition of fractions by using information presented in line plots.	Identify a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots.
Proficient	**Highly Proficient**
Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots.	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Create problems involving addition and subtraction of fractions by using information presented in line plots.

4.MD.C.5, 4.MD.C.5a, and 4.MD.C.5b

Content Standards	**4.MD.C.5** Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: **4.MD.C.5a** An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. **4.MD.C.5b** An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.
Explanations	Geometric measurement: understand concepts of angle and measure angles.
Content Limits	Whole-number degree measures. Angles are less than or equal to 360º.
Context	Context is allowed

Sample Task Demands	**Common Item Formats**
Students will be required to identify an angle.	• Graphic Response • Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to sort angles from other geometric objects.	
Students will be required to identify the unit used to measure angles.	
Students will be required to identify categories of angle measures.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. Recognize that a "one-degree angle" turns through 1/360 of a circle. b. Recognize that an "*n* degree angle" turns through *n*/360 of a circle.	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. Identify a one-degree angle, with its common endpoint at the center of a circle, as being 1/360 of the circle. b. Identify an "*n* degree angle," with its common endpoint at the center of a circle, as being *n*/360 of the circle.
Proficient	**Highly Proficient**
Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. Explain how an angle is measured with reference to a circle with its center at the common endpoint of the rays and how the angle measure is the same as the fraction of the circular arc between the points where the two rays intersect the circle. b. Explain why an angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

4.MD.C.6

Content Standards	Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
Explanations	Before students begin measuring angles with protractors, they need to have some experiences with benchmark angles. They transfer their understanding that a 360º rotation about a point makes a complete circle to recognize and sketch angles that measure approximately 90º and 180º. They extend this understanding and recognize and sketch angles that measure approximately 45º and 30º. They use appropriate terminology (acute, right, and obtuse) to describe angles and rays (perpendicular).
Content Limits	Whole-number degree measures. For identification, angles are less than 360º. For construction, angles are less than 180º.
Context	Context is not allowed

Sample Task Demands	Common Item Formats
Students will be required to measure a given angle.	• Equation Response • Graphic Response
Students will be required to construct an angle based on a given measure.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify angles measures in whole-number degrees using a protractor, when one of the rays is horizontal.	Identify angles measures in whole-number degrees using a protractor. Add a second ray to sketch angles of specified measure when given a horizontal ray.
Proficient	**Highly Proficient**
Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.	Measure angles in whole-number degrees using a protractor, including when the angle does not have a horizontal ray.

4.MD.C.7

Content Standards	Understand angle measures as additive. (When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts.) Solve addition and subtraction problems to find unknown angles on a diagram within mathematical problems as well as problems in real-world contexts.
Explanations	Geometric measurement: understand concepts of angle and measure angles.
Content Limits	Angles are less than or equal to 360º.
Context	Context is allowed

Sample Task Demands	Common Item Formats
Students will be required to calculate an angle measure from a given sum or difference and/or a decomposed larger angle.	• Equation Response • Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to identify angles that can be used to construct other angles.	
Students will be required to show how to find an angle measure from a given sum or difference using an equation.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Solve addition problems to find unknown angles on a diagram within mathematical problems as well as problems in real-world contexts.	Solve addition and subtraction problems to find unknown angles on a diagram within mathematical problems as well as problems in real-world contexts.
Proficient	**Highly Proficient**
Understand angle measures as additive. (When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts.) Solve addition and subtraction problems to find unknown angles on a diagram within mathematical problems as well as problems in real-world contexts.	Understand angle measures as additive. (When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts.) Create addition and subtraction problems, mathematical problems as well as problems in real-world contexts, for angles represented on a diagram.

Numbers and Operations – Fractions

4.NF.A.1

Content Standards	Explain why a fraction *a*/*b* is equivalent to a fraction (*n* x *a*)/(*n* x *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to understand and generate equivalent fractions.
Explanations	This standard extends the work in third grade by using additional denominators (5, 10, 12, and 100).
Content Limits	Denominators limited to 2, 3, 4, 5, 6, 8, 10, 12, 100 For denominators of 10 and 100, focus should not be on equivalence between these 2 denominators since this is addressed specifically in standards 4.NF.5 – 7, but should be more on equivalence between fractions with denominators of 2, 4, and 5 and fractions with denominators of 10 and 100. E.g. ½ = 5/10, 2/5 = 40/100, etc. Refer to the same whole Fraction models are limited to number lines, rectangles, circles, and squares. (The focus should not be on complex visual models.) Fractions a/bcan be improper fractions and students should not be guided to put fractions in lowest terms or to simplify.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify/recognize fractions that are equivalent to a given fraction.	• Equation Response • Graphic Response • Multiple Choice Response • Matching Item Response • Multi-Select Response • Proposition Response
Students will be required to identify/recognize fraction models that represent equivalent fractions.	
Students will be required to generate fractions that are equivalent to a given fraction or equivalent to fractions represented by a given fraction model.	
Students will be required to construct models representing fractions that are equivalent to given fractions or equivalent to fractions represented by given fraction models.	
Students will be required to give evidence or an explanation to support why fractions are equivalent or why fractions represented by models are equivalent.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify equivalent fractions.	Generate equivalent fractions.
Proficient	**Highly Proficient**
Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to understand and generate equivalent fractions.	Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models. Explain why the number and size of the parts is important in determining if two fractions are the same size. Use this principle to explain and generate equivalent fractions.

4.NF.A.2, 4.NF.A.2a, and 4.NF.A.2b

Content Standards	**4.NF.A.2** Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators and by comparing to a benchmark fraction). **4.NF.A.2a** Understand that comparisons are valid only when the two fractions refer to the same size whole. **4.NF.A.2b** Record the results of comparisons with symbols >, =, or <, and justify the conclusions.
Explanations	Benchmark fractions include common fractions between 0 and 1 such as halves, thirds, fourths, fifths, sixths, eighths, tenths, twelfths, and hundredths. Fractions can be compared using benchmarks, common denominators, or common numerators. Symbols used to describe comparisons include <, >, =.
Content Limits	Denominators limited to 2, 3, 4, 5, 6, 8, 10, 12, 100 Benchmarks limited to 0, 1/4, 1/2, 3/4, 1 Fractions a/bcan be improper fractions and students should not be guided to put fractions in lowest terms or to simplify.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to compare fractions relating them to benchmark fractions using visual models (e.g. number lines) and/or numeric reasoning.	• Equation Response • Graphic Response • Multiple Choice Response • Matching Item Response • Multi-Select Response • Proposition Response
Students will be required to interpret information about fractions to compare fractions using visual models or numeric reasoning.	
Students will be required to compare fractions using symbols <, >, and = with no situational context or visual model.	
Students will be required to develop logical arguments, draw conclusions, and relate use of models to numeric strategies to compare fractional quantities	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators and by comparing to a benchmark fraction). a. Determine whether or not two fractions refer to the same size whole. b. Compare two fraction models using the symbols >, =, or <.	Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators and by comparing to a benchmark fraction). a. Determine whether or not comparing two fractions is valid based on whether or not the fractions refer to the same size whole. b. Compare two fractions using the symbols >, =, or <.
Proficient	**Highly Proficient**
Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators and by comparing to a benchmark fraction). a. Understand that comparisons are valid only when the two fractions refer to the same size whole. b. Record the results of comparisons with symbols >, =, or <, and justify the conclusions.	Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators and by comparing to a benchmark fraction). a. Explain why comparisons are valid only when two fractions refer to the same size whole. b. Record the results of comparing multiple fractions with symbols >, =, or <, and justify the conclusions.

4.NF.B.3, 4.NF.B.3a, 4.NF.B.3b, 4.NF.B.3c, and 4.NF.B.3d

Content Standards	**4.NF.B.3** Understand a fraction *a/b* with a > 1 as a sum of unit fractions (1/*b*). **4.NF.B.3a** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. **4.NF.B.3b** Decompose a fraction into a sum of fractions with the same denominator in more than one way (e.g., 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 2/8 + 1/8; 2 1/8 = 1 + 1 + 1/8 + or 2 1/8 = 8/8 + 8/8 + 1/8). **4.NF.B.3c** Add and subtract mixed numbers with like denominators (e.g., by using properties of operations and the relationship between addition and subtraction and/or by replacing each mixed number with an equivalent fraction). **4.NF.B.3d** Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.
Explanations	A fraction with a numerator of one is called a unit fraction. When students investigate fractions other than unit fractions, such as 2/3, they should be able to decompose the non-unit fraction into a combination of several unit fractions. A separate algorithm for mixed numbers in addition and subtraction is not necessary. Students will tend to add or subtract the whole numbers first and then work with the fractions using the same strategies they have applied to problems that contained only fractions.
Content Limits	Denominators limited to 2, 3, 4, 5, 6, 8, 10, 12, 100 Use mixed numbers and fractions with like denominators Incorporate the concept of the same whole. Circle based models, rectangular models, and numbers line models, do not over use circle based area food models (i.e., pizza).
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to add or subtract fractions with like denominators.	• Equation Response • Graphic Response • Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to decompose a fraction into a sum of fractions in multiple ways.	
Students will be required to add or subtract mixed numbers.	
Students will be required to solve word problems involving fractions or mixed numbers and represent sums and differences of fractions or mixed numbers.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Understand a fraction *a/b* with *a* > 1 as a sum of unit fractions (1/*b*). a. Recognize addition of fractions as joining parts referring to the same whole. b. Identify a correct decomposition of a fraction into a sum of fractions with the same denominator in one way (e.g., 3/8 = 1/8 + 1/8+1/8). c. Add mixed numbers with like denominators, where regrouping is not necessary. d. Identify the solution to word problems involving addition of fractions referring to the same whole and having like denominators.	Understand a fraction *a/b* with *a* > 1 as a sum of unit fractions (1/*b*). a. Recognize addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Identify a correct decomposition of a fraction into a sum of fractions with the same denominator in more than one way (e.g., 3/8 = 1/8 + 1/8+1/8; 3/8 = 2/8 + 1/8; 2 1/8 = 1 + 1 + 1/8 + or 2 1/8 = 8/8 + 8/8 + 1/8). c. Add and subtract mixed numbers with like denominators where regrouping is not necessary. d. Identify the solution to word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.
Proficient	**Highly Proficient**
Understand a fraction *a/b* with *a* > 1 as a sum of unit fractions (1/*b*). a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way (e.g., 3/8 = 1/8 + 1/8+1/8; 3/8 = 2/8 + 1/8; 2 1/8 = 1 + 1 + 1/8 + or 2 1/8 = 8/8 + 8/8 + 1/8). c. Add and subtract mixed numbers with like denominators (e.g., by using properties of operations and the relationship between addition and subtraction and/or by replacing each mixed number with an equivalent fraction). d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.	Understand a fraction *a/b* with *a* > 1 as a sum of unit fractions (1/*b*). a. Explain how addition and subtraction of fractions is joining and separating parts referring to the same whole. b. Explain how to decompose a fraction into a sum of fractions with the same denominator in more than one way (e.g., 3/8 = 1/8 + 1/8+1/8; 3/8 = 2/8 + 1/8; 2 1/8 = 1 + 1 + 1/8 + or 2 1/8 = 8/8 + 8/8 + 1/8). c. Explain how to add and subtract mixed numbers with like denominators (e.g., by using properties of operations and the relationship between addition and subtraction and/or by replacing each mixed number with an equivalent fraction). d. Solve word problems involving addition and subtraction of fractions referring to the same whole but having different denominators.

4.NF.B.4, 4.NF.B.4a, 4.NF.B.4b, and 4.NF.B.4c

Content Standards	**4.NF.B.4** Build fractions from unit fractions. **4.NF.B.4a** Understand a fraction a/b as a multiple of a unit fraction $1/b$. In general, $a/b = a \times 1/b$. **4.NF.B.4b** Understand a multiple of a/b as a multiple of a unit fraction $1/b$, and use this understanding to multiply a whole number by a fraction. In general, $n \times a/b = (n \times a)/b$. **4.NF.B.4c** Solve word problems involving multiplication of a whole number by a fraction. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*
Explanations	Students need many opportunities to work with problems in context to understand the connections between models and corresponding equations. Contexts involving a whole number times a fraction lend themselves to modeling and examining patterns.
Content Limits	Fractions will only be multiplied by a whole number. Limit denominators to 2, 3, 4, 5, 6, 8, 10, 12, 100
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to model a non-unit fraction as the product of a whole number and a unit fraction.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response
Students will be required to multiply a fraction by a whole number.	
Students will be required to identify a missing number in an equation that multiplies a fraction by a whole number.	
Students will be required to solve a word problem that involves multiplying a fraction by a whole number within a real-world context.	
Students will be required to create and/or solve an equation that models a word problem involving multiplying a fraction by a whole number within a real-world context.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Build fractions from unit fractions. a. Identify the product when a whole number is multiplied by a unit fraction. In general, $a/b = a \times 1/b$. b. Identify the product when a whole number is multiplied by a fraction. In general, $n \times a/b = (n \times a)/b$. c. Identify the solution to word problems involving multiplication of a whole number by a fraction.	Build fractions from unit fractions. a. Determine the product when a whole number is multiplied by a unit fraction. In general, $a/b = a \times 1/b$. b. Determine the product when a whole number is multiplied by a fraction. In general, $n \times a/b = (n \times a)/b$. c. Determine the solution to word problems involving multiplication of a whole number by a fraction.
Proficient	**Highly Proficient**
Build fractions from unit fractions. a. Understand a fraction a/b as a multiple of a unit fraction $1/b$. In general, $a/b = a \times 1/b$. b. Understand a multiple of a/b as a multiple of a unit fraction $1/b$, and use this understanding to multiply a whole number by a fraction. In general, $n \times a/b = (n \times a)/b$. c. Solve word problems involving multiplication of a whole number by a fraction. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*	Build fractions from unit fractions. a. Explain why a fraction a/b is a multiple of a unit fraction $1/b$. b. Understand a multiple of a/b as a multiple of a unit fraction $1/b$, and use this understanding to multiply a whole number by a fraction. In general, $n \times a/b = (n \times a)/b$. c. Create word problems involving multiplication of a whole number by a fraction.

4.NF.C.5

Content Standards	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 (tenths) and 100 (hundredths). *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* (Note: Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators, in general, is not a requirement at this grade.)
Explanations	Students can use base ten blocks, graph paper, and other place value models to explore the relationship between fractions with denominators of 10 and denominators of 100. Students may represent 3/10 with 3 longs and may also write the fraction as 30/100 with the whole in this case being the flat (the flat represents one hundred units with each unit equal to one hundredth). Students begin to make connections to the place value chart as shown in 4.NF.6. This work in fourth grade lays the foundation for performing operations with decimal numbers in fifth grade.
Content Limits	Denominators must be either 10 or 100 Decimal notation is not assessed in this standard Equivalent fractions is an acceptable vocab word
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to express a fraction with denominator 10 as a fraction with denominator 100, and vice-versa.	• Equation Response • Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to add two fractions with different denominators of 10 and 100.	
Students will be required to determine a fraction equivalent to another fraction represented by a model.	
Students will be required to identify a missing addend.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify equivalent fractions, one with denominator 10 and one with denominator 100. *For example, identify 3/10 as equivalent to 30/100.*	Identify equivalent fractions, one with denominator 10 and one with denominator 100. Identify the sum of two fractions with respective denominators 10 (tenths) and 100 (hundredths). *For example, identify 3/10 as equivalent to 30/100, and identify that 3/10 + 4/100 = 34/100.*
Proficient	**Highly Proficient**
Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 (tenths) and 100 (hundredths). *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* (Note: Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators, in general, is not a requirement at this grade.)	Express a fraction with denominator 10 as an equivalent fraction with denominator a multiple of 10, and use this technique to add two fractions with the respective denominators. *For example, express 3/10 as 300/1000, and add 3/10 + 40/1000 = 340/1000.*

4.NF.C.6

Content Standards	Use decimal notation for fractions with denominators 10 (tenths) or 100 (hundredths), and locate these decimals on a number line.
Explanations	Students make connections between fractions with denominators of 10 and 100 and the place value chart. By reading fraction names, students say 32/100 as thirty-two hundredths and rewrite this as 0.32 or represent it on a place value model. Students use the representations explored in 4.NF.5 to understand 32/100 can be expanded to 3/10 and 2/100. Students represent values such as 0.32 or 32/100 on a number line. 32/100 is more than 30/100 (or 3/10) and less than 40/100 (or 4/10). It is closer to 30/100 so it would be placed on the number line near that value.
Content Limits	Denominators of 10 and 100 Decimal notation to tenths and hundredths
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to express a fraction or mixed number in decimal notation in 10ths or 100ths.	• Equation Response • Graphic Response • Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to locate or plot a decimal on a number line/model.	
Students will be required to relate two fractional representations (denominators of 10 and 100) to one decimal representation. (Medium and Hard difficulty only)	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify decimal notation for fractions with denominators 10 (tenths) or 100 (hundredths).	Identify decimal notation for fractions with denominators 10 (tenths) or 100 (hundredths), and locate these decimals on a number line.
Proficient	**Highly Proficient**
Use decimal notation for fractions with denominators 10 (tenths) or 100 (hundredths), and locate these decimals on a number line.	Use decimal notation for fractions and mixed numbers with denominators a multiple of 10. Explain the location of these decimals on a number line.

4.NF.C.7

Content Standards	Compare two decimals to hundredths by reasoning about their size. Understand that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <.
Explanations	Students build area and other models to compare decimals. Through these experiences and their work with fraction models, they build the understanding that comparisons between decimals or fractions are only valid when the whole is the same for both cases. When the wholes are the same, the decimals or fractions can be compared.
Content Limits	Examples reference the same whole value. Decimals limited to 10ths and 100ths Decimals should not be limited to values less than 1 Use mathematical symbols appropriately to compare values represented by models and not to compare models. (e.g., 0.62<0.89 instead of [model] < [model])
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to compare two decimals using a model (i.e., numerical, number line, visual model) - can vary models (10ths and 100ths) as long as they both relate to the same whole.	• Equation Response • Graphic Response • Multiple Choice Response • Matching Item Response • Multi-Select Response • Table Response
Students will be required to compare decimals by converting decimals to fractions with common denominators and/or by reasoning about place value.	
Students will be required to write or identify true comparisons between decimal numbers using symbols <, >, and =. Enter decimals or symbols to complete comparisons.	
Students will be required to explain conclusions about relationships and comparisons between decimals.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Compare two decimals, referring to the same whole, to hundredths.	Compare two decimals, referring to the same whole, to hundredths. Record the results of comparisons with the symbols >, =, or <.

Proficient	Highly Proficient
Compare two decimals to hundredths by reasoning about their size. Understand that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <.	Compare two decimals to hundredths by reasoning about their size. Explain why comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <.

Operations and Algebraic Thinking & Numbers in Base Ten

4.NBT.A.1

Content Standards	Apply concepts of place value, multiplication, and division to understand that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.
Explanations	Students should be familiar with and use place value as they work with numbers.
Content Limits	Whole numbers within 1,000,000
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to when presented with a multiplication problem, identify the power of 10 by which one number is greater than another.	• Equation Response
Students will be required to compare the value of a digit in different place values of two given numbers and identify the power of 10 by which one number is greater.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify which place value in a multi-digit whole number represents ten times the value of a given place value.	Given two multi-digit whole numbers, with a digit in different place values in each number, identify how many times the value of the digit is in one number compared to the other number.
Proficient	**Highly Proficient**
Apply concepts of place value, multiplication, and division to understand that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.	Apply concepts of place value, multiplication, and division to explain why a digit in one place represents ten times what it represents in the place to its right.

4.NBT.A.2

Content Standards	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
Explanations	The expanded form of 275 is 200 + 70 + 5. Students use place value to compare numbers. For example, in comparing 34,570 and 34,192, a student might say, both numbers have the same value of 10,000s and the same value of 1000s however, the value in the 100s place is different so that is where I would compare the two numbers.
Content Limits	Whole numbers within 1,000,000
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to write a number with a given name in numeric form.	• Equation Response • Graphic Response • Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to identify the name of a given number.	
Students will be required to write a number given in expanded form in numeric form or vice versa.	
Students will be required to compare two whole numbers in numeric form.	
Students will be required to order more than two whole numbers in numeric form.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify three-digit whole numbers using base-ten numerals and number names. Compare two three-digit numbers based on meanings of the digits in each place.	Identify multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place.
Proficient	**Highly Proficient**
Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	Read, write, and order multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare more than two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

4.NBT.A.3

Content Standards	Use place value understanding to round multi-digit whole numbers to any place.
Explanations	When students are asked to round large numbers, they first need to identify which digit is in the appropriate place.
Content Limits	Greater than 1000 and within 1,000,000
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify the value of a given number rounded to the nearest place value.	• Equation Response • Matching Item Response • Multi-Select Response • Table Response
Students will be required to identify the numbers that round to a given value.	
Students will be required to identify what place value a number was rounded to.	
Students will be required to interpret and distinguish between different rounding procedures used in rounding to a number in order to create a number that fits certain parameters.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Use place value understanding to round three-digit whole numbers to the hundreds place.	Use place value understanding to round multi-digit whole numbers to the largest place.
Proficient	**Highly Proficient**
Use place value understanding to round multi-digit whole numbers to any place.	Explain how to round multi-digit whole numbers to any place.

4.NBT.B.4

Content Standards	Fluently add and subtract multi-digit whole numbers using a standard algorithm.
Explanations	Students build on their understanding of addition and subtraction, their use of place value and their flexibility with multiple strategies to make sense of the standard algorithm. They continue to use place value in describing and justifying the processes they use to add and subtract. When students begin using the standard algorithm their explanation may be quite lengthy. After much practice with using place value to justify their steps, they will develop fluency with the algorithm. Students should be able to explain why the algorithm works. Note: Students should know that it is mathematically possible to subtract a larger number from a smaller number but that their work with whole numbers does not allow this as the difference would result in a negative number.
Content Limits	Whole numbers greater than 1,000 and within 1,000,000
Context	Context is not allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to calculate the sum or difference of two or more numbers.	• Equation Response
Students will be required to identify a missing digit in an addition or subtraction problem.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Fluently add and subtract multi-digit whole numbers using strategies and algorithms based on the relationship between addition and subtraction.	Fluently add and subtract multi-digit whole numbers using strategies and algorithms based on place value and/or the relationship between addition and subtraction.
Proficient	**Highly Proficient**
Fluently add and subtract multi-digit whole numbers using a standard algorithm.	Recognize and explain an error made while finding a sum or a difference, and give the correct answer.

4.NBT.B.5

Content Standards	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Explanations	Students who develop flexibility in breaking numbers apart have a better understanding of the importance of place value and the distributive property in multi-digit multiplication. Students use base ten blocks, area models, partitioning, compensation strategies, etc. when multiplying whole numbers and use words and diagrams to explain their thinking. They use the terms factor and product when communicating their reasoning. Multiple strategies enable students to develop fluency with multiplication and transfer that understanding to division. Use of the standard algorithm for multiplication is an expectation in the 5th grade.
Content Limits	Products up to 89,991 (9,999 x 9). Multiply four digits by one digit, three digits by one digit, two digits by one digit, and two digits by two digits
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to calculate the product of 2 numbers.	• Equation Response • Multi-Select Response
Students will be required to select expressions that are equivalent to a given product.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and visual models.	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate the calculation by using rectangular arrays and/or area models.
Proficient	**Highly Proficient**
Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers. Explain the calculation by using equations.

4.NBT.B.6

Content Standards	Demonstrate understanding of division by finding whole-number quotients and remainders with up to four-digit dividends and one-digit divisors.
Explanations	In fourth grade, students build on their third grade work with division within 100. Students need opportunities to develop their understandings by using problems in and out of context.
Content Limits	3-digit dividend and 1-digit divisor and 4-digit dividend and 1-digit divisor
Context	Context is not allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to calculate the quotient of 2 numbers.	• Equation Response • Multi-Select Response
Students will be required to select expressions that are equivalent to a given quotient.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify whole-number quotients with up to four-digit dividends and one-digit divisors.	Demonstrate understanding of division by identifying whole-number quotients and remainders with up to four-digit dividends and one-digit divisors.
Proficient	**Highly Proficient**
Demonstrate understanding of division by finding whole-number quotients and remainders with up to four-digit dividends and one-digit divisors.	Demonstrate understanding of division by explaining the meaning of whole-number quotients and remainders with up to four-digit dividends and one-digit divisors.

4.OA.A.1

Content Standards	Represent verbal statements of multiplicative comparisons as multiplication equations. Interpret a multiplication equation as a comparison (e.g., 35 is the number of objects in 5 groups, each containing 7 objects, and is also the number of objects in 7 groups, each containing 5 objects).
Explanations	A multiplicative comparison is a situation in which one quantity is multiplied by a specified number to get another quantity (e.g., “a is n times as much as b”). Students should be able to identify and verbalize which quantity is being multiplied and which number tells how many times.
Content Limits	Whole numbers within 100. Item must either include a verbal description of a multiplication equation or a division equation. Multiplication situation must be a comparison, e.g. three times as many
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to given a verbal description, create an equation that models the multiplication context.	• Equation Response • Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to given a multiplication equation, select a multiplicative comparison that describes the equation or vice versa.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify multiplication equations that represent verbal statements of multiplicative comparisons with visual support.	Interpret multiplication equations that represent verbal statements of multiplicative comparisons with visual support. Recognize that a multiplication equation is a comparison.
Proficient	**Highly Proficient**
Represent verbal statements of multiplicative comparisons as multiplication equations. Interpret a multiplication equation as a comparison (e.g., 35 is the number of objects in 5 groups, each containing 7 objects, and is also the number of objects in 7 groups, each containing 5 objects).	Create verbal statements of multiplicative comparisons to represent a given multiplication equation. Explain how a multiplication equation is a comparison.

4.OA.A.2

Content Standards	Multiply or divide within 1000 to solve word problems involving multiplicative comparison (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison).
Explanations	Students need many opportunities to solve contextual problems.
Content Limits	Multiplication situation must be a comparison, e.g. three times as many Operations limited to multiplication and division. Whole numbers within 100.
Context	Context is required.

Sample Task Demands	Common Item Formats
Students will be required to given a situation involving multiplicative comparison, create a multiplication or division equation (with an unknown value) to represent the situation.	• Equation Response • Multiple Choice Response
Students will be required to given a situation involving multiplicative comparison, solve a multiplication or division word problem.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify products and quotients within 1000 to solve word problems involving multiplicative comparison when a visual model is given.	Multiply or divide within 1000 to solve word problems involving multiplicative comparison when a visual model is given.
Proficient	**Highly Proficient**
Multiply or divide within 1000 to solve word problems involving multiplicative comparison (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison).	Identify a word problem involving multiplicative comparison within 1000 that is solved by a given multiplication or division expression.

4.OA.A.3

Content Standards	Solve multistep word problems using the four operations, including problems in which remainders must be interpreted. Understand how the remainder is a fraction of the divisor. Represent these problems using equations with a letter standing for the unknown quantity.
Explanations	Students need many opportunities solving multistep story problems using all four operations. In division problems, the remainder is the whole number left over when as large a multiple of the divisor as possible has been subtracted. Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimations.
Content Limits	Whole numbers Only easy- and medium- difficulty addition and subtraction problems of numbers up to 1 million Multiplication of numbers of up to four digits by a one-digit number or of two numbers with two digits Quotients and remainders with up to four-digit dividends and one-digit divisors Only 2- and 3-step problems Problems involving remainders should require the student to interpret and use the remainder with respect to context Variables must be represented by a letter. Variables should be introduced in a separate phrase like "Use p to represent the number of pages in the book" rather than using an appositive clause.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to interpret remainders within the context of a division situation by giving a numeric answer or interpretation.	• Equation Response • Multiple Choice Response • Multi-Select Response • Proposition Response
Students will be required to explain the reasonableness of a solution in words.	
Students will be required to reason through a word problem to find an unknown value (either the final answer or a key piece of information, given the final solution – e.g., working backward).	
Students will be required to reason through a word problem to find an unknown value given only some information.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Solve two-step word problems using the four operations with visual support. Identify the remainder as a fraction of the divisor. Identify equations with a letter standing for the unknown quantity that represents these problems.	Solve multistep word problems using the four operations. Identify the remainder as a fraction of the divisor. Identify equations with a letter standing for the unknown quantity that represents these problems.
Proficient	**Highly Proficient**
Solve multistep word problems using the four operations, including problems in which remainders must be interpreted. Understand how the remainder is a fraction of the divisor. Represent these problems using equations with a letter standing for the unknown quantity.	Solve multistep word problems using the four operations, including problems in which remainders must be interpreted. Explain why the remainder is a fraction of the divisor. Create word problems that can be solved using equations with a letter standing for the unknown quantity.

4.OA.B.4

Content Standards	Find all factor pairs for a whole number in the range 1 to 100 and understand that a whole number is a multiple of each of its factors.
Explanations	Students should understand the process of finding factor pairs so they can do this for any number 1 -100. Multiples can be thought of as the result of skip counting by each of the factors. When skip counting, students should be able to identify the number of factors counted e.g., 5, 10, 15, 20 (there are 4 fives in 20). A prime number is a number greater than 1 that has only 2 factors, 1 and itself. Composite numbers have more than 2 factors. Students investigate whether numbers are prime or composite by building rectangles (arrays) within the given area and finding which numbers have more than two rectangles (e.g. 7 can be made into only 2 rectangles, 1 x 7 and 7 x 1, therefore it is a prime number) or finding factors of the number.
Content Limits	Whole numbers in the range 1-100 Vocabulary includes prime, composite, factor or multiple
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify factors or multiples of a given number.	• Equation Response • Graphic Response • Multiple Choice Response • Matching Item Response • Multi-Select Response • Table Response
Students will be required to given a set of conditions (related to prime/composite, and factors), identify a number (or numbers) that meets those criteria.	
Students will be required to classify numbers as prime or composite.	
Students will be required to apply the concepts of prime numbers, composite numbers, and factors in problem-solving contexts.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify a factor pair for a whole number in the range 1 to 100.	Identify all factor pairs for a whole number in the range 1 to 100 and identify whole numbers that are a multiple of a given factor.
Proficient	**Highly Proficient**
Find all factor pairs for a whole number in the range 1 to 100 and understand that a whole number is a multiple of each of its factors.	Explain why a whole number is a multiple of each of its factors.

4.OA.C.5

Content Standards	Generate a number pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself and explain the pattern informally (e.g., given the rule "add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers).
Explanations	Patterns involving numbers or symbols either repeat or grow. Students need multiple opportunities creating and extending number and shape patterns. Numerical patterns allow students to reinforce facts and develop fluency with operations. Patterns and rules are related. A pattern is a sequence that repeats the same process over and over. A rule dictates what that process will look like. Students investigate different patterns to find rules, identify features in the patterns, and justify the reason for those features. After students have identified rules and features from patterns, they need to generate a numerical or shape pattern from a given rule.
Content Limits	Whole numbers Operations in patterns limited to addition, subtraction, multiplication, and division Growing shape patterns If generating a pattern from a given rule, ask for the next two to four terms.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to generate a number or shape pattern that follows a given rule.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response • Proposition Response • Table Response
Students will be required to identify apparent features (such as the pattern of odd and even numbers, all numbers are even, all numbers are odd, etc.) of the pattern.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify a number pattern that follows a given rule.	Identify a number pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

Proficient	Highly Proficient
Generate a number pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself and explain the pattern informally (e.g., given the rule "add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers).	Create a rule for a given number pattern. Explain features of the pattern that are not explicit in the rule and explain the rule informally.

4.OA.C.6

Content Standards	When solving problems, assess the reasonableness of answers using mental computation and estimation strategies including rounding.
Explanations	Generate and analyze pattern.
Content Limits	Multiplication is within 1000, up to 4 digits by 1 digit or 2 digits by 2 digits Addition and subtraction within 1,000,000 Can add fractions with common denominators.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to determine the best estimation strategy given the context of a situation.	• Equation Response • Multiple Choice Response • Editing Task Response
Students will be required to determine whether an answer is appropriate in a given context.	
Students will be required to recognize when an estimation strategy is or is not appropriate.	
Students will be required to use estimation strategies to solve a problem.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Recognize whether an answer is reasonable or not when rounding.	Use rounding to determine the reasonableness of answers when using the four operations to solve problems.
Proficient	**Highly Proficient**
When solving problems, assess the reasonableness of answers using mental computation and estimation strategies including rounding.	Recognize the reasonableness of answers using different types of estimation strategies when using the four operations to solve problems. Choose the best estimation strategy for a specific purpose.

AzM2 | Arizona's Statewide Achievement Assessment for English Language Arts and Mathematics

Mathematics Item Specifications

GRADE 5

Arizona Department of Education with American Institutes for Research - 2019

Table of Contents

Introduction

The Arizona Statewide Achievement Assessment for English Language Arts and Mathematics (AzM2) is Arizona's statewide achievement test. AzM2 assesses the Arizona English Language Arts Standards and Arizona Mathematics Standards adopted by the Arizona State Board of Education in December 2016. AzM2 will inform students, teachers, and parents about preparedness for college and careers upon graduating from high school. AzM2 tests are computer-based, meaning that they can better assess students' critical thinking skills and provide them with opportunities to demonstrate a deeper understanding of the materials. Computer-based testing also allows for the use of a variety of innovative items types.

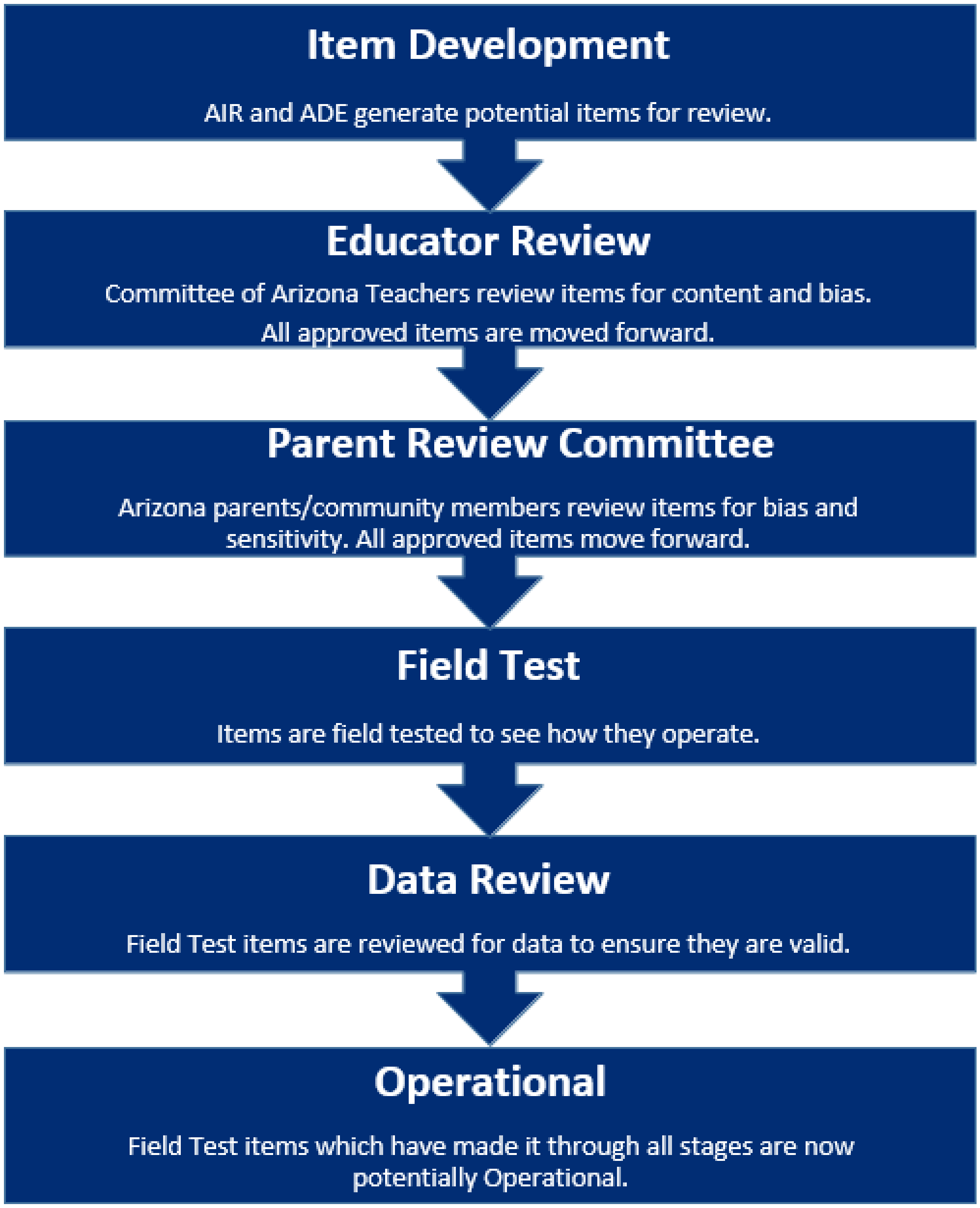
During the item-development process, all AzM2 items are written in accordance with the Item Specifications and are reviewed and approved by a committee of Arizona educators to confirm alignment and appropriateness for inclusion in the test. AzM2 items are generally representative of Arizona's geographic regions and culturally diverse population. Items are reviewed for the following kinds of bias: gender, racial, ethnic, linguistic, religious, geographic, and socioeconomic. Item reviews also include consideration of issues related to individuals with disabilities. Arizona community members also have an opportunity to review items for issues of potential concern to members of the community at large. Reviewers are asked to consider the variety of cultural, regional, philosophical, political, and religious backgrounds throughout Arizona, and then to determine whether the subject matter will be acceptable to Arizona students, families, and other members of Arizona communities.

This *AzM2 Item Specifications* is a resource document that defines the content and format of the test and test items for item writers and reviewers. Each *Item Specifications* document indicates the alignment of items with the Arizona Mathematics Standards. It also serves to provide all stakeholders with information about the scope and function of assessment items. This document can also serve to assist educators to understand how assessment items are developed in alignment with the standards for English language arts and math. These item specifications for AzM2 are intended to provide information regarding standards, item formats and response types. The descriptions of blueprints, and depth of knowledge in this document are meant to provide an overview of the test. Item specifications are meant for the purposes of assessment, not instruction. They are not intended to be tools for instruction or the basis for curricula. AzM2 has a test blueprint that was developed by Arizona and is different from any other state or consortium test blueprint.

For the math portion of AzM2, all of the test questions are aligned to the mathematic content standards for these subject areas. Any item specifications that are absent for standards listed in this document may be under development. This document does not endorse the exclusion of the instruction of any grade-level content standards. The test will ask questions that check a student's conceptual understanding of math as well as their procedural skills. These items have been written to be free from bias and sensitivity, and widely vary in their degree of difficulty.

Item Development Process

AzM2 items go through a rigorous review before they are operational. When an item is "operational" it means it is used to determine a student's score on the assessment. This is a description of the process every item must go through before it is operational on AzM2.



Sample tests are available online for the math portion of AzM2. For more information view the Guide to the Sample Tests at www.AzM2portal.org.

Test Construction Guidelines

The construction of the AzM2 assessment is guided by the depth and rigor of the Arizona College and Career Ready Standards. Items are created to address key components of the standards and assess a range of important skills. The AzM2 Blueprint provides an overview of the distribution of items on the AzM2 according to the standards. The standards for Math Practices are embedded within all AzM2 items. Further, the AzM2 blueprint outlines the Depth of Knowledge distribution of items.

Blueprint

Grade 5 AzM2 Math Blueprint 2016 Standards		
Reporting Category	**Min.**	**Max.**
Operations & Algebraic Thinking and Numbers & Operations in Base Ten	**38%**	**42%**
Numbers in Base Ten	31%	35%
Algebraic Thinking	4%	8%
Number and Operations-Fractions	**31%**	**35%**
Measurement, Data, and Geometry	**24%**	**28%**
Measurement and Data	18%	20%
Geometry	7%	11%

Depth of Knowledge (DOK)

DOK refers to the level of rigor or sophistication of the task in a given item, designed to reflect the complexity of the Arizona Mathematics Standards. Items at DOK level 1 focus on the recall of information, such as definitions, terms, and simple procedures. Items at DOK 2 require students to make decisions, solve problems, or recognize patterns; in general, they require a greater degree of engagement and cognitive processing than items at DOK 1. Items at DOK 3 feature higher-order cognitive tasks that assess students' capacities to approach abstract or complex problems.

Percentage of Points by Depth of Knowledge (DOK) Level			
Grade 5	DOK Level 1	DOK Level 2	DOK Level 3
	10% - 20%	60% - 70%	12% - 30%

For more information on DOK go to www.azed.gov/AzM2.

Calculators

Arizona Desmos Graphing Calculator is not permitted for the paper-based and computer-based assessment for Grade 5 Math.

Item Formats

The AzM2 Assessments are composed of item formats that include traditional multiple- choice response items and technology-enhanced response items (TEI). TEIs are computer- delivered response items that require students to interact with test content to select, construct, and/or support their responses. TEIs are better able to assess a deeper level of understanding.

Currently, there are nine types of TEIs that may appear on the Math computer-based assessment for AzM2:

- Editing Tasks (ET)
- Editing Task Choice (ETC)
- Equation Editor (EQ)
- Graphic Response Item Display (GRID)
- Hot Text (HT)
  - Selectable Hot Text
  - Drag-and-Drop Hot Text
- Matching Item (MI)
- Multi-Select (MS)
- Open Response
- Table Item (TI)

For paper-based assessments (including those for students with an IEP or 504 plan that specifies a paper based accommodation), TEIs will be modified so that they can be scanned and scored electronically or hand-scored.

See the table below for a description of each TEI. In addition, for examples of each response item format described, see the AzM2 Training Tests at www.AzM2portal.org.

Item Format	Description
Editing Task (ET)	The student clicks on a highlighted word or phrase that may be incorrect, which reveals a text box. The directions in the text box direct the student to replace the highlighted word or phrase with the correct word or phrase. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Editing Task Choice (ETC)	The student clicks a highlighted word or phrase, which reveals a drop-down menu containing options for correcting an error as well as the highlighted word or phrase as it is shown in the sentence to indicate that no correction is needed. The student then selects the correct word or phrase from the drop-down menu. For paper-based assessments, the item is modified so that it can be scanned and scored electronically. The student fills in a circle to indicate the correct word or phrase.

Item Format	Description
Equation Editor (EQ)	The student is presented with a toolbar that includes a variety of mathematical symbols that can be used to create a response. Responses may be in the form of a number, variable, expression, or equation, as appropriate to the test item. For paper-based assessments, this item type may be replaced with a modified version of the item that can be scanned and scored electronically or replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Graphic Response Item Display (GRID)	The student selects numbers, words, phrases, or images and uses the drag-and-drop feature to place them into a graphic. This item type may also require the student to use the point, line, or arrow tools to create a response on a graph. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Hot Text (HT)	**Selectable Hot Text -** Excerpted sentences from the text are presented in this item type. When the student hovers over certain words, phrases, or sentences, the options highlight. This indicates that the text is selectable (“hot”). The student can then click on an option to select it. For paper- based assessments, a “selectable” hot text item is modified so that it can be scanned and scored electronically. In this version, the student fills in a circle to indicate a selection.
	Drag-and-Drop Hot Text - Certain numbers, words, phrases, or sentences may be designated “draggable” in this item type. When the student hovers over these areas, the text highlights. The student can then click on the option, hold down the mouse button, and drag it to a graphic or other format. For paper-based assessments, drag-and-drop hot text items will be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Matching Item (MI)	The student checks a box to indicate if information from a column header matches information from a row. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Multi-Select (MS)	The student is directed to select all of the correct answers from among a number of options. These items are different from multiple-choice items, which allow the student to select only one correct answer. These items appear in the online and paper-based assessments.
Open Response	The student uses the keyboard to enter a response into a text field. These items can usually be answered in a sentence or two. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.

Item Format	Description
Table Item (TI)	The student types numeric values into a given table. The student may complete the entire table or portions of the table depending on what is being asked. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.

Arizona Math Standards

Operations and Algebraic Thinking (OA)		
5.OA.A **Write and interpret numerical expressions.**	**5.OA.A.1**	Use parentheses and brackets in numerical expressions, and evaluate expressions with these symbols (Order of Operations).
	5.OA.A.2	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them (e.g., express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x (18,932 + 921) is three times as large as 18,932 + 921, without having to calculate the indicated sum or product).
5.OA.B **Analyze patterns and relationships.**	**5.OA.B.3**	Generate two numerical patterns using two given rules (e.g., generate terms in the resulting sequences). Identify and explain the apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane (e.g., given the rule "add 3" and the starting number 0, and given the rule "add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence).
	5.OA.B.4	Understand primes have only two factors and decompose numbers into prime factors.
Number and Operations in Base Ten (NBT)		
5.NBT.A **Understand the place value system.**	**5.NBT.A.1**	Apply concepts of place value, multiplication, and division to understand that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
	5.NBT.A.2	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10.
	5.NBT.A.3	Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form. b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
	5.NBT.A.4	Use place value understanding to round decimals to any place.
5.NBT.B **Perform operations with multi-digit whole numbers and with decimals to hundredths.**	**5.NBT.B.5**	Fluently multiply multi-digit whole numbers using a standard algorithm.
	5.NBT.B.6	Apply and extend understanding of division to find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors.
	5.NBT.B.7	Add, subtract, multiply, and divide decimals to hundredths, connecting objects or drawings to strategies based on place value, properties of operations, and/or the relationship between operations. Relate the strategy to a written form.

Number and Operations – Fractions (NF)		
5.NF.A **Use equivalent fractions to add and subtract fractions.**	**5.NF.A.1**	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators (e.g., 2/3 + 5/4 = 8/12 + 15/12 = 23/12).
	5.NF.A.2	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by using a variety of representations, equations, and visual models to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers (e.g. recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2).
5.NF.B **Use previous understandings of multiplication and division to multiply and divide fractions.**	**5.NF.B.3**	Interpret a fraction as the number that results from dividing the whole number numerator by the whole number denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people, each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*
	5.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number and a fraction by a fraction. a. Interpret the product (*a*/*b*) x *q* as *a* parts of a partition of *q* into *b* equal parts. *For example, use a visual fraction model to show (2/3) x 4 = 8/3, and create a story context for this equation.* b. Interpret the product of a fraction multiplied by a fraction (*a*/*b*) x (*c*/*d*). Use a visual fraction model and create a story context for this equation. *For example, use a visual fraction model to show (2/3) x (4/5) = 8/15, and create a story context for this equation.* In general, (*a*/*b*) x (*c*/*d*) = *ac*/*bd*. c. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
	5.NF.B.5	Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number; explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $\frac{a}{b} = \frac{n\,x\,a}{n\,x\,b}$ to the effect of multiplying $\frac{a}{b}$ by 1.
5.NF.B (cont.)	**5.NF.B.6**	Solve problems in real-world contexts involving multiplication of fractions, including mixed numbers, by using a variety of representations including equations and models.
	5.NF.B.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. Use the relationship between multiplication and division to justify conclusions. b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient.* Use the relationship between multiplication and division to justify conclusions (e.g., 4 ÷ (1/5) = 20 because 20 x (1/5) = 4). c. Solve problems in real-world context involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, using a variety of representations.
Measurement and Data (MD)		
5.MD.A **Convert like measurement units within a given measurement system.**	**5.MD.A.1**	Convert among different-sized standard measurement units within a given measurement system, and use these conversions in solving multi-step, real-world problems.
5.MD.B **Represent and interpret data.**	**5.MD.B.2**	Make a line plot to display a data set of measurements in fractions of a unit (1/8, 1/2, 3/4). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*
5.MD.C **Geometric measurement: Understand concepts of volume and relate volume to multiplication and to addition.**	**5.MD.C.3**	Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.
	5.MD.C.4	Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

5.MD.C (cont.)	**5.MD.C.5**	Relate volume to the operations of multiplication and addition and solve mathematical problems and problems in real-world contexts involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes (e.g., to represent the associative property of multiplication). b. Understand and use the formulas $V = l \times w \times h$ and $V = B \times h$, where in this case B is the area of the base ($B = l \times w$), for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths to solve mathematical problems and problems in real-world contexts. c. Understand volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms, applying this technique to solve mathematical problems and problems in real-world contexts.
		Geometry (G)
5.G.A **Graph points on the coordinate plane to solve mathematical problems as well as problems in real-world context.**	**5.G.A.1**	Understand and describe a coordinate system as perpendicular number lines, called axes, that intersect at the origin (0 , 0). Identify a given point in the first quadrant of the coordinate plane using an ordered pair of numbers, called coordinates. Understand that the first number (x) indicates the distance traveled on the horizontal axis, and the second number (y) indicates the distance traveled on the vertical axis.
	5.G.A.2	Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.
5.G.B **Classify two-dimensional figures into categories based on their properties.**	**5.G.B.3**	Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.
	5.G.B.4	Classify two-dimensional figures in a hierarchy based on properties.

Grade 5 Item Specifications

Measurement and Data & Geometry

5.G.A.1

Content Standards	Understand and describe a coordinate system as perpendicular number lines, called axes, that intersect at the origin (0, 0). Identify a given point in the first quadrant of the coordinate plane using an ordered pair of numbers, called coordinates. Understand that the first number (*x*) indicates the distance traveled on the horizontal axis, and the second number (*y*) indicates the distance traveled on the vertical axis.
Explanations	Graph points on the coordinate plane to solve real-world and mathematical problems.
Content Limits	Whole numbers Use only points located in the first quadrant of the coordinate plane. Plotting points given the ordered pair is aligned to 5.G.2
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to find the coordinates of a point based on its distance from the origin in the direction of the axes.	• Graphic Response • Multiple Choice Response • Multi-Select Response
Students will be required to plot a point based on its distance from the origin in the direction of the axes.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the axes and the origin (0 , 0) of a coordinate system. Identify the *x*- and *y*-coordinates of an ordered pair.	Describe a coordinate system as having 2 axes that intersect at the origin (0 , 0). Identify an ordered pair and the x- and y-coordinates of an ordered pair.
Proficient	**Highly Proficient**
Understand and describe a coordinate system as perpendicular number lines, called axes, that intersect at the origin (0 , 0). Identify a given point in the first quadrant of the coordinate plane using an ordered pair of numbers, called coordinates. Understand that the first number (*x*) indicates the distance traveled on the horizontal axis, and the second number (*y*) indicates the distance traveled on the vertical axis.	Understand and describe a coordinate system. Identify points in the coordinate plane using coordinates. Explain that the x-coordinate indicates the distance traveled on the horizontal axis, and the y-coordinate indicates the distance traveled on the vertical axis.

5.G.A.2

Content Standards	Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.
Explanations	Graph points on the coordinate plane to solve real-world and mathematical problems.
Content Limits	Whole numbers Use only points located in the first quadrant of the coordinate plane. Mathematical and real-world problems must have axes scaled to whole numbers (not letters).
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to find the coordinates of a point based on a graphed point in a coordinate plane.	• Graphic Response • Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to plot points based on given coordinates.	
Students will be required to plot points based on the relationship between their locations on the coordinate plane.	
Students will be required to identify how many units and which direction one point is from another point.	
Students will be required to interpret meaning of coordinate values within a context (axes indicate specific units).	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify points graphed in the first quadrant of the coordinate plane.	Graph points in the first quadrant of the coordinate plane, and identify the coordinate values of points in the context of the situation.
Proficient	**Highly Proficient**
Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	Create real-world and mathematical problems that can be solved by graphing points in the first quadrant of the coordinate plane. Explain the meaning of the coordinate values of points in the context of the situation.

5.G.B.3

Content Standards	Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.
Explanations	Geometric properties include properties of sides (parallel, perpendicular, congruent), properties of angles (type, measurement, congruent), and properties of symmetry (point and line). Properties of figure may include: Properties of sides - parallel, perpendicular, congruent, number of sides - or properties of angles - types of angles, congruent.
Content Limits	Focus should be on quadrilaterals, although other polygons can be included as well. There are two competing definitions for trapezoids - one that requires exactly one pair or parallel sides, and another that requires at least one pair of parallel sides (using this definition, parallelograms are trapezoids). Some students are taught one definition, others, the other. Thus, items that require the student to choose a definition in order to arrive at the correct answer should be avoided. Do not use Venn diagrams to represent hierarchy.
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to select shapes based on the attributes of a specific category.	• Graphic Response • Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to select attributes that categories share.	
Students will be required to select shapes that can be treated the same way as shapes in an upper category.	
Students will be required to show a hierarchy of shapes categorized by their attributes.	
Students will be required to select the categories a shape belongs.	
Students will be required to select shapes belonging to a particular subcategory.	
Students will be required to support/refute statements about categorizing shapes.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify attributes belonging to a category of two-dimensional figures.	Recognize that attributes belonging to a category of two-dimensional figures also belong to a subcategory of that category.
Proficient	**Highly Proficient**
Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.	Explain why attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.

5.G.B.4

Content Standards	Classify two-dimensional figures in a hierarchy based on properties.
Explanations	Geometric properties include properties of sides (parallel, perpendicular, congruent), properties of angles (type, measurement, congruent), and properties of symmetry (point and line). Properties of figure may include: Properties of sides - parallel, perpendicular, congruent, number of sides - or properties of angles - types of angles, congruent
Content Limits	Focus should be on quadrilaterals, although other polygons can be included as well. There are two competing definitions for trapezoids - one that requires exactly one pair or parallel sides, and another that requires at least one pair of parallel sides (using this definition, parallelograms are trapezoids). Some students are taught one definition, others, the other. Thus, items that require the student to choose a definition in order to arrive at the correct answer should be avoided. Do not use Venn diagrams to represent hierarchy.
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to select shapes based on the attributes of a specific category.	• Graphic Response • Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to select attributes that categories share.	
Students will be required to select shapes that can be treated the same way as shapes in an upper category.	
Students will be required to show a hierarchy of shapes categorized by their attributes.	
Students will be required to select the categories a shape belongs.	
Students will be required to select shapes belonging to a particular subcategory.	
Students will be required to support/refute statements about categorizing shapes.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify two-dimensional figures based on properties limited to sides and angles.	Classify two-dimensional figures based on properties limited to sides and angles.
Proficient	**Highly Proficient**
Classify two-dimensional figures in a hierarchy based on properties.	Draw or construct two-dimensional figures based on properties or classifications.

5.MD.A.1

Content Standards	Convert among different-sized standard measurement units within a given measurement system, and use these conversions in solving multi-step, real-world problems.
Explanations	In fifth grade, students build on their prior knowledge of related measurement units to determine equivalent measurements. Prior to making actual conversions, they examine the units to be converted, determine if the converted amount will be more or less units than the original unit, and explain their reasoning. They use several strategies to convert measurements. When converting metric measurement, students apply their understanding of place value and decimals.
Content Limits	Measurement values can be whole, decimal, and/or fractional values. Conversion is within the same system. Units of measurement include: kilometer, meter, centimeter, millimeter, liter, milliliter, kilogram, gram, milligram, mile, yard, foot, inch, gallon, quart, pint, cup, ton, pound, and ounce.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to calculate a measurement conversion within a problem with no context.	• Equation Response • Multiple Choice Response • Multi-Select Response • Table Response
Students will be required to solve a real world problem involving measurement conversions.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify equivalent, different-sized standard measurement units within a given measurement system, and use these conversions in solving one-step, real-world problems.	Convert among different-sized standard measurement units within a given measurement system, and use these conversions in solving two-step, real-world problems.
Proficient	**Highly Proficient**
Convert among different-sized standard measurement units within a given measurement system, and use these conversions in solving multi-step, real-world problems.	Create multi-step, real-world problems that require converting among different-sized standard measurement units within a given measurement system.

5.MD.B.2

Content Standards	Make a line plot to display a data set of measurements in fractions of a unit (1/8, 1/2, 3/4). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*
Explanations	Students apply their understanding of operations with fractions. They use either addition and/or multiplication to determine the total number of liters in the beakers. Then the sum of the liters is shared evenly among the ten beakers.
Content Limits	Measurement units are limited to halves, quarters, and eighths. Division is limited to a whole number divided by a unit fraction or a unit fraction divided by a whole number.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to construct a line plot based on given data comprised of fractions.	• Equation Response • Graphic Response • Multiple Choice Response • Table Response
Students will be required to interpret data in a line plot to solve problems involving addition, subtraction, multiplication, and division of fractions.	
Students will be required to interpret data in a line plot to solve problems involving addition, subtraction, multiplication, and division of unit fractions, where information is not fully provided.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify a line plot that displays a data set of measurements in fractions of a unit (1/2, 1/4). Use operations on fractions for this grade to identify solutions to one-step problems involving information presented in line plots.	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4). Use operations on fractions for this grade to solve one- or two-step problems involving information presented in line plots.
Proficient	**Highly Proficient**
Make a line plot to display a data set of measurements in fractions of a unit (1/8, 1/2, 3/4). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*	Make a line plot to display a data set of measurements in fractions of a unit. Use operations on fractions for this grade to solve multi-step problems involving information presented in line plots.

5.MD.C.3, 5.MD.C.3a, and 5.MD.C.3b

Content Standards	**5.MD.C.3** Recognize volume as an attribute of solid figures and understand concepts of volume measurement. **5.MD.C.3a** A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. **5.MD.C.3b** A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of *n* cubic units.
Explanations	Students' prior experiences with volume were restricted to liquid volume. As students develop their understanding volume they understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. This cube has a length of 1 unit, a width of 1 unit and a height of 1 unit and is called a cubic unit. This cubic unit is written with an exponent of 3 (e.g., in3, m3). Students connect this notation to their understanding of powers of 10 in our place value system. Models of cubic inches, centimeters, cubic feet, etc., are helpful in developing an image of a cubic unit. Student's estimate how many cubic yards would be needed to fill the classroom or how many cubic centimeters would be needed to fill a pencil box.
Content Limits	Right rectangular prisms with whole-number side lengths Graphics include unit cube Labels can include cubic units (i.e. cubic centimeters, cubic feet, etc) or exponential units (i.e. cm3, ft3, etc.)
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to recognize volume as an attribute of solid figures. (3a, 3b)	• Equation Response • Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to identify a unit cube as 1 cubic unit of volume. (3a)	
Students will be required to recognize the use of n unit cubes packed in a solid figure to find the volume of that figure in n cubic units. (3b)	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. Identify a “unit cube,” and know that it can be used to measure volume. b. Match the number of unit cubes it takes to pack a solid figure without gaps or overlaps to the volume of the figure.	Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. Define a “unit cube” and "one cubic unit." b. Identify that a solid figure which can be packed without gaps or overlaps using n unit cubes, and thus has a volume of n cubic units.
Proficient	**Highly Proficient**
Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.	Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. Explain why a cube with side length 1 unit, called a “unit cube,” and why it is said to have “one cubic unit” of volume. b. Explain why a solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

5.MD.C.4

Content Standards	Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
Explanations	Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.
Content Limits	Right rectangular prisms with whole-number side lengths Graphics include unit cube Labels can include cubic units (i.e. cubic centimeters, cubic feet, etc) or exponential units (i.e. cm3, ft3, etc.)
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to identify the volume of a rectangular prism by counting unit cubes, and compare volumes of multiple prisms.	• Equation Response • Multiple Choice Response • Matching Item Response • Multi-Select Response

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify volumes by counting unit cubes.	Measure volumes by counting unit cubes.
Proficient	**Highly Proficient**
Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	Look for patterns in measuring volumes of prisms by counting unit cubes. Fluently use cubic cm, cubic in, cubic ft, and improvised units.

5.MD.C.5, 5.MD.C.5a, 5.MD.C.5b, and 5.MD.C.5c

Content Standards	**5.MD.C.5** Relate volume to the operations of multiplication and addition and solve mathematical problems and problems in real-world contexts involving volume. **5.MD.C.5a** Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes (e.g., to represent the associative property of multiplication). **5.MD.C.5b** Understand and use the formulas $V = l \times w \times h$ and $V = B \times h$, where in this case B is the area of the base ($B = l \times w$), for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths to solve mathematical problems and problems in real-world contexts. **5.MD.C.5c** Understand volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms, applying this technique to solve mathematical problems and problems in real-world contexts.
Explanations	Students need multiple opportunities to measure volume by filling rectangular prisms with cubes and looking at the relationship between the total volume and the area of the base. They derive the volume formula (volume equals the area of the base times the height) and explore how this idea would apply to other prisms. Students use the associative property of multiplication and decomposition of numbers using factors to investigate rectangular prisms with a given number of cubic units.
Content Limits	Whole number side lengths Right rectangular prisms No more than two non-overlapping prisms - non-overlapping means that two prisms may share a face, but they do not share the same volume
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to calculate the volume of a right rectangular prism when given the formula.	• Equation Response • Graphic Response • Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to determine the volume of a right rectangular prism without the formula given.	
Students will be required to determine the dimensions of a right rectangular prism given the volume.	
Students will be required to compare volumes of rectangular prisms using the formula for volume.	
Students will be required to show how to determine the volume of a solid composed of 2 non-overlapping rectangular prisms (e.g. by writing an expression with an unknown.)	
Students will be required to calculate the volume of a solid figure that is composed of 2 non-overlapping rectangular prisms.	
Students will be required to identify an additional volume needed to complete a larger volume.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Relate volume to the operations of multiplication and addition and solve mathematical problems and problems in real-world contexts involving volume. a. Identify the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, or by multiplying the edge lengths. b. Understand and use the formula $V = l \times w \times h$ for rectangular prisms to identify volumes of right rectangular prisms with whole-number edge lengths. c. Understand volume as additive. Identify volumes of solid figures composed of two non-overlapping right rectangular prisms.	Relate volume to the operations of multiplication and addition and solve mathematical problems and problems in real-world contexts involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, or by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. b. Understand and use the formulas $V = l \times w \times h$ and $V = B \times h$, where in this case B is the area of the base ($B = l \times w$), for rectangular prisms to identify volumes of right rectangular prisms with whole-number edge lengths to solve mathematical problems. c. Understand volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms, applying this technique to solve mathematical problems.
Proficient	**Highly Proficient**
Relate volume to the operations of multiplication and addition and solve mathematical problems and problems in real-world contexts involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes (e.g., to represent the associative property of multiplication). b. Understand and use the formulas $V = l \times w \times h$ and $V = B \times h$, where in this case B is the area of the base ($B = l \times w$), for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths to solve mathematical problems and problems in real-world contexts. c. Understand volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms, applying this technique to solve mathematical problems and problems in real-world contexts.	Relate volume to the operations of multiplication and addition and solve mathematical problems and problems in real-world contexts involving volume. a. Explain why the volume of a right rectangular prism can be calculated by multiplying the edge lengths, and explain why this is equivalent to multiplying the height by the area of the base. Represent threefold whole-number products as volumes (e.g., to represent the associative property of multiplication). b. Create problems in real-world contexts that require understanding and using the formulas $V = l \times w \times h$ and $V = B \times h$. c. Understand volume as additive. Find volumes of solid figures composed of more than two non-overlapping right rectangular prisms, applying this technique to solve mathematical problems and problems in real-world contexts.

Numbers and Operations – Fractions

5.NF.A.1

Content Standards	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators (e.g., 2/3 + 5/4 = 8/12 + 15/12 = 23/12).
Explanations	Students should apply their understanding of equivalent fractions developed in fourth grade and their ability to rewrite fractions in an equivalent form to find common denominators. They should know that multiplying the denominators will always give a common denominator but may not result in the smallest denominator.
Content Limits	Improper fractions and mixed numbers included. Least common denominator is not necessary to calculate sums of fractions. Do not use the terms "simplify" or "lowest terms". Denominators should be one-digit or two-digit.
Context	Context is not allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to calculate the sum or difference of two or more fractions with unlike denominators.	• Equation Response • Multiple Choice Response

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify the sum or difference of fractions with unlike denominators.	Identify the sum or difference of fractions with unlike denominators (including mixed numbers).
Proficient	**Highly Proficient**
Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators (e.g., 2/3 + 5/4 = 8/12 + 15/12 = 23/12).	Explain how to find the sum or difference of fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.

5.NF.A.2

Content Standards	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by using a variety of representations, equations, and visual models to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers (e.g. recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2).
Explanations	Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies for calculations with fractions extend from students' work with whole number operations and can be supported through the use of physical models.
Content Limits	Improper fractions and mixed numbers included. Least common denominator is not necessary to calculate sums of fractions. Do not use the terms "simplify" or "lowest terms".
Context	Context is required.

<table>
<tr><th>Sample Task Demands</th><th>Common Item Formats</th></tr>
<tr><td>Students will be required to calculate the sum or difference of two or more fractions with like and/or unlike denominators in a given word problem.</td><td rowspan="3">Equation Response
Multiple Choice Response</td></tr>
<tr><td>Students will be required to determine a missing numerator or denominator in the addend, subtrahend, or minuend of an addition or subtraction problem with fractions in a given word problem.</td></tr>
<tr><td>Students will be required to use benchmark fractions to explain why an assertion is or is not reasonable.</td></tr>
</table>

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the solution to word problems involving addition and subtraction of fractions referring to the same whole, by using visual models to represent the problem. Use benchmark fractions and number sense of fractions to identify an estimate.	Identify the solution to word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by using a variety of representations, equations, and visual models to represent the problem. Use benchmark fractions and number sense of fractions to identify an estimate and assess the reasonableness of answers.

Proficient	Highly Proficient
Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by using a variety of representations, equations, and visual models to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers (e.g., recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2).	Create word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators. Explain how to estimate mentally and assess the reasonableness of answers.

5.NF.B.3

<table>
<tr><td>Content Standards</td><td colspan="2">Interpret a fraction as the number that results from dividing the whole number numerator by the whole number denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people, each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?</td></tr>
<tr><td>Explanations</td><td colspan="2">Students are expected to demonstrate their understanding using concrete materials, drawing models, and explaining their thinking when working with fractions in multiple contexts. They read 3/5 as “three fifths” and after many experiences with sharing problems, learn that 3/5 can also be interpreted as “3 divided by 5.”</td></tr>
<tr><td>Content Limits</td><td colspan="2">Quotients in division problems should not be equivalent to a whole number.
Only use whole numbers for the divisor and dividend of a fraction.</td></tr>
<tr><td>Context</td><td colspan="2">Context is required.</td></tr>
<tr><th colspan="2">Sample Task Demands</th><th>Common Item Formats</th></tr>
<tr><td colspan="2">Students will be required to express a given division problem as a fraction.</td><td rowspan="5">Equation Response
Multiple Choice Response
Table Response</td></tr>
<tr><td colspan="2">Students will be required to identify a given fraction as a division problem.</td></tr>
<tr><td colspan="2">Students will be required to find the solution to a division word problem and express the quotient as a fraction.</td></tr>
<tr><td colspan="2">Students will be required to with or without context, determine the two consecutive whole numbers between which the answer lies in a given division problem.</td></tr>
<tr><td colspan="2">Students will be required to identify an area model or number line model that shows the solution to a division word problem.</td></tr>
</table>

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify a fraction that results from dividing the whole number numerator by the whole number denominator. Identify the solution to word problems involving division of whole numbers leading to answers in the form of fractions.	Determine the fraction that results from dividing the whole number numerator by the whole number denominator. Identify the solution to word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers.
Proficient	**Highly Proficient**
Interpret a fraction as the number that results from dividing the whole number numerator by the whole number denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people, each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*	Explain the meaning of a fraction as the number that results from dividing the whole number numerator by the whole number denominator, and why multiplying a fraction by the denominator results in the numerator. Create word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers.

5.NF.B.4, 5.NF.B.4a, 5.NF.B.4b, and 5.NF.B.4c

<table>
<tr><td>Content Standards</td><td colspan="2">5.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number and a fraction by a fraction.

5.NF.B.4a Interpret the product (a/b) x q as a parts of a partition of q into b equal parts. For example, use a visual fraction model to show (2/3) x 4 = 8/3, and create a story context for this equation.

5.NF.B.4b Interpret the product of a fraction multiplied by a fraction (a/b) x (c/d). Use a visual fraction model and create a story context for this equation. For example, use a visual fraction model to show (2/3) x (4/5) = 8/15, and create a story context for this equation. In general, (a/b) x (c/d) = ac/bd.

5.NF.B.4c Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.</td></tr>
<tr><td>Explanations</td><td colspan="2">Students are expected to multiply fractions including proper fractions, improper fractions, and mixed numbers. They multiply fractions efficiently and accurately as well as solve problems in both contextual and non-contextual situations.</td></tr>
<tr><td>Content Limits</td><td colspan="2">Multiply whole numbers by fractions or fractions by fractions

Visual models Part a - any appropriate fraction model. (e.g. circles, tape, polygons, etc…) Part b - rectangle models only, tile with unit squares

For tiling, the dimensions of the tile should be unit fractions with the same denominator as the given rectangular shape (see p. 13 of the progression document for demonstration)

Problems do not require simplifying or lowest form.</td></tr>
<tr><td>Context</td><td colspan="2">Context is allowed.</td></tr>
<tr><th colspan="2">Sample Task Demands</th><th>Common Item Formats</th></tr>
<tr><td colspan="2">Students will be required to multiply a fraction by a whole number or a fraction.</td><td rowspan="5">Equation Response
Graphic Response
Multiple Choice Response
Multi-Select Response</td></tr>
<tr><td colspan="2">Students will be required to relate a scenario that describes “a parts of a partition of q into b equal parts” to an expression of the form (a/b) x q and/or a x q ÷ b.</td></tr>
<tr><td colspan="2">Students will be required to tile a rectangular shape to find the area, either given the dimensions of the tile, or the dimensions of the shape.</td></tr>
<tr><td colspan="2">Students will be required to multiply length and width to find the area of a rectangular shape with fractional side lengths.</td></tr>
<tr><td colspan="2">Students will be required to identify rectangular shape(s) with a given area, where the shapes have given side lengths or are tiled with tiles of given dimensions.</td></tr>
</table>

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Apply and extend previous understandings of multiplication to multiply a fraction by a whole number and a fraction by a fraction. a. Identify the product (a/b) x q as a parts of a partition of q into b equal parts using a visual fraction model. b. Identify the product of a fraction multiplied by a fraction (a/b) x (c/d) as (ac/bd) using a visual fraction model. c. Identify the area of a rectangle with fractional side lengths that has been tiled with unit squares of the appropriate unit fraction side lengths. Identify the product of fractional side lengths to find areas of rectangles.	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number and a fraction by a fraction. a. Identify the product (a/b) x q as a parts of a partition of q into b equal parts. b. Identify the product of a fraction multiplied by a fraction (a/b) x (c/d) as ac/bd. Identify the correct story context for a given equation in the form (a/b) x (c/d) = ac/bd. c. Find the area of a rectangle with fractional side lengths that has been tiled with unit squares of the appropriate unit fraction side lengths. Identify the product of fractional side lengths to find areas of rectangles. Recognize that fraction products are rectangular areas.
Proficient	**Highly Proficient**
Apply and extend previous understandings of multiplication to multiply a fraction by a whole number and a fraction by a fraction. a. Interpret the product (a/b) x q as a parts of a partition of q into b equal parts. *For example, use a visual fraction model to show (2/3) x 4 = 8/3, and create a story context for this equation.* b. Interpret the product of a fraction multiplied by a fraction (a/b) x (c/d). Use a visual fraction model and create a story context for this equation. *For example, use a visual fraction model to show (2/3) x (4/5) = 8/15, and create a story context for this equation.* In general, (a/b) x (c/d) = ac/bd. c. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number and a fraction by a fraction. a. Explain why the product (a/b) x q is a parts of a partition of q into b equal parts, and create a word problem for an equation given in the form (a/b) x q. b. Explain why the product of a fraction multiplied by a fraction (a/b) x (c/d) is the product of the numerators divided by the product of the denominators ac/bd. Create a story context for a given equation in the form (a/b) x (c/d) = ac/bd. c. Given a rectangle with fractional side lengths, explain how tiling the rectangle with unit squares of the appropriate fractional side lengths and calculating the sum of area of those tiles is the same as multiplying the side lengths of the rectangle. Explain the connection between the product of two fractions and the area of a rectangle with side lengths equal to those fractions.

5.NF.B.5, 5.NF.B.5a, and 5.NF.B.5b

Content Standards	**5.NF.B.5** Interpret multiplication as scaling (resizing), by: **5.NF.B.5a** Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. **5.NF.B.5b** Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number; explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.
Explanations	Use previous understandings of multiplication and division to multiply and divide fractions.
Content Limits	Fractions greater than or equal to 0 Base numbers should be large and unwieldy to discourage students from calculating products Scaling is explored or demonstrated only in terms of quantity. Scaling geometric figures should not be assessed at this standard. Scaling quantities of any kind in 2 dimensions is strictly beyond the scope of this standard.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to identify a statement comparing the value of a given multiplication expression to one of its factors.	• Multiple Choice Response • Multi-Select Response
Students will be required to identify an expression that represents a given statement comparing a product to one of its factors.	
Students will be required to identify expressions that have a value less than or greater than a given number, where the expressions are that number multiplied by another number.	
Students will be required to identify a possible factor of a given expression, given one factor and a comparison of the value of the product to that factor.	
Students will be required to identify an expression that is equivalent to multiplying a given number by 1.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Interpret multiplication as scaling (resizing), by: a. Identifying how the size of the product relates to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication, given a visual model. b. Identifying that multiplying a given number by a fraction greater than 1 results in a product greater than the given number; identifying that multiplying a given number by a fraction less than 1 results in a product smaller than the given number.	Interpret multiplication as scaling (resizing), by: a. Identifying how the size of the product relates to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication, given a visual model. b. Identifying that multiplying a given number by a fraction greater than 1 results in a product greater than the given number; identifying that multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and identifying that multiplying a given fraction by a fraction equal to 1 results in an equivalent fraction.
Proficient	**Highly Proficient**
Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number; explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.	Interpret multiplication as scaling (resizing), by: a. Explaining how the size of a product compares to the size of one factor on the basis of the size of the other factor. b. Demonstrating how multiplying a given number by a fraction greater than 1 results in a product greater than the given number; demonstrating how multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and demonstrating how the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ relates to the effect of multiplying a/b by 1.

5.NF.B.6

<table>
<tr><td>Content Standards</td><td colspan="2">Solve problems in real-world contexts involving multiplication of fractions, including mixed numbers, by using a variety of representations including equations and models.</td></tr>
<tr><td>Explanations</td><td colspan="2">Use previous understandings of multiplication and division to multiply and divide fractions.</td></tr>
<tr><td>Content Limits</td><td colspan="2">Items should require student to interpret the context to determine operations.</td></tr>
<tr><td>Context</td><td colspan="2">Context is required.</td></tr>
<tr><th colspan="2">Sample Task Demands</th><th>Common Item Formats</th></tr>
<tr><td colspan="2">Students will be required to solve simple word problems involving multiplication of fractions .(i.e., multiplying two given values)</td><td rowspan="2">• Equation Response
• Multiple Choice Response</td></tr>
<tr><td colspan="2">Students will be required to solve complex word problems involving multiplication of fractions (e.g., multiplying three numbers, involving other operations, finding an unknown. (numerator, denominator, etc.)</td></tr>
</table>

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the solutions to problems in real-world contexts involving multiplication of fractions, by using visual models.	Identify the solutions to problems in real-world contexts involving multiplication of fractions, by using a variety of representations including equations and models.
Proficient	**Highly Proficient**
Solve problems in real-world contexts involving multiplication of fractions, including mixed numbers, by using a variety of representations including equations and models.	Create problems in real-world contexts involving multiplication of fractions, including mixed numbers, given a representation such as an equation or a model.

5.NF.B.7, 5.NF.B.7a, 5.NF.B.7b, and 5.NF.B.7c

Content Standards	**5.NF.B.7** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. **5.NF.B.7a** Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. Use the relationship between multiplication and division to justify conclusions. **5.NF.B.7b** Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient.* Use the relationship between multiplication and division to justify conclusions (e.g., 4 ÷ (1/5) = 20 because 20 x (1/5) = 4). **5.NF.B.7c** Solve problems in real-world context involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, using a variety of representations.
Explanations	In fifth grade, students experience division problems with whole number divisors and unit fraction dividends (fractions with a numerator of 1) or with unit fraction divisors and whole number dividends. Students extend their understanding of the meaning of fractions, how many unit fractions are in a whole, and their understanding of multiplication and division as involving equal groups or shares and the number of objects in each group/share. In sixth grade, they will use this foundational understanding to divide into and by more complex fractions and develop abstract methods of dividing by fractions.
Content Limits	All problems should have either: A) Division of unit fractions by a non-zero whole number, or B) Division of a non-zero whole number by a unit fraction
Context	Context is required.

Sample Task Demands	Common Item Formats
Students will be required to divide a fraction by a whole number.	• Equation Response • Graphic Response • Multiple Choice Response
Students will be required to divide a whole number by a fraction.	
Students will be required to select a division expression that represents the scenario of a given division problem.	
Students will be required to select an equivalent multiplication equation for a given division equation.	
Students will be required to solve a simple word problem that involves division, a whole number, and a fraction.	
Students will be required to solve a simple word problem that involves division, and justify the solution using an equation or number line.	

Performance Level Descriptors

<table>
<tr><th>Minimally Proficient</th><th>Partially Proficient</th></tr>
<tr><td>Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

a. Identify the quotient of a unit fraction by a non-zero whole number.

b. Identify the quotient of a whole number by a unit fraction.

c. Identify the solutions to problems in real-world context involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, using visual models.</td><td>Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

a. Compute the quotient of a unit fraction by a non-zero whole number.

b. Compute the quotient of a whole number by a unit fraction.

c. Identify the solutions to problems in real-world context involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, using a variety of representations.</td></tr>
<tr><th>Proficient</th><th>Highly Proficient</th></tr>
<tr><td>Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. Use the relationship between multiplication and division to justify conclusions.

b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to justify conclusions (e.g., 4 ÷ (1/5) = 20 because 20 x (1/5) = 4).

c. Solve problems in real-world context involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, using a variety of representations.</td><td>Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

a. Use the relationship between multiplication and division to explain how to divide a unit fraction by a non-zero whole number.

b. Use the relationship between multiplication and division to explain how to divide a whole number by a unit fraction.

c. Create problems in real-world context involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions.</td></tr>
</table>

5.NBT.A.1

Content Standards	Apply concepts of place value, multiplication, and division to understand that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
Explanations	In fourth grade, students examined the relationships of the digits in numbers for whole numbers only. This standard extends this understanding to the relationship of decimal fractions. Students use base ten blocks, pictures of base ten blocks, and interactive images of base ten blocks to manipulate and investigate the place value relationships. They use their understanding of unit fractions to compare decimal places and fractional language to describe those comparisons. Before considering the relationship of decimal fractions, students express their understanding that in multi-digit whole numbers, a digit in one place represents 10 times what it represents in the place to its right and 1/10 of what it represents in the place to its left.
Content Limits	Whole numbers to any place value and decimals to thousandths
Context	Context is not allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to identify the factor by which one number is greater or less than another.	• Equation Response • Multiple Choice Response
Students will be required to compare the value of a digit in different place values of two given numbers and identify the power of 10 by which one number is greater than another.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify which place value in a multi-digit whole number represents 10 times the value of a given place value, or identify which place value in a multi-digit whole number represents 1/10 the value of a given place value.	Given two multi-digit numbers, with a digit in different place values in each number, identify how many times the value of the digit is in one number compared to the other number (e.g. the value of the 4 in 29,143 is 1/100 times the value of the 4 in 74,851).
Proficient	**Highly Proficient**
Apply concepts of place value, multiplication, and division to understand that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	Apply concepts of place value, multiplication, and division to explain why a digit in one place represents ten times what it represents in the place to its right and 1/10 of what it represents in the place to its left.

5.NBT.A.2

Content Standards	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10.
Explanations	Understand the place value system.
Content Limits	Whole number exponents with a base of 10. Decimals to thousandths.
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to calculate a power of 10.	• Equation Response • Multiple Choice Response • Multi-Select Response
Students will be required to multiply or divide a decimal by a power of ten.	
Students will be required to find a missing exponent when multiplying or dividing a decimal by a power of ten.	
Students will be required to identify patterns when multiplying or dividing by a power of 10.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify patterns in the number of zeros of the product when multiplying a number by powers of 10, and identify patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10.	Find patterns in the number of zeros of the product when multiplying a number by powers of 10, and find patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10.
Proficient	**Highly Proficient**
Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10.	Given a pattern in the number of zeros of the product when multiplying a number by powers of 10, or a pattern in the placement of the decimal point when multiplying or dividing a number by a power of 10, create a possible equation that represents the pattern and explain why there are multiple correct equations.

5.NBT.A.3, 5.NBT.A.3a, and 5.NBT.A.3b

Content Standards	**5.NBT.A.3** Read, write, and compare decimals to thousandths. **5.NBT.A.3a** Read and write decimals to thousandths using base-ten numerals, number names, and expanded form. **5.NBT.A.3b** Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
Explanations	Students build on the understanding they developed in fourth grade to read, write, and compare decimals to thousandths. They connect their prior experiences with using decimal notation for fractions and addition of fractions with denominators of 10 and 100. They use concrete models and number lines to extend this understanding to decimals to the thousandths. Models may include base ten blocks, place value charts, grids, pictures, drawings, manipulatives, technology-based, etc. They read decimals using fractional language and write decimals in fractional form, as well as in expanded notation as show in the standard 3a. This investigation leads them to understanding equivalence of decimals (0.8 = 0.80 = 0.800). Students need to understand the size of decimal numbers and relate them to common benchmarks such as 0, 0.5 (0.50 and 0.500), and 1. Comparing tenths to tenths, hundredths to hundredths, and thousandths to thousandths is simplified if students use their understanding of fractions to compare decimals.
Content Limits	Decimals to thousandths
Context	Context is allowed.

<table>
<tr><th>Sample Task Demands</th><th>Common Item Formats</th></tr>
<tr><td>Students will be required to write a number with a given name in numeric form.</td><td rowspan="6">Equation Response
Graphic Response
Multiple Choice Response
Matching Item Response
Multi-Select Response</td></tr>
<tr><td>Students will be required to identify the name of a given number.</td></tr>
<tr><td>Students will be required to write a number given in traditional expanded form in numeric form or vice versa.</td></tr>
<tr><td>Students will be required to compare two decimals.</td></tr>
<tr><td>Students will be required to order more than two decimals in numeric form.</td></tr>
<tr><td>Students will be required to identify numbers in non-traditional expanded form (e.g., 47.389 = 9 × (1/1000) + 7 × 1 + 3 × (1/10) + 4 × 10 + 8 x (1/100)).</td></tr>
</table>

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Read and write, decimals to tenths. a. Identify decimals to tenths using base-ten numerals and number names. b. Compare two decimals to tenths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	Read, write, and compare decimals to hundredths. a. Identify decimals to hundredths using base-ten numerals, number names, and expanded form. b. Compare two decimals to hundredths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
Proficient	**Highly Proficient**
Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form. b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	Read, write, and compare decimals to thousandths. a. Order multiple decimals to thousandths using base-ten numerals, number names, and expanded form. b. Compare more than two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

5.NBT.A.4

Content Standards	Use place value understanding to round decimals to any place.
Explanations	When rounding a decimal to a given place, students may identify the two possible answers, and use their understanding of place value to compare the given number to the possible answers.
Content Limits	Decimals to thousandths
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify the value of a decimal number rounded to a place value.	• Equation Response • Multiple Choice Response • Matching Item Response • Multi-Select Response • Table Response
Students will be required to identify the decimal numbers that round to a given value.	
Students will be required to distinguish between different rounding procedures used in order to create a number that fits certain parameters.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Use place value understanding to round decimals to the tenths place.	Use place value understanding to round decimals to the hundredths place.
Proficient	**Highly Proficient**
Use place value understanding to round decimals to any place.	Explain how to use place value understanding to round decimals to any place.

5.NBT.B.5

Content Standards	Fluently multiply multi-digit whole numbers using a standard algorithm.
Explanations	In prior grades, students used various strategies to multiply. Students can continue to use these different strategies as long as they are efficient, but must also understand and be able to use the standard algorithm. In applying the standard algorithm, students recognize the importance of place value.
Content Limits	Multiplication should not exceed 5 digits by 2 digits.
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to calculate the product of two numbers.	• Equation Response
Students will be required to identify a missing factor or digit in a multiplication problem.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the product of two multi-digit whole numbers.	Calculate the product of two multi-digit whole numbers.
Proficient	**Highly Proficient**
Fluently multiply multi-digit whole numbers using a standard algorithm.	Explain how to use a standard algorithm to multiply multi-digit whole numbers.

5.NBT.B.6

Content Standards	Apply and extend understanding of division to find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors.
Explanations	In fourth grade, students' experiences with division were limited to dividing by one-digit divisors. This standard extends students' prior experiences with strategies, illustrations, and explanations. When the two-digit divisor is a "familiar" number, a student might decompose the dividend using place value.
Content Limits	Only 3-digit or 4-digit dividend and 2-digit divisor
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to calculate the quotient of 2 numbers.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response
Students will be required to select expressions that are equivalent to a given quotient.	
Students will be required to illustrate and explain quotients of 2 numbers using equations, rectangular arrays, or area models.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Apply understanding of division to identify whole-number quotients of whole numbers with up to three-digit dividends and two-digit divisors.	Apply understanding of division to identify whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors.
Proficient	**Highly Proficient**
Apply and extend understanding of division to find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors.	Apply and extend understanding of division to find whole-number quotients of whole numbers with more than four-digit dividends and two-digit divisors.

5.NBT.B.7

Content Standards	Add, subtract, multiply, and divide decimals to hundredths, connecting objects or drawings to strategies based on place value, properties of operations, and/or the relationship between operations. Relate the strategy to a written form.
Explanations	This standard requires students to extend the models and strategies they developed for whole numbers in grades 1-4 to decimal values. Before students are asked to give exact answers, they should estimate answers based on their understanding of operations and the value of the numbers. Students should be able to express that when they add decimals they add tenths to tenths and hundredths to hundredths. So, when they are adding in a vertical format (numbers beneath each other), it is important that they write numbers with the same place value beneath each other. This understanding can be reinforced by connecting addition of decimals to their understanding of addition of fractions. Adding fractions with denominators of 10 and 100 is a standard in fourth grade.
Content Limits	Decimals within hundredths place in all numbers involved (divisors, dividends, quotients and likewise for other operations)
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to perform a calculation involving decimals.	• Equation Response • Multi-Select Response
Students will be required to solve a problem involving decimals and the four operations given a scenario.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Add and subtract decimals (without regrouping) to hundredths, connecting objects or drawings to strategies based on place value, properties of operations, and/or the relationship between operations.	Add, subtract, and multiply decimals to hundredths, connecting objects or drawings to strategies based on place value, properties of operations, and/or the relationship between operations.
Proficient	**Highly Proficient**
Add, subtract, multiply, and divide decimals to hundredths, connecting objects or drawings to strategies based on place value, properties of operations, and/or the relationship between operations. Relate the strategy to a written form.	Add, subtract, multiply, and divide decimals to hundredths. Relate the strategy to a written form. Apply this to real-world context.

5.OA.A.1

Content Standards	Use parentheses and brackets in numerical expressions, and evaluate expressions with these symbols (Order of Operations).
Explanations	This standard builds on the expectations of third grade where students are expected to start learning the conventional order. Students need experiences with multiple expressions that use grouping symbols throughout the year to develop understanding of when and how to use parentheses, brackets, and braces. First, students use these symbols with whole numbers. Then the symbols can be used as students add, subtract, multiply and divide decimals and fractions. To further develop students' understanding of grouping symbols and facility with operations, students place grouping symbols in equations to make the equations true or they compare expressions that are grouped differently.
Content Limits	Whole numbers & simple fraction expressions (single digit denominators, fraction multiplied by a whole number) Do not use nested grouping symbols (based on the progressions document for K-5 operations and algebraic thinking) Expressions should not be more complex than those used in associative or distributive property situations
Context	Context is not allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to evaluate a numerical expression with parentheses.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response
Students will be required to rewrite a given numerical expression with parentheses, brackets and/or braces (by inserting these grouping symbols) such that the expression evaluates to a given answer.	
Students will be required to identify a calculation error when evaluating a numerical expression.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Use parenthesis in numerical expressions and evaluate numeric expressions.	Evaluate numerical expressions with parentheses and brackets.
Proficient	**Highly Proficient**
Use parentheses and brackets in numerical expressions, and evaluate expressions with these symbols (Order of Operations).	Use parentheses and brackets to create multiple numerical expressions equivalent to a given value.

5.OA.A.2

Content Standards	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them (e.g., express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x (18,932 + 921) is three times as large as 18,932 + 921, without having to calculate the indicated sum or product).
Explanations	Students use their understanding of operations and grouping symbols to write expressions and interpret the meaning of a numerical expression.
Content Limits	Whole numbers Simple fraction expressions Do not use nested parentheses Use numeric expressions only. Multiplication cross symbol is the only acceptable symbol for multiplication. Do not use the c-dot. When grouping symbols are part of the expression, the associative property or distributive property should be found in that expression.
Context	Context is not allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to construct a numeric expression given a written statement of numerical values.	• Equation Response • Multiple Choice Response • Proposition Response
Students will be required to interpret the meaning of a written numerical statement without evaluating it.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify simple expressions that record calculations with numbers, and identify numerical expressions without evaluating them.	Write simple expressions that record calculations with numbers, and identify numerical expressions without evaluating them.
Proficient	**Highly Proficient**
Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them (e.g., express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x (18,932 + 921) is three times as large as 18,932 + 921, without having to calculate the indicated sum or product).	Write simple expressions that record multi-step calculations with numbers, and interpret multi-step numerical expressions without evaluating them.

5.OA.B.3

Content Standards	Generate two numerical patterns using two given rules (e.g., generate terms in the resulting sequences). Identify and explain the apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane (e.g., given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence).
Explanations	Analyze patterns and relationships.
Content Limits	Whole numbers & fractions with denominators less than 10 Quadrant I on coordinate plane Acceptable operations: addition, subtraction, multiplication, and division The rule should be no more complex than one finds in an application of the associative or distributive property. Examples should not contain nested grouping symbols.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to find terms of two numerical patterns given rules, including forming ordered pairs determined by the pattern.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response • Table Response
Students will be required to identify specific relationships between terms of two numerical patterns (term when the sequences are equal, where one is twice the other, etc.)	
Students will be required to graph ordered pairs corresponding to terms in two numerical patterns in a coordinate plane.	
Students will be required to identify relationships between two numerical patterns.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify two numerical patterns using two given rules (e.g., identify terms in the resulting sequences). Identify the apparent relationships between corresponding terms. Identify ordered pairs consisting of corresponding terms from the two patterns.	Determine the missing values in two numerical patterns using two given rules (e.g., determine the missing terms in the resulting sequences). Identify the apparent relationships between corresponding terms. Identify ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.
Proficient	**Highly Proficient**
Generate two numerical patterns using two given rules (e.g., generate terms in the resulting sequences). Identify and explain the apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane (e.g., given the rule "add 3" and the starting number 0, and given the rule "add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence).	Explain how the rules for two numerical patterns relate to the relationships between the corresponding terms in those patterns (e.g., given the rule "add 3" and the starting number 0, and given the rule "add 6" and the starting number 0, observe that the terms in one sequence are twice the corresponding terms in the other sequence, and recognize that "add 3" is twice "add 6").

5.OA.B.4

Content Standards	Understand primes have only two factors and decompose numbers into prime factors.
Explanations	A prime number is a whole number greater than 1 that has only 2 factors, 1 and itself. Composite numbers have more than 2 factors. Students investigate whether numbers are prime or composite by building rectangles (arrays) and finding which rectangles can be built using more than one equal row and one equal column. These rectangles represent composite numbers. Rectangles that cannot be built with more than one equal row and one equal column (e.g., 7 can be only be shown as a 1-by-7 or 7-by-1 array) represent prime numbers.
Content Limits	Vocabulary includes "prime," "composite," "factor," and "multiple."
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will determine whether a whole number is prime or composite.	• Equation Response • Graphic Response • Multiple Choice Response • Table Response
Students will be able to decompose whole numbers into prime factors.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify prime numbers.	Understand prime numbers have only two factors and identify the prime factorization of numbers.
Proficient	**Highly Proficient**
Understand primes have only two factors and decompose numbers into prime factors.	Explain how to decompose numbers into prime factors.

AzM2 | Arizona's Statewide Achievement Assessment for English Language Arts and Mathematics

Mathematics Item Specifications

GRADE 6

Arizona Department of Education with American Institutes for Research - 2019

Table of Contents

Introduction

The Arizona Statewide Achievement Assessment for English Language Arts and Mathematics (AzM2) is Arizona's statewide achievement test. AzM2 assesses the Arizona English Language Arts Standards and Arizona Mathematics Standards adopted by the Arizona State Board of Education in December 2016. AzM2 will inform students, teachers, and parents about preparedness for college and careers upon graduating from high school. AzM2 tests are computer-based, meaning that they can better assess students' critical thinking skills and provide them with opportunities to demonstrate a deeper understanding of the materials. Computer-based testing also allows for the use of a variety of innovative items types.

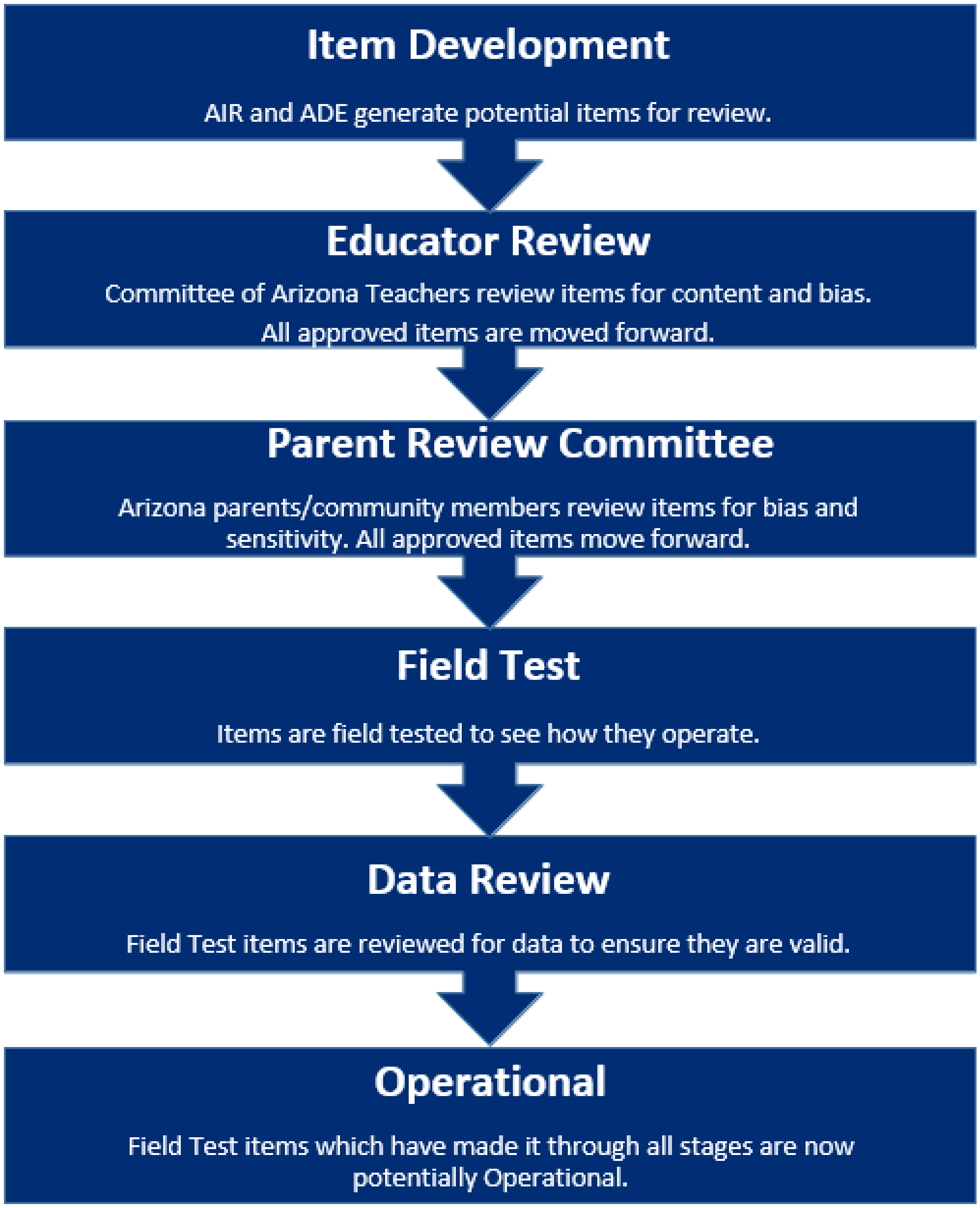
During the item-development process, all AzM2 items are written in accordance with the Item Specifications and are reviewed and approved by a committee of Arizona educators to confirm alignment and appropriateness for inclusion in the test. AzM2 items are generally representative of Arizona's geographic regions and culturally diverse population. Items are reviewed for the following kinds of bias: gender, racial, ethnic, linguistic, religious, geographic, and socioeconomic. Item reviews also include consideration of issues related to individuals with disabilities. Arizona community members also have an opportunity to review items for issues of potential concern to members of the community at large. Reviewers are asked to consider the variety of cultural, regional, philosophical, political, and religious backgrounds throughout Arizona, and then to determine whether the subject matter will be acceptable to Arizona students, families, and other members of Arizona communities.

This *AzM2 Item Specifications* is a resource document that defines the content and format of the test and test items for item writers and reviewers. Each *Item Specifications* document indicates the alignment of items with the Arizona Mathematics Standards. It also serves to provide all stakeholders with information about the scope and function of assessment items. This document can also serve to assist educators to understand how assessment items are developed in alignment with the standards for English language arts and math. These item specifications for AzM2 are intended to provide information regarding standards, item formats and response types. The descriptions of math blueprints and depth of knowledge in this document are meant to provide an overview of the test. Item specifications are meant for the purposes of assessment, not instruction. They are not intended to be tools for instruction or the basis for curricula. AzM2 has a test blueprint that was developed by Arizona and is different from any other state or consortium test blueprint.

For the math portion of AzM2, all of the test questions are aligned to the mathematic content standards for these subject areas. Any item specifications that are absent for standards listed in this document may be under development. This document does not endorse the exclusion of the instruction of any grade-level content standards. The test will ask questions that check a student's conceptual understanding of math as well as their procedural skills. These items have been written to be free from bias and sensitivity, and widely vary in their degree of difficulty.

Item Development Process

AzM2 items go through a rigorous review before they are operational. When an item is "operational" it means it is used to determine a student's score on the assessment. This is a description of the process every item must go through before it is operational on AzM2.



Sample tests are available online for the math portion of AzM2. For more information view the Guide to the Sample Tests at www.AzM2portal.org.

Test Construction Guidelines

The construction of the AzM2 assessment is guided by the depth and rigor of the Arizona College and Career Ready Standards. Items are created to address key components of the standards and assess a range of important skills. The AzM2 Blueprint provides an overview of the distribution of items on the AzM2 according to the standards. The standards for Math Practices are embedded within all AzM2 items. Further, the AzM2 blueprint outlines the Depth of Knowledge distribution of items.

Blueprint

Grade 6 AzM2 Blueprint 2016 Standards		
Reporting Category	**Min.**	**Max.**
Ratio and Proportional Relationships	19%	23%
Expressions and Equations	29%	33%
Geometry, Statistics & Probability	15%	19%
Geometry	6%	15%
Statistics and Probability	6%	11%
The Number System	28%	32%

Depth of Knowledge (DOK)

DOK refers to the level of rigor or sophistication of the task in a given item, designed to reflect the complexity of the Arizona Mathematics Standards. Items at DOK level 1 focus on the recall of information, such as definitions, terms, and simple procedures. Items at DOK 2 require students to make decisions, solve problems, or recognize patterns; in general, they require a greater degree of engagement and cognitive processing than items at DOK 1. Items at DOK 3 feature higher-order cognitive tasks that assess students' capacities to approach abstract or complex problems.

Percentage of Points by Depth of Knowledge (DOK) Level			
Grade 6	DOK Level 1	DOK Level 2	DOK Level 3
	10% - 20%	60% - 70%	12% - 30%

For more information on DOK go to www.azed.gov/AzM2.

Calculators

Arizona Desmos Graphing Calculator is not permitted for the paper-based and computer-based assessment for Grade 6 Math.

Item Formats

The AzM2 Assessments are composed of item formats that include traditional multiple-choice response items and technology-enhanced response items (TEI). TEIs are computer-delivered response items that require students to interact with test content to select, construct, and/or support their responses. TEIs are better able to assess a deeper level of understanding.

Currently, there are nine types of TEIs that may appear on the Math computer based assessment for AzM2:

- Editing Tasks (ET)
- Editing Task Choice (ETC)
- Equation Editor (EQ)
- Graphic Response Item Display (GRID)
- Hot Text (HT)
  - Selectable Hot Text
  - Drag-and-Drop Hot Text
- Matching Item (MI)
- Multi-Select (MS)
- Open Response
- Table Item (TI)

For paper-based assessments (including those for students with an IEP or 504 plan that specifies a paper-based accommodation), TEIs will be modified so that they can be scanned and scored electronically or hand-scored.

See the table below for a description of each TEI. In addition, for examples of each response item format described, see the AzM2 Training Tests at www.AzM2portal.org.

Item Format	Description
Editing Task (ET)	The student clicks on a highlighted word or phrase that may be incorrect, which reveals a text box. The directions in the text box direct the student to replace the highlighted word or phrase with the correct word or phrase. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Editing Task Choice (ETC)	The student clicks a highlighted word or phrase, which reveals a drop-down menu containing options for correcting an error as well as the highlighted word or phrase as it is shown in the sentence to indicate that no correction is needed. The student then selects the correct word or phrase from the drop-down menu. For paper-based assessments, the item is modified so that it can be scanned and scored electronically. The student fills in a circle to indicate the correct word or phrase.

Item Format	Description
Equation Editor (EQ)	The student is presented with a toolbar that includes a variety of mathematical symbols that can be used to create a response. Responses may be in the form of a number, variable, expression, or equation, as appropriate to the test item. For paper-based assessments, this item type may be replaced with a modified version of the item that can be scanned and scored electronically or replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Graphic Response Item Display (GRID)	The student selects numbers, words, phrases, or images and uses the drag-and-drop feature to place them into a graphic. This item type may also require the student to use the point, line, or arrow tools to create a response on a graph. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Hot Text (HT)	**Selectable Hot Text -** Excerpted sentences from the text are presented in this item type. When the student hovers over certain words, phrases, or sentences, the options highlight. This indicates that the text is selectable ("hot"). The student can then click on an option to select it. For paper- based assessments, a "selectable" hot text item is modified so that it can be scanned and scored electronically. In this version, the student fills in a circle to indicate a selection.
	Drag-and-Drop Hot Text - Certain numbers, words, phrases, or sentences may be designated "draggable" in this item type. When the student hovers over these areas, the text highlights. The student can then click on the option, hold down the mouse button, and drag it to a graphic or other format. For paper-based assessments, drag- and-drop hot text items will be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Matching Item (MI)	The student checks a box to indicate if information from a column header matches information from a row. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Multi-Select (MS)	The student is directed to select all of the correct answers from among a number of options. These items are different from multiple-choice items, which allow the student to select only one correct answer. These items appear in the online and paper-based assessments.
Open Response	The student uses the keyboard to enter a response into a text field. These items can usually be answered in a sentence or two. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.

Item Format	Description
Table Item (TI)	The student types numeric values into a given table. The student may complete the entire table or portions of the table depending on what is being asked. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.

Arizona Math Standards Grade 6

Ratio and Proportion (RP)		
6.RP.A Understand ratio concepts and use ratio reasoning to solve problems.	**6.RP.A.1**	Understand the concept of a ratio as comparing two quantities multiplicatively or joining/composing the two quantities in a way that preserves a multiplicative relationship. Use ratio language to describe a ratio relationship between two quantities. *For example, "There were 2/3 as many men as women at the concert."*
	6.RP.A.2	Understand the concept of a unit rate *a/b* associated with a ratio *a* : *b* with *b* ≠ 0, and use rate language (e.g., for every, for each, for each 1, per) in the context of a ratio relationship. (Complex fraction notation is not an expectation for unit rates in this grade level.)
	6.RP.A.3	Use ratio and rate reasoning to solve mathematical problems and problems in real-world context (e.g., by reasoning about data collected from measurements, tables of equivalent ratios, tape diagrams, double number line diagrams, or equations). a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity). Solve percent problems with the unknown in all positions of the equation. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
The Number System (NS)		
6.NS.A Apply and extend previous understanding of multiplication and division to divide fractions by fractions.	**6.NS.A.1**	Interpret and compute quotients of fractions to solve mathematical problems and problems in real-world context involving division of fractions by fractions using visual fraction models and equations to represent the problem. *For example, create a story context for 2/3 ÷ 3/4 and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that 2/3 ÷ 3/4 = 8/9 because 3/4 of 8/9 is 2/3. In general, a/b ÷ c/d = ad/bc.*
6.NS.B Compute fluently with multi-digit numbers and find common factors and multiples.	**6.NS.B.2**	Fluently divide multi-digit numbers using a standard algorithm.
	6.NS.B.3	Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation.
	6.NS.B.4	Use previous understanding of factors to find the greatest common factor and the least common multiple. a. Find the greatest common factor of two whole numbers less than or equal to 100. b. Find the least common multiple of two whole numbers less than or equal to 12. c. Use the distributive property to express a sum of two whole numbers 1 to 100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4(9+2).*

6.NS.C **Apply and extend previous understanding of numbers to the system of rational numbers.** *Note: Limit negative rational numbers to integers and fractions with denominators of 2, 3, 4, 5, 10.*	**6.NS.C.5**	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values. Use positive and negative numbers to represent quantities in real-world context, explaining the meaning of 0 in each situation.
	6.NS.C.6	Understand a rational number can be represented as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
	6.NS.C.7	Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line. b. Write, interpret, and explain statements of order for rational numbers in real-world context. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in real-world context. d. Distinguish comparisons of absolute value from statements about order in mathematical problems and problems in real-world context.
	6.NS.C.8	Solve mathematical problems and problems in real-world context by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Geometry (G)		
6.G.A **Solve mathematical problems and problems in real-world context involving area, surface area, and volume.**	**6.G.A.1**	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques to solve mathematical problems and problems in real-world context.
	6.G.A.2	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Understand and use the formula $V = B \cdot h$, where in this case, B is the area of the base ($B = l \times w$) to find volumes of right rectangular prisms with fractional edge lengths in mathematical problems and problems in real-world context.
	6.G.A.3	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques to solve mathematical problems and problems in a real-world context.
	6.G.A.4	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques to solve mathematical problems and problems in real-world context.

Statistics and Probability (SP)		
6.SP.A **Develop understanding of statistical variability.**	**6.SP.A.1**	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for variability in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*
	6.SP.A.2	Understand that a set of data collected to answer a statistical question has a distribution whose general characteristics can be described by its center, spread, and overall shape.
	6.SP.A.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation uses a single number to describe the spread of the data set.
6.SP.B **Summarize and describe distributions.**	**6.SP.B.4**	Display and interpret numerical data by creating plots on a number line including histograms, dot plots, and box plots.
	6.SP.B.5	Summarize numerical data sets in relation to their context by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Grade 6 Item Specifications

Expressions and Equations

6.EE.A.1

Content Standards	Write and evaluate numerical expressions involving whole-number exponents.
Explanations	Apply and extend previous understanding of arithmetic to algebraic expression.
Content Limits	Positive rational number bases Whole number exponents Expressions can contain operations that are not exponentiation, but should contain at least one exponentiation
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to evaluate numeric expressions involving whole number exponents.	• Equation Response • Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to create expressions using whole number exponents	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Write and evaluate numerical expressions involving a single number with a whole-number exponent.	Write and evaluate numerical expressions involving a single term and whole-number exponents.
Proficient	**Highly Proficient**
Write and evaluate numerical expressions involving whole-number exponents.	Write and evaluate numerical expressions involving multiple terms and whole-number exponents.

6.EE.A.2, 6.EE.A.2a, 6.EE.A.2b, 6.EE.A.2c

Content Standards	**6.EE.A.2** Write, read, and evaluate algebraic expressions. **6.EE.A.2a** Write expressions that record operations with numbers and variables. **6.EE.A.2b** Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, and coefficient); view one or more parts of an expression as a single entity. **6.EE.A.2c** Evaluate expressions given specific values of their variables. Include expressions that arise from formulas used to solve mathematical problems and problems in real-world context. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).
Explanations	It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number. Students should identify the parts of an algebraic expression including variables, coefficients, constants, and the names of operations (sum, difference, product, and quotient). Development of this common language helps students to understand the structure of expressions and explain their process for simplifying expressions. Terms are the parts of a sum. When the term is an explicit number, it is called a constant. When the term is a product of a number and a variable, the number is called the coefficient of the variable. Variables are letters that represent numbers. There are various possibilities for the numbers they can represent; students can substitute these possible numbers for the letters in the expression for various different purposes.
Content Limits	Rational numbers For items asking the student to evaluate, the student should be given the expression, or, in rare cases, be asked to create an expression from a context and then evaluate. The student should not be required to know real-world formulas for this standard. For standard 2b, in addition to the mathematical terms listed, “difference” may also be used
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify parts of an expression using mathematical terms.	• Equation Response • Matching Item Response • Multi-Select Response
Students will be required to evaluate given expressions, including real-world formulas, with variables by substituting numeric values.	
Students will be required to create, and also possibly evaluate, expressions with variables by analyzing the context.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Write, read, and evaluate algebraic expressions. a. Write expressions that record a single operation with numbers and variables. b. Match part of an expression to its mathematical term (sum, term, and product); view one part of an expression as a single entity. c. Identify the value of an expression with one variable given the specific value of the variable. Include expressions that arise from formulas used to solve mathematical problems and problems in real-world context. Perform arithmetic operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).	Write, read, and evaluate algebraic expressions. a. Write expressions that record two operations with numbers and variables. b. Identify parts of an expression using mathematical terms (sum, term, and product); view one or more parts of an expression as a single entity. c. Identify the value of an expression with two variables given specific values of their variables. Include expressions that arise from formulas used to solve mathematical problems and problems in real-world context. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).
Proficient	**Highly Proficient**
Write, read, and evaluate algebraic expressions. a. Write expressions that record operations with numbers and variables. b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, and coefficient); view one or more parts of an expression as a single entity. c. Evaluate expressions given specific values of their variables. Include expressions that arise from formulas used to solve mathematical problems and problems in real-world context. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).	Write, read, and evaluate algebraic expressions. a. Write expressions that record operations, including exponents, with numbers and variables. b. Create expressions given mathematical terms (sum, term, product, factor, quotient, and coefficient); explain how one part of an expression relates to other parts of the expression. c. Evaluate expressions with multiple variables and multiple operations given specific values of their variables. Include expressions that arise from formulas used to solve mathematical problems and problems in real-world context. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

6.EE.A.3

Content Standards	Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x.*
Explanations	Apply and extend previous understanding of arithmetic to algebraic expression.
Content Limits	Positive rational numbers, values may include exponents Variables must be included in the expression Collecting like terms limited to coefficients of 1
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to given an expression, construct an equivalent expression.	• Equation Response • Multiple Choice Response • Multi-Select Response

Performance Level Descriptions

Minimally Proficient	**Partially Proficient**
Apply the Associative and Commutative properties of operations to generate equivalent expressions involving whole-numbers.	Apply the properties of operations to generate equivalent expressions involving whole-numbers.
Proficient	**Highly Proficient**
Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x.	Apply the properties of operations to generate equivalent expressions involving rational numbers and whole-number exponents in real-world contexts.

6.EE.A.4

Content Standards	Identify when two expressions are equivalent. *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*
Explanations	Students connect their experiences with finding and identifying equivalent forms of whole numbers and can write expressions in various forms. Students generate equivalent expressions using the associative, commutative, and distributive properties. They can prove that the expressions are equivalent by simplifying each expression into the same form.
Content Limits	Positive rational numbers Variables must be included in the expression To distinguish from 6.EE.3, equivalent expressions do not necessarily need to be direct applications of the associative, commutative, and distributive properties - the focus should be on the student recognizing that equivalent expressions have the same va Collecting like terms limited to coefficients of 1
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify which expressions are equivalent.	• Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to determine that two expressions are equivalent by substitution.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify when two expressions are equivalent in cases of repeated addition.	Identify when two expressions are equivalent in cases where the resulting expression only has one term.
Proficient	**Highly Proficient**
Identify when two expressions are equivalent. For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.	Create equivalent expressions.

Content Standards	Identify when two expressions are equivalent. *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

6.EE.B.5

Content Standards	Understand solving an equation or inequality as a process of reasoning to find the value(s) of the variables that make that equation or inequality true. Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
Explanations	Beginning experiences in solving equations should require students to understand the meaning of the equation as well as the question being asked. Solving equations using reasoning and prior knowledge should be required of students to allow them to develop effective strategies such as using reasoning, fact families, and inverse operations. Students may use balance models in representing and solving equations and inequalities.
Content Limits	Nonnegative rational numbers One-variable linear equations and inequalities An equation or inequality should be given if a context is included
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to choose which value(s) satisfy an equation or inequality.	• Equation Response • Multiple Choice • Matching Item Response • Multi-Select Response
Students will be required to choose a set of numbers which contains only solutions to an inequality.	
Students will be required to determine the value of an expression that makes the equation true.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Understand solving an equation or inequality as a process of reasoning to find the value(s) of the variables that make that equation or inequality true. Use substitution to identify a whole number in a specified set that makes an equation or inequality true.	Understand solving an equation or inequality as a process of reasoning to find the value(s) of the variables that make that equation or inequality true. Use substitution to identify a number in a specified set that makes an equation or inequality true.
Proficient	**Highly Proficient**

Understand solving an equation or inequality as a process of reasoning to find the value(s) of the variables that make that equation or inequality true. Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	Explain how solving an equation or inequality is the process of reasoning to find the value(s) of the variables that make that equation or inequality true.

6.EE.B.6

Content Standards	Use variables to represent numbers and write expressions when solving mathematical problems and problems in real-world context; understand that a variable can represent an unknown number or any number in a specified set.
Explanations	Connecting writing expressions with story problems and/or drawing pictures will give students a context for this work. It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number.
Content Limits	Nonnegative rational numbers Expression must contain variables
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to construct an expression that represents a situation.	• Equation Response • Multiple Choice Response • Proposition Response
Students will be required to explain or choose what a variable in an expression represents in a real world context.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify what the variables represent when solving mathematical problems and problems in real-world context; understand that a variable can represent an unknown number.	Identify what the expressions represent when solving mathematical problems and problems in real-world context; understand that a variable can represent an unknown number or any number in a specified set.
Proficient	**Highly Proficient**
Use variables to represent numbers and write expressions when solving mathematical problems and problems in real-world context; understand that a variable can represent an unknown number or any number in a specified set.	Solve problems by writing an expression with a variable that represents several possible rational numbers within a mathematical or real-world context; understand that a variable can represent an unknown number or any number in a specified set.

6.EE.B.7

Content Standards	Solve mathematical problems and problems in real-world context by writing and solving equations of the form $x + p = q$, $x - p = q$, $px = q$, and $x/p = q$ for cases in which p, q and x are all non-negative rational numbers.
Explanations	Students create and solve equations that are based on real world situations. It may be beneficial for students to draw pictures that illustrate the equation in problem situations. Solving equations using reasoning and prior knowledge should be required of students to allow them to develop effective strategies.
Content Limits	Nonnegative rational numbers One-step linear equations of one variable
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to solve one step linear equations for purely mathematical problems.	• Equation Response
Students will be required to given a simple context, write and/or solve one step linear equations.	
Students will be required to write and/or solve one step linear equations where the given information can be simplified to a form given in the standard.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Solve mathematical equations of the form $x + p = q$, $x - p = q$, and $px = q$, for cases in which p, q and x are all non-negative whole numbers.	Solve mathematical problems and problems in real-world context by solving equations of the form $x + p = q$, $x - p = q$, $px = q$, and $x/p = q$ for cases which p, q and x are all non-negative whole numbers.
Proficient	**Highly Proficient**
Solve mathematical problems and problems in real-world context by writing and solving equations of the form $x + p = q$, $x - p = q$, $px = q$, and $x/p = q$ for cases in which p, q and x are all non-negative rational numbers.	Create mathematical problems and problems in real-world context that can be solved using equations of the form $x + p = q$, $x - p = q$, $px = q$, and $x/p = q$ for cases in which p, q and x are all non-negative rational numbers.

6.EE.B.8

Content Standards	Write an inequality of the form $x > c$, $x < c$, $x \geq c$, or $x \leq c$ to represent a constraint or condition to solve mathematical problems and problems in real-world context. Recognize that inequalities have infinitely many solutions; represent solutions of such inequalities on number lines.
Explanations	None
Content Limits	Nonnegative rational numbers Both strict and non-strict inequalities are acceptable When creating rubrics for items with real-world contexts, be wary that some students may create compound inequalities if a natural bound exists (e.g., when describing the weight of something, a student may create the inequality $x < 50$, or $0 < x < 50$)
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to write an inequality that represents a constraint or condition in a mathematical problem.	• Equation Response • Graphic Response • Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to relate a graph to an inequality or a description.	
Students will be required to represent a constraint or condition in a real-world or mathematical problem on a number line.	
Students will be required to write an inequality that represents a constraint or condition in a real-world problem.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Recognize that inequalities of the form $x > c$, $x < c$, $x \geq c$, or $x \leq c$ have infinitely many solutions; identify solutions of such inequalities on number lines.	Recognize that inequalities of the form $x > c$, $x < c$, $x \geq c$, or $x \leq c$ have infinitely many solutions; identify solutions of compound inequalities on number lines.
Proficient	**Highly Proficient**
Write an inequality of the form $x > c$, $x < c$, $x \geq c$, or $x \leq c$ to represent a constraint or condition to solve mathematical problems and problems in real-world context. Recognize that inequalities have infinitely many solutions; represent solutions of such inequalities on number lines.	Given an inequality of the form $x > c$, $x < c$, $x \geq c$, or $x \leq c$ create mathematical problems and problems in real-world context that could be represented by the inequality.

6.EE.C.9

Content Standards	Use variables to represent two quantities that change in relationship to one another to solve mathematical problems and problems in real-world context. Write an equation to express one quantity (the dependent variable) in terms of the other quantity (the independent variable). Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.
Explanations	Students can use many forms to represent relationships between quantities. Multiple representations include describing the relationship using language, a table, an equation, or a graph. Translating between multiple representations helps students understand that each form represents the same relationship and provides a different perspective on the function.
Content Limits	Equation of the form y=px or y=x+p Positive rational numbers (zero can be used in graph and table)
Context	Context is required.

Sample Task Demands	**Common Item Formats**
Students will be required to identify or model the relationship between an independent and a dependent variable by constructing or referring to a graph or a table, or by reviewing an equation.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response • Table Response
Students will be required to construct an equation that represents the relationship between the independent and dependent variables in a context or from a graph or table	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Given a graph or table representing two quantities that change in relationship to one another, identify an equation that expresses one quantity in terms of the other quantity.	Given a graph or table representing two quantities that change in relationship to one another, identify the dependent and independent variables, and write an equation that expresses one quantity in terms of the other quantity.
Proficient	**Highly Proficient**
Use variables to represent two quantities that change in relationship to one another to solve mathematical problems and problems in real-world context. Write an equation to express one quantity (the dependent variable) in terms of the other quantity (the independent variable). Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.	Given an equation where variables represent two quantities that change in relationship to one another, create a problem in real-world context that could be represented by the equation. Explain the relationship between the dependent and independent variables and relate these to the equation.

Geometry & Statistics and Probability

6.G.A.1

Content Standards	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques to solve mathematical problems and problems in real-world context.
Explanations	Special quadrilaterals include rectangles, squares, parallelograms, trapezoids, rhombi, and kites.
Content Limits	Positive rational numbers For drawing items, do not use scales - this concept is not mastered until 7th grade. Limit shapes to those that can be decomposed or composed into rectangles and/or right triangles.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to find the area of a shape (by composing/decomposing).	• Equation Response • Graphic Response
Students will be required to create an expression with an unknown to model the area of a shape as a composition/decomposition of rectangles and/or right triangles.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Find the area of right triangles and polygons decomposed into right triangles and rectangles, given all the measurements.	Find the area of triangles and polygons decomposed into right triangles and rectangles, given some of the measurements.
Proficient	**Highly Proficient**
Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques to solve mathematical problems and problems in real-world context.	Find the area of triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques to solve mathematical problems and problems in real-world context, including decimal and fractional measurements.

6.G.A.2

Content Standards	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Understand and use the formula $V = B \cdot h$, where in this case, B is the area of the base ($B = l \times w$) to find volumes of right rectangular prisms with fractional edge lengths in mathematical problems and problems in real-world context.
Explanations	Students need multiple opportunities to measure volume by filling rectangular prisms with blocks and looking at the relationship between the total volume and the area of the base. Through these experiences, students derive the volume formula (volume equals the area of the base times the height). In addition to filling boxes, students can draw diagrams to represent fractional side lengths, connecting with multiplication of fractions. This process is similar to composing and decomposing two dimensional shapes.
Content Limits	Right rectangular prisms Unit fractional edge lengths (numerator is 1).
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to find the volume of a right rectangular prism given its fractional dimensions.	• Equation Response • Graphic Response • Table Response
Students will be required to find the volume of a right rectangular prism when the number of unit cubes packed in it and their unit fraction edge length is given.	
Students will be required to find the edge lengths (and volume) of a rectangular prism given the number of unit cubes packed in the prism and their fractional edge length.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Use the formula $V = B \cdot h$, where in this case, B is the area of the base ($B = l \times w$) to find volumes of right rectangular prisms with whole number edge lengths in mathematical problems and problems in real-world context.	Use the formula $V = B \cdot h$, where in this case, B is the area of the base ($B = l \times w$) to find volumes of right rectangular prisms with one fractional edge length in mathematical problems and problems in real-world context.
Proficient	**Highly Proficient**
Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Understand and use the formula $V = B \cdot h$, where in this case, B is the area of the base ($B = l \times w$) to find volumes of right rectangular prisms with fractional edge lengths in mathematical problems and problems in real-world context.	Explain that the volume of a right rectangular prism with fractional edge lengths found by multiplying the edge lengths of the prism. Understand the formula $V = B \cdot h$, where in this case, B is the area of the base ($B = l \times w$). Given the volume, use the formula to find edge lengths of right rectangular prisms with fractional edge lengths in mathematical problems and problems in real-world context.

6.G.A.3

Content Standards	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques to solve mathematical problems and problems in a real-world context.
Explanations	Solve real-world and mathematical problems involving area, surface area, and volume.
Content Limits	Can use rational numbers Can use all four quadrants When finding side length, limit polygons to traditional orientation (side lengths perpendicular to axes).
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to draw a polygon on the coordinate plane given the coordinates that represent each of its vertices.	• Equation Response • Graphic Response • Multiple Choice Response
Students will be required to find the side length or perimeter of a polygon whose sides are parallel to the axes and its vertices are given as ordered pairs.	
Students will be required to identify the visual shape that is created if a set of given points would be plotted on a coordinate plane.	
Students will be required to compose a polygon when given some of its vertices and the name of the polygon.	
Students will be required to compose a polygon when given some of its vertices and other constraints which require strategic planning (such as perimeter, side lengths, area).	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Draw polygons in the coordinate plane given coordinates for the vertices.	Use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate.
Proficient	**Highly Proficient**
Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques to solve mathematical problems and problems in a real-world context.	Use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques to solve mathematical problems and problems in a real-world context. Finds a missing vertex of a polygon given other vertices.

6.G.A.4

<table>
<tr><td>Content Standards</td><td colspan="2">Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques to solve mathematical problems and problems in real-world context.</td></tr>
<tr><td>Explanations</td><td colspan="2">Students construct models and nets of three dimensional figures, describing them by the number of edges, vertices, and faces. Solids include rectangular and triangular prisms. Students are expected to use the net to calculate the surface area.

Students also describe the types of faces needed to create a three-dimensional figure. Students make and test conjectures by determining what is needed to create a specific three-dimensional figure.</td></tr>
<tr><td>Content Limits</td><td colspan="2">Positive rational numbers

3-dimensional figures are limited to rectangular prisms, triangular prisms, rectangular pyramids, and triangular pyramids.</td></tr>
<tr><td>Context</td><td colspan="2">Context is allowed.</td></tr>
<tr><td colspan="2">Sample Task Demands</td><td>Common Item Formats</td></tr>
<tr><td colspan="2">Students will be required to match net(s) to 3-D figure(s).</td><td rowspan="6">Equation Response
Graphic Response
Multiple Choice Response
Matching Item Response</td></tr>
<tr><td colspan="2">Students will be required to identify the set of shapes that can be arranged to form a net of a given 3-D figure.</td></tr>
<tr><td colspan="2">Students will be required to find the surface area of a 3-D figure given its net.</td></tr>
<tr><td colspan="2">Students will be required to draw a net of a given 3-D figure.</td></tr>
<tr><td colspan="2">Students will be required to create an expression with one unknown to model the surface area of a solid.</td></tr>
<tr><td colspan="2">Students will be required to given the surface area, net, and all but one dimension of a 3-D figure, determine the unknown dimension.</td></tr>
</table>

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Represent three-dimensional figures using nets made up of rectangles and triangles.	Use the nets representing three-dimensional figures to find the surface area of these figures.
Proficient	**Highly Proficient**
Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques to solve mathematical problems and problems in real-world context.	Represent three-dimensional figures with fractional edges using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques to solve mathematical problems and problems in real-world context.

6.SP.A.1

Content Standards	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for variability in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*
Explanations	Statistics are numerical data relating to an aggregate of individuals; statistics is also the name for the science of collecting, analyzing and interpreting such data. A statistical question anticipates an answer that varies from one individual to the next and is written to account for the variability in the data. Data are the numbers produced in response to a statistical question. Data are frequently collected from surveys or other sources (e.g., documents).
Content Limits	Data and contexts should be familiar to students at this grade.
Context	Context is required.

Sample Task Demands	Common Item Formats
Students will be required to recognize whether questions are statistical in nature.	• Multiple Choice Response • Multi-Select Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify a statistical question.	Change a non-statistical question into a statistical question.
Proficient	**Highly Proficient**
Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for variability in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*	Create a statistical question given a context.

6.SP.A.2

Content Standards	Understand that a set of data collected to answer a statistical question has a distribution whose general characteristics can be described by its center, spread, and overall shape.
Explanations	Develop understanding of statistical variability.
Content Limits	Rational numbers Dot plot, histogram, box plot Mode should not be referred to in any item
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify features, such as symmetry, clusters, peaks, and gaps, or common shapes and patterns of a set of data or data display.	• Graphic Response • Multiple Choice Response • Multi-Select Response
Students will be required to interpret features, such as symmetry, clusters, peaks, and gaps, or common shapes and patterns of a set of data or data display.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify a set of data by its center, spread, and overall shape.	Describe a set of data by its center, spread, and overall shape.
Proficient	**Highly Proficient**
Understand that a set of data collected to answer a statistical question has a distribution whose general characteristics can be described by its center, spread, and overall shape.	Create a set of data with a distribution whose general characteristics can be described by a given center, spread, and overall shape.

6.SP.A.3

Content Standards	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation uses a single number to describe the spread of the data set.
Explanations	When using measures of center (mean, median, and mode) and range, students are describing a data set in a single number. The range provides a single number that describes how the values vary across the data set. The range can also be expressed by stating the minimum and maximum values.
Content Limits	Rational numbers, only numerical data sets Mode should not be referred to in any item
Context	Context is subject to task demand.

Sample Task Demands	Common Item Formats
Students will be required to distinguish between situations where data is summarized by its measure of center (mean or median) or its variation (range, interquartile range, mean absolute deviation.) Context is allowed.	• Equation Response • Multiple Choice Response • Multi-Select Response
Students will be required to apply understanding of the qualitative properties of measures of center and/or variation. Context is required.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Recognize mean, median, and mode as measures of center and range as a measure of variation.	Calculate mean, median, and mode as measures of center and range as a measure of variation.
Proficient	**Highly Proficient**
Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation uses a single number to describe the spread of the data set.	Recognize how a measure of center or a measure of variation would be impacted by outliers in a numerical data set.

6.SP.B.4

Content Standards	Display and interpret numerical data by creating plots on a number line including histograms, dot plots, and box plots.
Explanations	In order to display numerical data in dot plots, histograms or box plots, students need to make decisions and perform calculations. Students are expected to display data graphically in a format appropriate for that data set as well as reading data from graphs generated by others students or contained in reference materials. Dot plots are simple plots on a number line where each dot represents a piece of data in the data set. Dot plots are suitable for small to moderate size data sets and are useful for highlighting the distribution of the data including clusters, gaps, and outliers. In most real data sets, there is a large amount of data and many numbers will be unique. A graph (such as a dot plot) that shows how many ones, how many twos, etc. would not be meaningful; however, a histogram can be used. Students organize the data into convenient ranges and use these intervals to generate a frequency table and histogram. Note that changing the size of the range changes the appearance of the graph and the conclusions you may draw from it. Box plots are another useful way to display data and are plotted horizontally or vertically on a number line. Box plots are generated from the five number summaries of a data set consisting of the minimum, maximum, median, and two quartile values. Students can readily compare two sets of data if they are displayed with side by side box plots on the same scale. Box plots display the degree of spread of the data and the skewness of the data.
Content Limits	Rational numbers Focus should be on dot plots, box plots, and histograms, but other data displays, such as bar graphs, can be used as distractors
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify the graph(s) for a given set of data.	• Graphic Response • Multiple Choice Response • Multi-Select Response
Students will be required to determine the best and/or appropriate type(s) of graph(s) to display data sets.	
Students will be required to create number lines, dot plots, histograms, and/or box plots to display given data.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify an appropriate display for numerical data including histograms, dot plots, and box plots.	Construct an appropriate display for numerical data including histograms, dot plots, and box plots.
Proficient	**Highly Proficient**
Display and interpret numerical data by creating plots on a number line including histograms, dot plots, and box plots.	Display and interpret numerical data by creating plots on a number line including histograms, dot plots, and box plots, and explaining what the display indicates about the data.

6.SP.B.5, 6.SP.B.5a, 6.SP.B.5b, 6.SP.B.5c, 6.SP.B.5d

Content Standard	**6.SP.B.5** Summarize numerical data sets in relation to their context by: **6.SP.B.5a** Reporting the number of observations. **6.SP.B.5b** Describing the nature of the attribute under investigation including how it was measured and its units of measurement. **6.SP.B.5c** Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. **6.SP.B.5d** Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
Explanations	Students summarize numerical data by providing background information about the attribute being measured, methods and unit of measurement, the context of data collection activities, the number of observations, and summary statistics. Summary statistics include quantitative measures of center, spread, and variability including extreme values (minimum and maximum), mean, median, mode, range, quartiles, interquartile ranges, and mean absolute deviation. The measure of center that a student chooses to describe a data set will depend upon the shape of the data distribution and context of data collection. The mode is the value in the data set that occurs most frequently. The mode is the least frequently used as a measure of center because data sets may not have a mode, may have more than one mode, or the mode may not be descriptive of the data set. The mean is a very common measure of center computed by adding all the numbers in the set and dividing by the number of values. The mean can be affected greatly by a few data points that are very low or very high. In this case, the median or middle value of the data set might be more descriptive. In data sets that are symmetrically distributed, the mean and median will be very close to the same. In data sets that are skewed, the mean and median will be different, with the median frequently providing a better overall description of the data set. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students develop understanding of what the mean represents by redistributing data sets to be level or fair. The leveling process can be connected to and used to develop understanding of the computation of the mean. The use of mean absolute deviation in 6th grade is mainly exploratory. The intent is to build a deeper understanding of variability. Students would understand the mean distance between the pieces of data and the mean of the data set expresses the spread of the data set. Students can see that the

	larger the mean distance, the greater the variability. Comparisons can be made between different data sets. Students can also summarize and describe the center and variability in data sets using the median and a five number summary consisting of the minimum, quartiles, and maximum as seen in the box plot example in 6.SP.4. The median is the middle number of the data set with half the number below the median and half the numbers above the median. The quartiles partition the data set into four parts by dividing each of the halves of the data set into half again. Quartile 1 (Q1 or the lower quartile) is the middle value of the lower half of the data set and quartile 3 (Q3 or the upper quartile) is the middle value of the upper half of the data set. The median can also be referred to as quartile 2 (Q2). The range of the data is the difference between the minimum and maximum values. The interquartile range of the data is the difference between the lower and upper quartiles (Q3 – Q1). The interquartile range is a measure of the dispersion or spread of the data set: a small value indicates values that are clustered near the median whereas a larger value indicates values that are more distributed.
Content Limits	Histograms, dot plots, or box plots may be used when appropriate Rational numbers Mode should not be referred to in any item

<table>
<tr><th>Sample Task Demands</th><th>Common Item Formats</th></tr>
<tr><td>Students will be required to identify measures of center and variability from a given graph.</td><td rowspan="7">Equation Response
Graphic Response
Multiple Choice Response
Multi-Select Response
Table Response</td></tr>
<tr><td>Students will be required to identify the center and/or spread of data set.</td></tr>
<tr><td>Students will be required to identify the number of observations from a data set.</td></tr>
<tr><td>Students will be required to solve problems involving measures of center and spread.</td></tr>
<tr><td>Students will be required to describe overall pattern of a data set including clusters, peaks, and gaps in distributions, within a context.</td></tr>
<tr><td>Students will be required to create or complete data sets given certain attributes and information about spread of data and/or measure of center.</td></tr>
<tr><td>Students will be required to draw conclusions about a data set and select the most appropriate measure to answer a question.</td></tr>
</table>

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Summarize numerical data sets in relation to their context by: a. Reporting the number of observations in a dot plot. b. For the attribute under investigation, identify its units of measurement. c. Distinguish between measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation). d. Identify mean and mean absolute deviation as the best choice of measures of center and variability for a symmetric data distribution.	Summarize numerical data sets in relation to their context by: a. Reporting the number of observations in a histogram. b. For the attribute under investigation, identify how it was measured. c. Calculate measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation. d. Identify median and interquartile range as the best choice of measures of center and variability for a skewed data distribution.
Proficient	**Highly Proficient**
Summarize numerical data sets in relation to their context by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	Summarize numerical data sets in relation to their context by: a. Reporting the number of observations given calculations for a measure of center or variability. b. Describing the nature of the attribute under investigation including explaining why it was measured a particular way and why certain units of measurement were used. c. Comparing data sets using measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Choose the appropriate measure of center and variability for data set and explains the reasoning for the choice.

The Number System

6.NS.A.1

Content Standards	Interpret and compute quotients of fractions to solve mathematical problems and problems in real-world context involving division of fractions by fractions using visual fraction models and equations to represent the problem. *For example, create a story context for 2/3 ÷ 3/4 and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that 2/3 ÷ 3/4 = 8/9 because 3/4 of 8/9 is 2/3. In general, a/b ÷ c/d = ad/bc.*
Explanations	Contexts and visual models can help students to understand quotients of fractions and begin to develop the relationship between multiplication and division. Model development can be facilitated by building from familiar scenarios with whole or friendly number dividends or divisors. Computing quotients of fractions build upon and extends student understandings developed in Grade 5. Students make drawings, model situations with manipulatives, or manipulate computer generated models.
Content Limits	Dividing a unit fraction by a whole number or vice versa (e.g., $[1/a] \div q$ or $q \div [1/a]$) is below grade level.
Context	Context is not allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to calculate the quotient of two fractions or a non-unit fraction and whole number.	• Equation Response • Multiple Choice Response
Students will be required to use context cues from a story to represent or calculate the quotient of two fractions or a non-unit fraction and whole number.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Compute quotients of fractions to solve mathematical problems using visual fraction models to represent the problem.	Compute quotients of fractions to solve mathematical problems using visual fraction models and equations to represent the problem.
Proficient	**Highly Proficient**
Interpret and compute quotients of fractions to solve mathematical problems and problems in real-world context involving division of fractions by fractions using visual fraction models and equations to represent the problem. *For example, create a story context for 2/3 ÷ 3/4 and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that 2/3 ÷ 3/4 = 8/9 because 3/4 of 8/9 is 2/3. In general, a/b ÷ c/d = ad/bc.*	Compute quotients of fractions to solve mathematical problems and problems in real-world context involving mixed numbers using visual fraction models and equations to represent the problem. Interpret the solution in the context of the problem.

6.NS.B.2

Content Standards	Fluently divide multi-digit numbers using a standard algorithm.
Explanations	Students are expected to fluently and accurately divide multi-digit whole numbers. Divisors can be any number of digits at this grade level. As students divide they should continue to use their understanding of place value to describe what they are doing. When using the standard algorithm, students' language should reference place value.
Content Limits	5-digit dividend by 2-digit divisor and 4-digit dividend by 2- or 3-digit divisor
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to calculate the quotient of 2 numbers.	• Equation Response • Multiple Choice Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Fluently divide three-digit numbers by two-digit numbers using a standard algorithm.	Fluently divide four-digit numbers by two-digit numbers using a standard algorithm.
Proficient	**Highly Proficient**
Fluently divide multi-digit numbers using a standard algorithm.	Fluently divide multi-digit numbers to solve real-world problems, not including multi-digit decimals, using a standard algorithm and assess the reasonableness of the result.

6.NS.B.3

Content Standards	Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation.
Explanations	The use of estimation strategies supports student understanding of operating on decimals. Students use the understanding they developed in Grade 5 related to the patterns involved when multiplying and dividing by powers of ten to develop fluency with operations with multi-digit decimals.
Content Limits	Positive rational numbers only Limit to one type of operation per problem
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to perform calculations involving all 4 operations.	• Equation Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Fluently add, subtract, and multiply multi-digit decimals, where decimals are limited to the hundredths, using a standard algorithm for each operation.	Fluently add, subtract, multiply, and divide multi-digit decimals, where the divisor is a whole number, using a standard algorithm for each operation.
Proficient	**Highly Proficient**
Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation.	Fluently add, subtract, multiply, and divide multi-digit decimals to solve real world problems, using a standard algorithm for each operation, and assess the reasonableness of the result.

6.NS.B.4, 6.NS.B.4a, 6.NS.B.4b, 6.NS.B.4c

Content Standards	**6.NS.B.4** Use previous understanding of factors to find the greatest common factor and the least common multiple. **6.NS.B.4a** Find the greatest common factor of two whole numbers less than or equal to 100. **6.NS.B.4b** Find the least common multiple of two whole numbers less than or equal to 12. **6.NS.B.4c** Use the distributive property to express a sum of two whole numbers 1 to 100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4(9 + 2).*
Explanations	Compute fluently with multi-digit numbers and find common factors and multiples.
Content Limits	Whole numbers less than or equal to 100 Least common multiple of two whole numbers less than or equal to 12
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify the greatest common factor (GCF) of two numbers given.	• Equation Response • Multiple Choice Response • Table Response
Students will be required to identify the least common multiple (LCM) of two given numbers.	
Students will be required to recongize equivalent expressions that express the same sum.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Use previous understanding of factors to find the greatest common factor and the least common multiple. a. Select the greatest common factor of two whole numbers less than or equal to 100 using visual models. b. Select the least common multiple of two whole numbers less than or equal to 12 using visual models. c. Identify the distributive property to express a sum of two whole numbers 1 to 100 with a common factor as a multiple of a sum of two whole numbers. For example, express 16 + 8 as 2(8 + 4).using visual models.	Use previous understanding of factors to find the greatest common factor and the least common multiple. a. Identify the greatest common factor of two whole numbers less than or equal to 100. b. Identify the least common multiple of two whole numbers less than or equal to 12. c. Identify the distributive property to express a sum of two whole numbers 1 to 100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 16 + 8 as 8(2 + 1).
Proficient	**Highly Proficient**
Use previous understanding of factors to find the greatest common factor and the least common multiple. a. Find the greatest common factor of two whole numbers less than or equal to 100. b. Find the least common multiple of two whole numbers less than or equal to 12. c. Use the distributive property to express a sum of two whole numbers 1 to 100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4(9 + 2).	Use previous understanding of factors to find the greatest common factor and the least common multiple. a. Find two whole numbers when given their greatest common factor. b. Find two whole numbers when given their least common multiple. c. Use the greatest common factor and the distributive property to express a sum of two whole numbers greater than 100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 336 + 270 as 6(56 + 45).

6.NS.C.5

Content Standards	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values. Use positive and negative numbers to represent quantities in real-world context, explaining the meaning of 0 in each situation.
Explanations	Apply and extend previous understandings of numbers to the system of rational number.
Content Limits	Rational numbers Items should not require the student to perform an operation
Context	Context is required.

Sample Task Demands	Common Item Formats
Students will be required to identify a rational number which represents a given situation.	• Equation Response • Multiple Choice Response • Multi-Select Response • Proposition Response
Students will be required to interpret a rational number in terms of a context.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Understand that positive and negative numbers are used together to describe quantities having opposite directions or values. Identify positive and negative numbers that represent quantities in real-world context, identifying the meaning of 0 in each situation.	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values. Identify real-world context that can be represented with positive and negative numbers, defining the meaning of 0 in each situation.
Proficient	**Highly Proficient**
Understand that positive and negative numbers are used together to describe quantities having opposite directions or values. Use positive and negative numbers to represent quantities in real-world context, explaining the meaning of 0 in each situation.	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values. Use positive and negative numbers to represent quantities in real-world context, explaining the meaning of 0 in each situation. Interpret and represent changes in positive and negative numbers representing quantities in real-world situations in terms of the context.

6.NS.C.6, 6.NS.C.6a, 6.NS.C.6b, 6.NS.C.6c

Content Standards	**6.NS.C.6** Understand a rational number can be represented as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **6.NS.C.6a** Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself and that 0 is its own opposite. **6.NS.C.6b** Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. **6.NS.C.6c** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	
Explanations	Number lines can be used to show numbers and their opposites. Both 3 and -3 are 3 units from zero on the number line. Graphing points and reflecting across zero on a number line extends to graphing and reflecting points across axes on a coordinate grid. The use of both horizontal and vertical number line models facilitates the movement from number lines to coordinate grids.	
Content Limits	Rational numbers Plotting of points in the coordinate plane should include some negative values (not just first quadrant)	
Context	Context is not allowed.	

Sample Task Demands	**Common Item Formats**
Students will be required to locate rational numbers on the number line.	• Equation Response • Graphic Response
Students will be required to plot points on the coordinate plane.	
Students will be required to identify the opposite of a number, including the opposite of a negative number.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Understand a rational number can be represented as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Identify the opposite of a number. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize a negative coordinate indicates left or down while a positive coordinate indicates up or right. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram.	Understand a rational number can be represented as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; indicate the quadrant a point lies in based on the sign of the coordinates. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and on a coordinate plane.
Proficient	**Highly Proficient**
Understand a rational number can be represented as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	Understand a rational number can be represented as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself and that 0 is its own opposite. Indicate whether a number will be to the left or right of 0 on the number line, given the number of negative symbols it has. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; explain why it is that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Create real world problems that are solved by given rational numbers on a number line diagram; create real world problems that are solved by given pairs of integers and other rational numbers on a coordinate plane.

6.NS.C.7, 6.NS.C.7a, 6.NS.C.7b, 6.NS.C.7c, 6.NS.C.7d

Content Standards	**6.NS.C.7** Understand ordering and absolute value of rational numbers. **6.NS.C.7a** Interpret statements of inequality as statements about the relative position of two numbers on a number line. **6.NS.C.7b** Write, interpret, and explain statements of order for rational numbers in real-world context. **6.NS.C.7c** Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in real-world context. **6.NS.C.7d** Distinguish comparisons of absolute value from statements about order in mathematical problems and problems in real-world context.
Explanations	Common models to represent and compare integers include number line models, temperature models and the profit-loss model. On a number line model, the number is represented by an arrow drawn from zero to the location of the number on the number line; the absolute value is the length of this arrow. The number line can also be viewed as a thermometer where each point of on the number line is a specific temperature. In the profit-loss model, a positive number corresponds to profit and the negative number corresponds to a loss. Each of these models is useful for examining values but can also be used in later grades when students begin to perform operations on integers. In working with number line models, students internalize the order of the numbers; larger numbers on the right or top of the number line and smaller numbers to the left or bottom of the number line. They use the order to correctly locate integers and other rational numbers on the number line. By placing two numbers on the same number line, they are able to write inequalities and make statements about the relationships between the numbers. Comparative statements generate informal experience with operations and lay the foundation for formal work with operations on integers in Grade 7. Students recognize the distance from zero as the absolute value or magnitude of a rational number. Students need multiple experiences to understand the relationships between numbers, absolute value, and statements about order.
Content Limits	Positive and negative rational numbers
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to compare integers in terms of relative locations on the number line.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response
Students will be required to compare values of rational numbers in a context.	
Students will be required to order rational numbers.	
Students will be required to compare integers and absolute value of integers in terms of relative locations on the number line.	
Students will be required to distinguish between order and magnitude of rational numbers.	
Students will be required to compare integers and/or absolute values of integers for abstract values represented by variables.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Understand ordering and absolute value of rational numbers. a. Identify a statement of inequality given the position of the two numbers on a number line. b. Identify correct statements of order for rational numbers in real-world context. c. Understand the absolute value of a rational number is always positive. d. Compare the absolute value of two positive numbers in mathematical problems and problems in real-world context.	Understand ordering and absolute value of rational numbers. a. Create a statement of inequality given the position of the two numbers on a number line. b. Write statements of order for rational numbers in real-world context. c. Understand the absolute value of a rational number as its distance from 0 on the number line. d. Compare the absolute value of two numbers in mathematical problems and problems in real-world context.
Proficient	**Highly Proficient**
Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line. b. Write, interpret, and explain statements of order for rational numbers in real-world context. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in real-world context. d. Distinguish comparisons of absolute value from statements about order in mathematical problems and problems in real-world context.	Understand ordering and absolute value of rational numbers. a. Justify the relative position of multiple numbers on a number line given statements of inequality about their relative positions. b. Create scenarios in real-world context that fit statements of order for rational numbers. c. Solve problems involving understanding the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in real-world context. d. Explain comparisons of absolute value from statements about order in mathematical problems and problems in real-world context.

6.NS.C.8

Content Standards	Solve mathematical problems and problems in real-world context by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.
Explanations	Apply and extend previous understandings of numbers to the system of rational number.
Content Limits	Positive and negative whole numbers Do not use polygons/vertices for this standard Do not exceed 10x10 coordinate grid, though scales can vary
Context	Context is required.

<table>
<tr><th>Sample Task Demands</th><th>Common Item Formats</th></tr>
<tr><td>Students will be required to identify the location of a point that is a specified distance from another point.</td><td rowspan="3">Equation Response
Graphic Response</td></tr>
<tr><td>Students will be required to calculate the distance between two points.</td></tr>
<tr><td>Students will be required to solve problems related to location and distance in the coordinate plane.</td></tr>
</table>

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Solve mathematical problems by graphing points in all one quadrant of the coordinate plane. Count spaces between coordinates to find whole number distances between points with the same first coordinate or the same second coordinate.	Solve mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates to find whole number distances between points with the same first coordinate or the same second coordinate.
Proficient	**Highly Proficient**
Solve mathematical problems and problems in real-world context by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	Justify solutions to mathematical problems and problems in real-world context solved by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Ratio and Proportional Relationships

6.RP.A.1

Content Standards	Understand the concept of a ratio as comparing two quantities multiplicatively or joining/composing the two quantities in a way that preserves a multiplicative relationship. Use ratio language to describe a ratio relationship between two quantities. *For example, "There were 2/3 as many men as women at the concert."*
Explanations	A ratio is a comparison of two quantities which can be written as *a* to *b*, *a*/*b*, or *a*:*b*. A rate is a ratio where two measurements are related to each other. When discussing measurement of different units, the word rate is used rather than ratio. Understanding rate, however, is complicated and there is no universally accepted definition. When using the term rate, contextual understanding is critical. Students need many opportunities to use models to demonstrate the relationships between quantities before they are expected to work with rates numerically. Students should be able to identify all these ratios and describe them using "For every…, there are …"
Content Limits	Whole numbers Ratios can be expressed with a colon (1:5) or with words such as per, to, each, for each, for every, etc (1 to 5); be sure to vary these representations across items at this standard Quantities/Units can be discrete or continuous and can be the same or different across the two quantities. Be precise in describing relationships such as "the ratio of the number of *x* to the number of *y*" or "the ratio of the length of *x* to the length of *y*", or explicitly reference types of quantities. Limit use of percents to 6.RP.3c
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to recognize correct ratio notation (1:2, 1 to 2, ½) for a given ratio relationship - items should focus on notation, meaning that all options should contain the same numbers.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response • Table Response
Students will be required to represent a ratio relationship described in situational contexts or shown in tape diagrams, double number line diagrams, or graphics, etc. using ratio notation or descriptions.	
Students will be required to distinguish between part-to-part and part-to-whole ratio relationships described in situational contexts or shown in tape diagrams, double number line diagrams, or graphics, etc. using ratio notation or descriptions.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Understand the concept of a ratio as comparing two quantities. Use ratio language to identify a ratio relationship between two quantities.	Understand the concept of a ratio as comparing two quantities multiplicatively. Use ratio language to describe a ratio relationship between two quantities using a limited variety of representations.
Proficient	**Highly Proficient**
Understand the concept of a ratio as comparing two quantities multiplicatively or joining/composing the two quantities in a way that preserves a multiplicative relationship. Use ratio language to describe a ratio relationship between two quantities. *For example, "There were 2/3 as many men as women at the concert."*	Explain the concept of a ratio as comparing two quantities multiplicatively or joining/composing the two quantities in a way that preserves a multiplicative relationship. Use ratio language to describe a ratio relationship between two quantities.

6.RP.A.2

Content Standards	Understand the concept of a unit rate *a/b* associated with a ratio *a* : *b* with *b* ≠ 0, and use rate language (e.g., for every, for each, for each 1, per) in the context of a ratio relationship. (Complex fraction notation is not an expectation for unit rates in this grade level.)
Explanations	A unit rate compares a quantity in terms of one unit of another quantity. Students will often use unit rates to solve missing value problems. Cost per item or distance per time unit are common unit rates, however, students should be able to flexibly use unit rates to name the amount of either quantity in terms of the other quantity. Students will begin to notice that related unit rates are reciprocals as in the first example. It is not intended that this be taught as an algorithm or rule because at this level, students should primarily use reasoning to find these unit rates. In Grade 6, students are not expected to work with unit rates expressed as complex fractions. Both the numerator and denominator of the original ratio will be whole numbers.
Content Limits	Whole numbers except when identifying a unit rate. Rates can be expressed as fractions, with ":" or with words. Units can be the same or different across the two quantities. Context itself does not determine the order Name the amount of either quantity in terms of the other as long as one of the values is one unit Expectations for unit rates in this grade are limited to non-complex fractions, as stated in the standards.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify unit rates.	• Equation Response • Multiple Choice Response • Multi-Select Response • Table Response
Students will be required to find the unit rate given a ratio or ratio relationship expressed as a tape diagram or double number line diagram.	
Students will be required to solve word problems where the solution is in terms of a unit rate.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify a unit rate associated with a ratio and use basic unit rate language to describe it.	Determine a unit rate associated with a ratio and use unit rate language to describe it.
Proficient	**Highly Proficient**
Understand the concept of a unit rate a/b associated with a ratio a : b with b ≠ 0, and use rate language (e.g., for every, for each, for each 1, per) in the context of a ratio relationship. (Complex fraction notation is not an expectation for unit rates in this grade level.)	Explain the concept of a unit rate a/b associated with a ratio a : b with b ≠ 0, and use rate language in the context of a ratio relationship.

6.RP.A.3, 6.RP.A.3a, 6.RP.A.3b, 6.RP.A.3c, 6.RP.A.3d

<table>
<tr><td>Content Standards</td><td colspan="2">6.RP.A.3 Use ratio and rate reasoning to solve mathematical problems and problems in real-world context (e.g., by reasoning about data collected from measurements, tables of equivalent ratios, tape diagrams, double number line diagrams, or equations).

6.RP.A.3a Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

6.RP.A.3b Solve unit rate problems including those involving unit pricing and constant speed.

6.RP.A.3c Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity). Solve percent problems with the unknown in all positions of the equation.

6.RP.A.3d Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.</td></tr>
<tr><td>Explanations</td><td colspan="2">Understand ratio and concepts and use ratio reasoning to solve problems.</td></tr>
<tr><td>Content Limits</td><td colspan="2">Whole numbers except when identifying a unit rate.

Rates can be expressed as fractions, with “:” or with words.

Units can be the same or different across the two quantities.

Percent found as a rate per 100.</td></tr>
<tr><td>Context</td><td colspan="2">Context is allowed.</td></tr>
<tr><th colspan="2">Sample Task Demands</th><th>Common Item Formats</th></tr>
<tr><td colspan="2">Students will be required to generate tables of equivalent ratios. (a,b)</td><td rowspan="8">• Equation Response
• Graphic Response
• Multiple Choice Response
• Table Response</td></tr>
<tr><td colspan="2">Students will be required to plot ordered pairs of equivalent ratios. (a)</td></tr>
<tr><td colspan="2">Students will be required to solve a unit rate problem by finding a missing quantity based on that unit rate. (b)</td></tr>
<tr><td colspan="2">Students will be required to given a unit rate, add to a set to create an equivalent ratio.</td></tr>
<tr><td colspan="2">Students will be required to find a specified percent of a given quantity. (c)</td></tr>
<tr><td colspan="2">Students will be required to find a total quantity from a given quantity that is a percent of the whole. (c)</td></tr>
<tr><td colspan="2">Students will be required to apply a unit rate as a conversion factor to transform units when multiplying or dividing quantities. (d)</td></tr>
<tr><td colspan="2">Students will be required to given two criteria based on unit rates (part-to-part and/or part-to-whole), create a set of objects that satisfies both criteria.</td></tr>
</table>

Performance Level Descriptors

<table>
<tr><th>Minimally Proficient</th><th>Partially Proficient</th></tr>
<tr><td>Use ratio and rate reasoning to solve mathematical problems and problems in real-world context (e.g., by reasoning about data collected from measurements, tables of equivalent ratios, tape diagrams, double number line diagrams, or equations).

a. Use tables of equivalent ratios relating quantities with whole-number measurements, identify missing values in the tables, and identify the pairs of values plotted on the coordinate plane. Use tables to compare ratios.

b. Identify the unit rate for unit rate problems including those involving unit pricing and constant speed.

c. Identify a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity). Identify solutions to percent problems when the percent is the unknown.

d. Use ratio reasoning to match measurement units; transform units appropriately when multiplying quantities.</td><td>Use ratio and rate reasoning to solve mathematical problems and problems in real-world context (e.g., by reasoning about data collected from measurements, tables of equivalent ratios, tape diagrams, double number line diagrams, or equations).

a. Use tables of equivalent ratios relating quantities with whole-number measurements, determine missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

b. Define unit rate for unit rate problems including those involving unit pricing and constant speed.

c. Identify a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity). Identify solutions to percent problems when the percent or the part is the unknown.

d. Use ratio reasoning to identify measurement units; transform units appropriately when multiplying or dividing quantities.</td></tr>
<tr><th>Proficient</th><th>Highly Proficient</th></tr>
<tr><td>Use ratio and rate reasoning to solve mathematical problems and problems in real-world context (e.g., by reasoning about data collected from measurements, tables of equivalent ratios, tape diagrams, double number line diagrams, or equations).

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

b. Solve unit rate problems including those involving unit pricing and constant speed.

c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity). Solve percent problems with the unknown in all positions of the equation.

d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.</td><td>Use ratio and rate reasoning to solve mathematical problems and problems in real-world context (e.g., by reasoning about data collected from measurements, tables of equivalent ratios, tape diagrams, double number line diagrams, or equations).

a. Explain the pattern in tables of equivalent ratios relating quantities with whole-number measurements, explain how to find missing values in the tables, and how to plot the pairs of values on the coordinate plane. Use tables to compare ratios.

b. Solve unit rate problems involving more than one unit rate.

c. Explain why a percent of a quantity is a rate per 100. Create and solve percent problems with the unknown in all positions of the equation.

d. Use ratio reasoning to convert measurement units when more than one conversion is required; manipulate and transform units appropriately when multiplying or dividing quantities.</td></tr>
</table>

AzM2 | Arizona's Statewide Achievement Assessment for English Language Arts and Mathematics

Mathematics Item Specifications

GRADE 7

Arizona Department of Education with American Institutes for Research - 2019

Table of Contents

Introduction

The Arizona Statewide Achievement Assessment for English Language Arts and Mathematics (AzM2) is Arizona's statewide achievement test. AzM2 assesses the Arizona English Language Arts Standards and Arizona Mathematics Standards adopted by the Arizona State Board of Education in December 2016. AzM2 will inform students, teachers, and parents about preparedness for college and careers upon graduating from high school. AzM2 tests are computer-based, meaning that they can better assess students' critical thinking skills and provide them with opportunities to demonstrate a deeper understanding of the materials. Computer-based testing also allows for the use of a variety of innovative items types.

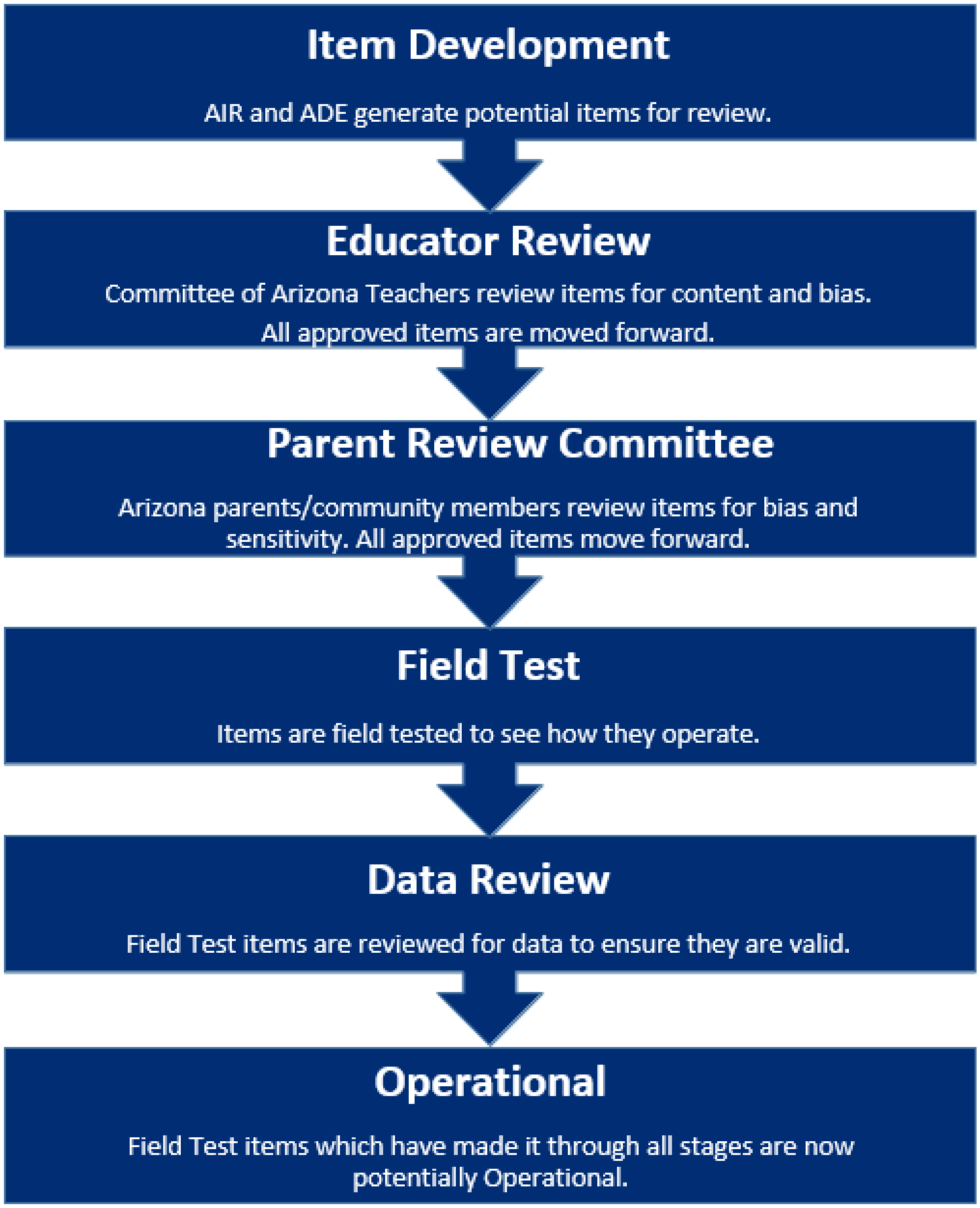
During the item-development process, all AzM2 items are written in accordance with the Item Specifications and are reviewed and approved by a committee of Arizona educators to confirm alignment and appropriateness for inclusion in the test. AzM2 items are generally representative of Arizona's geographic regions and culturally diverse population. Items are reviewed for the following kinds of bias: gender, racial, ethnic, linguistic, religious, geographic, and socioeconomic. Item reviews also include consideration of issues related to individuals with disabilities. Arizona community members also have an opportunity to review items for issues of potential concern to members of the community at large. Reviewers are asked to consider the variety of cultural, regional, philosophical, political, and religious backgrounds throughout Arizona, and then to determine whether the subject matter will be acceptable to Arizona students, families, and other members of Arizona communities.

This *AzM2 Item Specifications* is a resource document that defines the content and format of the test and test items for item writers and reviewers. Each *Item Specifications* document indicates the alignment of items with the Arizona Mathematics Standards. It also serves to provide all stakeholders with information about the scope and function of assessment items. This document can also serve to assist educators to understand how assessment items are developed in alignment with the standards for English language arts and math. These item specifications for AzM2 are intended to provide information regarding standards, item formats and response types. The descriptions of math blueprints and depth of knowledge in this document are meant to provide an overview of the test. Item specifications are meant for the purposes of assessment, not instruction. They are not intended to be tools for instruction or the basis for curricula. AzM2 has a test blueprint that was developed by Arizona and is different from any other state or consortium test blueprint.

For the math portion of AzM2, all of the test questions are aligned to the mathematic content standards for these subject areas. Any item specifications that are absent for standards listed in this document may be under development. This document does not endorse the exclusion of the instruction of any grade-level content standards. The test will ask questions that check a student's conceptual understanding of math as well as their procedural skills. These items have been written to be free from bias and sensitivity, and widely vary in their degree of difficulty.

Item Development Process

AzM2 items go through a rigorous review before they are operational. When an item is "operational" it means it is used to determine a student's score on the assessment. This is a description of the process every item must go through before it is operational on AzM2.



Sample tests are available online for the math portion of AzM2. For more information view the Guide to the Sample Tests at www.AzM2portal.org.

Test Construction Guidelines

The construction of the AzM2 assessment is guided by the depth and rigor of the Arizona College and Career Ready Standards. Items are created to address key components of the standards and assess a range of important skills. The AzM2 Blueprint provides an overview of the distribution of items on the AzM2 according to the standards. The standards for Math Practices are embedded within all AzM2 items. Further, the AzM2 blueprint outlines the Depth of Knowledge distribution of items.

Blueprint

Grade 7 AzM2 Blueprint 2016 Standards		
Reporting Category	**Min.**	**Max.**
Ratios & Proportions	19%	23%
The Number System	19%	23%
Expressions & Equations	23%	27%
Geometry and Statistics & Probability	27%	35%
Geometry	*15%*	*19%*
Statistics and Probability	*12%*	*16%*

Depth of Knowledge (DOK)

DOK refers to the level of rigor or sophistication of the task in a given item, designed to reflect the complexity of the Arizona Mathematics Standards. Items at DOK level 1 focus on the recall of information, such as definitions, terms, and simple procedures. Items at DOK 2 require students to make decisions, solve problems, or recognize patterns; in general, they require a greater degree of engagement and cognitive processing than items at DOK 1. Items at DOK 3 feature higher-order cognitive tasks that assess students' capacities to approach abstract or complex problems.

Percentage of Points by Depth of Knowledge (DOK) Level			
Grade 7	DOK Level 1	DOK Level 2	DOK Level 3
	10% - 20%	60% - 70%	12% - 30%

For more information on DOK go to www.azed.gov/AzM2.

Calculators

Arizona Desmos Scientific Calculator is permitted for the paper-based and computer-based assessment for Grade 7 Math.

Item Formats

The AzM2 Assessments are composed of item formats that include traditional multiple-choice response items and technology-enhanced response items (TEI). TEIs are computer-delivered response items that require students to interact with test content to select, construct, and/or support their responses. TEIs are better able to assess a deeper level of understanding.

Currently, there are nine types of TEIs that may appear on the Math computer based assessment for AzM2:

- Editing Tasks (ET)
- Editing Task Choice (ETC)
- Equation Editor (EQ)
- Graphic Response Item Display (GRID)
- Hot Text (HT)
  - Selectable Hot Text
  - Drag-and-Drop Hot Text
- Matching Item (MI)
- Multi-Select (MS)
- Open Response
- Table Item (TI)

For paper-based assessments (including those for students with an IEP or 504 plan that specifies a paper-based accommodation), TEIs will be modified so that they can be scanned and scored electronically or hand-scored.

See the table below for a description of each TEI. In addition, for examples of each response item format described, see the AzM2 Training Tests at www.AzM2portal.org.

Item Format	Description
Editing Task (ET)	The student clicks on a highlighted word or phrase that may be incorrect, which reveals a text box. The directions in the text box direct the student to replace the highlighted word or phrase with the correct word or phrase. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Editing Task Choice (ETC)	The student clicks a highlighted word or phrase, which reveals a drop-down menu containing options for correcting an error as well as the highlighted word or phrase as it is shown in the sentence to indicate that no correction is needed. The student then selects the correct word or phrase from the drop-down menu. For paper-based assessments, the item is modified so that it can be scanned and scored electronically. The student fills in a circle to indicate the correct word or phrase.

Item Format	Description
Equation Editor (EQ)	The student is presented with a toolbar that includes a variety of mathematical symbols that can be used to create a response. Responses may be in the form of a number, variable, expression, or equation, as appropriate to the test item. For paper-based assessments, this item type may be replaced with a modified version of the item that can be scanned and scored electronically or replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Graphic Response Item Display (GRID)	The student selects numbers, words, phrases, or images and uses the drag-and-drop feature to place them into a graphic. This item type may also require the student to use the point, line, or arrow tools to create a response on a graph. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Hot Text (HT)	**Selectable Hot Text -** Excerpted sentences from the text are presented in this item type. When the student hovers over certain words, phrases, or sentences, the options highlight. This indicates that the text is selectable ("hot"). The student can then click on an option to select it. For paper- based assessments, a "selectable" hot text item is modified so that it can be scanned and scored electronically. In this version, the student fills in a circle to indicate a selection.
	Drag-and-Drop Hot Text - Certain numbers, words, phrases, or sentences may be designated "draggable" in this item type. When the student hovers over these areas, the text highlights. The student can then click on the option, hold down the mouse button, and drag it to a graphic or other format. For paper-based assessments, drag- and-drop hot text items will be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Matching Item (MI)	The student checks a box to indicate if information from a column header matches information from a row. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Multi-Select (MS)	The student is directed to select all of the correct answers from among a number of options. These items are different from multiple-choice items, which allow the student to select only one correct answer. These items appear in the online and paper-based assessments.
Open Response	The student uses the keyboard to enter a response into a text field. These items can usually be answered in a sentence or two. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.

Item Format	Description
Table Item (TI)	The student types numeric values into a given table. The student may complete the entire table or portions of the table depending on what is being asked. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.

Arizona Math Standards Grade 7

Ratio and Proportion (RP)		
7.RP.A **Analyze proportional relationships and use them to solve mathematical problems and problems in real-world context.**	**7.RP.A.1**	Compute unit rates associated with ratios involving both simple and complex fractions, including ratios of quantities measured in like or different units.
	7.RP.A.2	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin). b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.* d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate.
	7.RP.A.3	Use proportional relationships to solve multi-step ratio and percent problems (e.g., simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error).
The Number System (NS)		
7.NS.A **Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers except division by zero.**	**7.NS.A.1**	Add and subtract integers and other rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world context. c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world context. d. Apply properties of operations as strategies to add and subtract rational numbers.

7.NS.A (cont.)	**7.NS.A.2**	Multiply and divide integers and other rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world context. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world context. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to decimal form using long division; know that the decimal form of a rational number terminates in 0's or eventually repeats.
	7.NS.A.3	Solve mathematical problems and problems in real-world context involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions where $a/b \div c/d$ when $a,b,c,$and d are all integers and $b,c,$ and $d \neq 0$.
Expressions and Equations (EE)		
7.EE.A **Use properties of operations to generate equivalent expressions.**	**7.EE.A.1**	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
	7.EE.A.2	Rewrite an expression in different forms, and understand the relationship between the different forms and their meanings in a problem context. *For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."*
7.EE.B **Solve mathematical problems and problems in real-world context using numerical and algebraic expressions and equations.**	**7.EE.B.3**	Solve multi-step mathematical problems and problems in real-world context posed with positive and negative rational numbers in any form. Convert between forms as appropriate and assess the reasonableness of answers. *For example, If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50 per hour.*
	7.EE.B.4	Use variables to represent quantities in mathematical problems and problems in real-world context, and construct simple equations and inequalities to solve problems. a. Solve word problems leading to equations of the form $px+q = r$ and $p(x+q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. b. Solve word problems leading to inequalities of the form $px+q > r$ or $px+q < r$, where p, q, and r are rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

Geometry (G)		
7.G.A **Draw, construct, and describe geometrical figures, and describe the relationships between them.**	**7.G.A.1**	Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
	7.G.A.2	Draw geometric shapes with given conditions using a variety of methods. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
	7.G.A.3	Describe the two-dimensional figures that result from slicing three-dimensional figures.
7.G.B **Solve mathematical problems and problems in real-world context involving angle measure, area, surface area, and volume.**	**7.G.B.4**	Understand and use the formulas for the area and circumference of a circle to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
	7.G.B.5	Use facts about supplementary, complementary, vertical, and adjacent angles in multi-step problems to write and solve simple equations for an unknown angle in a figure.
	7.G.B.6	Solve mathematical problems and problems in a real-world context involving area of two-dimensional objects composed of triangles, quadrilaterals, and other polygons. Solve mathematical problems and problems in real-world context involving volume and surface area of three-dimensional objects composed of cubes and right prisms.
Statistics and Probability (SP)		
7.SP.A **Use random sampling to draw inferences about a population.**	**7.SP.A.1**	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
	7.SP.A.2	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*
7.SP.B **Draw informal comparative inferences about two populations.**	**7.SP.B.3**	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*

7.SP.B (cont.)	**7.SP.B.4**	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*
7.SP.C **Investigate chance processes and develop, use and evaluate probability models.**	**7.SP.C.5**	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
	7.SP.C.6	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*
	7.SP.C.7	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies. If the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

Grade 7 Item Specifications

Expressions and Equations

7.EE.A.1

Content Standards	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
Explanations	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers except division by zero.
Content Limits	Using negative numbers and multiple operations should be emphasized to distinguish from 6.EE.3 Linear expressions Do not use the word "simplify" in items - wording for items using the EQ response mechanism must be precise in order to elicit a correct form of the expression (i.e. use "by combining all like terms" so that the given expression is not a correct answer)
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to perform operations to construct equivalent expressions.	• Equation Response • Multiple Choice Response • Multi-Select Reponse

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify properties of operations used to add, subtract, factor, and expand linear expressions with integer coefficients.	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with integer coefficients.
Proficient	**Highly Proficient**
Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients and interpret the meaning in a real-world context.

7.EE.A.2

Content Standards	Rewrite an expression in different forms and understand the relationship between the different forms and their meanings in a problem context. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*
Explanations	Use properties of operations to generate equivalent expressions.
Content Limits	Rational numbers Linear expressions with an unknown
Context	Context is required.

Sample Task Demands	Common Item Formats
Students will be required to given an expression within a context, identify an equivalent expression that shows a feature of that context.	• Equation Response • Multiple Choice Response • Multi-Select Reponse • Proposition Response
Students will be required to given a context and an expression with different values than given in the context, interpret part of the expression that is not found in the context.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify an expression in different forms.	Identify an expression in different forms and understand the relationship between the different forms and their meanings in a problem context. For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."
Proficient	**Highly Proficient**
Rewrite an expression in different forms and understand the relationship between the different forms and their meanings in a problem context. For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."	Rewrite an expression in different forms and explain the relationship between the different forms and their meanings in a problem context. For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."

7.EE.B.3

Content Standards	Solve multi-step mathematical problems and problems in real-world context posed with positive and negative rational numbers in any form. Convert between forms as appropriate and assess the reasonableness of answers. *For example, If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50 per hour.*
Explanations	Estimation strategies for calculations with fractions and decimals extend from students' work with whole number operations.
Content Limits	Rational numbers No variables Items involving estimation to assess reasonableness should not allow the student to respond with the exact answer.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to solve a problem where only the information needed is given.	• Equation Response • Multiple Choice Response
Students will be required to choose which value is reasonable based on estimation.	
Students will be required to solve a problem where extra information not needed to find the solution is given.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Solve multi-step mathematical problems and problems in real-world context posed with positive and negative rational numbers in one form.	Solve multi-step mathematical problems and problems in real-world context posed with positive and negative rational numbers in any form. Convert between forms as appropriate.
Proficient	**Highly Proficient**
Solve multi-step mathematical problems and problems in real-world context posed with positive and negative rational numbers in any form. Convert between forms as appropriate and assess the reasonableness of answers. For example, If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50 per hour.	Create problems with a real-world context given multi-step equations with positive and negative rational numbers. Convert between forms as appropriate and interpret the reasonableness of answers.

7.EE.B.4, 7.EE.B.4a, and 7.EE.B.4b

<table>
<tr><td>Content Standards</td><td>7.EE.B.4 Use variables to represent quantities in mathematical problems and problems in real-world context, and construct simple equations and inequalities to solve problems.

7.EE.B.4a Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

7.EE.B.4b Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.</td></tr>
<tr><td>Explanations</td><td>Solve mathematical problems and problems in real-world context using numerical and algebraic expressions and equations.</td></tr>
<tr><td>Content Limits</td><td>Rational numbers</td></tr>
<tr><td>Context</td><td>Context is allowed.</td></tr>
</table>

<table>
<tr><th>Sample Task Demands</th><th>Common Item Formats</th></tr>
<tr><td>Students will be required to solve an equation or inequality for an unknown value, without context. (a, b)</td><td rowspan="5">Equation Response
Graphic Response
Multiple Choice Response
Multi-Select Response</td></tr>
<tr><td>Students will be required to create an equation or inequality that models the situation, and then find the solution. (a, b)</td></tr>
<tr><td>Students will be required to construct an algebraic equation or inequality that models a word problem. (a, b)</td></tr>
<tr><td>Students will be required to graph the solution set of an inequality. (b)</td></tr>
<tr><td>Students will be required to interpret the solution set of an inequality in the context of the problem. (b)</td></tr>
</table>

Performance Level Descriptors

<table>
<tr><th>Minimally Proficient</th><th>Partially Proficient</th></tr>
<tr><td>Use variables to represent quantities in mathematical problems and problems in real-world context, and construct simple equations and inequalities to solve problems.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where $p, q,$ and r are integers.

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where $p, q,$ and r are integers.</td><td>Use variables to represent quantities in mathematical problems and problems in real-world context, and construct simple equations and inequalities to solve problems.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are integers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where $p, q,$ and r are rational numbers. Graph the solution set of the inequality.</td></tr>
<tr><th>Proficient</th><th>Highly Proficient</th></tr>
<tr><td>Use variables to represent quantities in mathematical problems and problems in real-world context, and construct simple equations and inequalities to solve problems.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where $p, q,$ and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where $p, q,$ and r are rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.</td><td>Use variables to represent quantities in mathematical problems and problems in real-world context, and construct simple equations and inequalities to solve problems.

a. Solve real-world problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where $p, q,$ and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, explaining the sequence of the operations used in each approach.

b. Solve real-world problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.</td></tr>
</table>

Geometry & Statistics and Probability

7.G.A.1

Content Standards	Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
Explanations	Draw, construct and describe geometrical figures and describe the relationships between them.
Content Limits	Two-dimensional polygons Keep any conversions within one system (e.g., inches to feet is okay but inches to meters is not okay).
Context	Context is allowed.

<table>
<tr><th>Sample Task Demands</th><th>Common Item Formats</th></tr>
<tr><td>Students will be required to find the length of a side or measure of area/perimeter given a shape and a scale factor.</td><td rowspan="4">• Equation Response
• Graphic Response</td></tr>
<tr><td>Students will be required to find the scale factor given two figures where one is a scale drawing of the other.</td></tr>
<tr><td>Students will be required to draw a scale model given a shape and a scale factor.</td></tr>
<tr><td>Students will be required to determine a scale factor and create a scale model based on given parameters.</td></tr>
</table>

Performance Level Descriptors

<table>
<tr><th>Minimally Proficient</th><th>Partially Proficient</th></tr>
<tr><td>Solve problems involving scale drawings of geometric figures, by identifying the scale.</td><td>Solve problems involving scale drawings of geometric figures, with a given scale.</td></tr>
<tr><th>Proficient</th><th>Highly Proficient</th></tr>
<tr><td>Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.</td><td>Solve complex problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.</td></tr>
</table>

7.G.A.2

Content Standards	Draw geometric shapes with given conditions using a variety of methods. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
Explanations	Conditions may involve points, line segments, angles, parallelism, congruence, angles, and perpendicularity.
Content Limits	Limit to two-dimensional polygons Include questions about the sum of two side lengths of a triangle is greater than the third side length. Given conditions should not focus on similarity or congruence or that sum of angles in a triangle is 180 degrees. Be aware of the scoring capabilities for the graphic response tool when designing these items. To distinguish from previous grades, conditions should include factors outside of parallel/perpendicular lines and angle measure, such as symmetry and side length.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to draw a figure based on given conditions.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response
Students will be required to select appropriate side lengths that will determine a triangle.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Classify geometric shapes with given conditions using a variety of methods. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	Identify geometric shapes with given conditions using a variety of methods. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
Proficient	**Highly Proficient**
Draw geometric shapes with given conditions using a variety of methods. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	Draw complex geometric shapes with given conditions using a variety of methods. Focus on constructing triangles from three measures of angles or sides, explaining when the conditions determine a unique triangle, more than one triangle, or no triangle.

7.G.A.3

Content Standards	Describe the two-dimensional figures that result from slicing three-dimensional figures.
Explanations	Draw, construct, and describe geometrical figures, and describe the relationships between them.
Content Limits	Limited to right prisms and pyramids up to ones with a hexagonal base. Spheres, cones and cylinders are allowed. Diagonals are limited to slices which will result in shapes that have been described in previous grade level standards.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to match a two-dimensional cross section with its (possible) 3-D figure(s).	• Graphic Response • Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to draw a two-dimensional figure that represents the cross section of a 3-D figure.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the two-dimensional figures that result from slicing three-dimensional figures parallel or perpendicular to the base.	Identify the two-dimensional figures that result from slicing three-dimensional figures.
Proficient	**Highly Proficient**
Describe the two-dimensional figures that result from slicing three-dimensional figures.	Describe the two-dimensional figures that result from slicing irregular three-dimensional figures.

7.G.B.4

Content Standards	Understand and use the formulas for the area and circumference of a circle to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
Explanations	Solve mathematical problems and problems in real-world context involving angle measure, area, surface area, and volume.
Content Limits	Circles and semi-circles.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to find the area of a circle given diameter or radius.	• Equation Response
Students will be required to find the radius or diameter of a circle given the circumference.	
Students will be required to find circumference given the radius or diameter.	
Students will be required to find the area of shapes created by the intersection of circles and other shapes.	
Students will be required to find the area given the circumference.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify area and circumference of a circle to solve problems.	Understand and use the formulas for the area and circumference of a circle to solve problems.
Proficient	**Highly Proficient**
Understand and use the formulas for the area and circumference of a circle to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	Understand and use the formulas for the area and circumference of a circle to solve problems and interpret the solution; explain the relationship between the circumference and area of a circle.

7.G.B.5

Content Standards	Use facts about supplementary, complementary, vertical, and adjacent angles in multi-step problems to write and solve simple equations for an unknown angle in a figure.
Explanations	Angle relationships that can be explored include but are not limited to: same-side (consecutive) interior and same-side (consecutive) exterior angles are supplementary.
Content Limits	Angle measurements are shown only in degrees and should not be greater than 180. Students should not be required to know the sum of the interior angles of any polygon.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to find the unknown measure of a supplementary, complementary, vertical, or adjacent angle.	• Equation Response
Students will be required to create an expression that can be used to find an unknown angle measurement.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify supplementary, complementary, vertical, and adjacent angles in a figure.	Use facts about supplementary, complementary, vertical, and adjacent angles in multi-step problems to solve simple equations for an unknown angle in a figure.
Proficient	**Highly Proficient**
Use facts about supplementary, complementary, vertical, and adjacent angles in multi-step problems to write and solve simple equations for an unknown angle in a figure.	Use facts about supplementary, complementary, vertical, and adjacent angles in multi-step problems to write and solve simple equations for an unknown angle in a figure and explain the solution.

7.G.B.6

Content Standards	Solve mathematical problems and problems in a real-world context involving area of two-dimensional objects composed of triangles, quadrilaterals, and other polygons. Solve mathematical problems and problems in real-world context involving volume and surface area of three-dimensional objects composed of cubes and right prisms.
Explanations	Students understanding of volume can be supported by focusing on the area of base times the height to calculate volume. Students understanding of surface area can be supported by focusing on the sum of the area of the faces. Nets can be used to evaluate surface area calculations.
Content Limits	3D shapes include right prisms and pyramids. If the base of the right prism has more than four sides, then the area of the base should be given.
Context	Context is allowed.

<table>
<tr><th>Sample Task Demands</th><th>Common Item Formats</th></tr>
<tr><td>Students will be required to find the volume or surface area of an object.</td><td rowspan="4">Equation Response
Table Response</td></tr>
<tr><td>Students will be required to find dimensions when the area, surface area, or volume is given.</td></tr>
<tr><td>Students will be required to find the volume of a cube given the surface area. (Area of each face is limited to 100 square units)</td></tr>
<tr><td>Students will be required to find the volume of an object composed of two objects.</td></tr>
</table>

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify solutions mathematical problems and problems in a real-world context involving area of two-dimensional objects composed of triangles, quadrilaterals, and other polygons.	Solve mathematical problems and problems in a real-world context involving area of two-dimensional objects composed of triangles, quadrilaterals, and other polygons. Identify solutions to mathematical problems and problems in real-world context involving volume and surface area of three-dimensional objects composed of cubes and right prisms.
Proficient	**Highly Proficient**
Solve mathematical problems and problems in a real-world context involving area of two-dimensional objects composed of triangles, quadrilaterals, and other polygons. Solve mathematical problems and problems in real-world context involving volume and surface area of three-dimensional objects composed of cubes and right prisms.	Solve mathematical problems and problems in a real-world context involving area of two-dimensional objects composed of triangles, quadrilaterals, and other polygons. Solve mathematical problems and problems in real-world context involving volume and surface area of three-dimensional objects.

7.SP.A.1

Content Standards	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
Explanations	Use random sampling to draw inferences about a population.
Content Limits	Use random sampling to draw inferences about a population.
Context	Context is required.

Sample Task Demands	Common Item Formats
Students will be required to identify a valid sample (random, representative, and proportional to population).	• EBSR Response • Multiple Choice Response • Multi-Select Response
Students will be required to justify a chosen sampling method.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify statistics that can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population.	Recognize that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Recognize that random sampling tends to produce representative samples and support valid inferences.
Proficient	**Highly Proficient**
Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.	Interpret statistics that can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

7.SP.A.2

Content Standards	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*
Explanations	Use random sampling to draw inferences about a population.
Content Limits	Rational numbers Given dot plots should have an approximately normal distribution
Context	Context is required.

Sample Task Demands	**Common Item Formats**
Students will be required to draw inferences about a population based on a set of random samples.	• Equation Response • Graphic Response • Multiple Choice Response
Students will be required to explore the variation among a set of random samples.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Use data from a random sample to identify inferences about a population with an unknown characteristic of interest.	Use data from a random sample to identify inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.
Proficient	**Highly Proficient**
Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.	Interpret data from a random sample to draw inferences about multiple populations with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

7.SP.B.3

<table>
<tr><td>Content Standards</td><td colspan="2">Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.</td></tr>
<tr><td>Explanations</td><td colspan="2">Researching data sets provides opportunities to connect mathematics to their interests and other academic subjects. Students can utilize statistic functions in graphing calculators or spreadsheets for calculations with larger data sets or to check their computations. Students calculate mean absolute deviations in preparation for later work with standard deviations.

Measures of center include mean, median, and mode. The measures of variability include range, mean absolute deviation, and interquartile range.</td></tr>
<tr><td>Content Limits</td><td colspan="2">Data displays should be dot plots or box plots with approximately normal distributions</td></tr>
<tr><td>Context</td><td colspan="2">Context is allowed.</td></tr>
<tr><th colspan="2">Sample Task Demands</th><th>Common Item Formats</th></tr>
<tr><td colspan="2">Students will be required to given sets of data displays that represent data distributions, select the set that shows the most visual overlap.</td><td rowspan="2">• Equation Response
• Graphic Response
• Multiple Choice Response
• Multi-Select Response</td></tr>
<tr><td colspan="2">Students will be required to given two data displays with different centers but the same variability, compute the difference in centers in terms of the mean absolute deviation and informally assess the degree of overlap.</td></tr>
</table>

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Compare the degree of visual overlap of two numerical data distributions with similar variabilities.	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities.
Proficient	**Highly Proficient**
Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.	Interpret the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.

7.SP.B.4

Content Standards	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*
Explanations	Researching data sets provides opportunities to connect mathematics to their interests and other academic subjects. Students can utilize statistic functions in graphing calculators or spreadsheets for calculations with larger data sets or to check their computations. Students calculate mean absolute deviations in preparation for later work with standard deviations. Measures of center include mean, median, and mode. The measures of variability include range, mean absolute deviation, and interquartile range.
Content Limits	Data displays should be dot plots or box plots with approximately normal distributions
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to evaluate data displays or measures regarding evidence (center and variation, based on overlap of the data) that the data for one population is greater than another.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify measures of center and measures of variability for numerical data from random samples for two populations.	Use measures of center and measures of variability for numerical data from random samples to identify informal comparative inferences about two populations.
Proficient	**Highly Proficient**
Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.	Interpret measures of center and measures of variability for numerical data from random samples to draw comparative inferences about two populations.

7.SP.C.5

Content Standards	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
Explanations	Probability can be expressed in terms such as impossible, unlikely, likely, or certain or as a number between 0 and 1 as illustrated on the number line.
Content Limits	Rational numbers Probabilities should not be given as percentages
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify the likelihood of a chance event occurring.	• Equation Response • Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to given a likelihood of an event occurring, identify a possible probability.	
Students will be required to compare probabilities as being more or less likely.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify that a probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.	Identify that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
Proficient	**Highly Proficient**
Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring and use this to solve real-world problems.

7.SP.C.6

Content Standards	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.
Explanations	Students can perform experiments multiple times, pool data with other groups, or increase the number of trials in a simulation to look at the long-run relative frequencies.
Content Limits	Probabilities should not be given as percentages All numbers are whole, other than probabilities For TD1, the student should only be required to find one probability
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to approximate/estimate the probability of a chance event by observing collected data (empirical/experimental probability).	• Equation Response
Students will be required to predict the approximate relative frequency given the theoretical probability.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the approximate probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency.	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and identify the approximate relative frequency given the probability.
Proficient	**Highly Proficient**
Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.	Explain the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.

7.SP.C.7, 7.SP.C.7a, and 7.SP.C.7b

Content Standards	**7.SP.C.7** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies. If the agreement is not good, explain possible sources of the discrepancy. **7.SP.C.7a** Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* **7.SP.C.7b** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*
Explanations	Students need multiple opportunities to perform probability experiments and compare these results to theoretical probabilities. Critical components of the experiment process are making predictions about the outcomes by applying the principles of theoretical probability, comparing the predictions to the outcomes of the experiments, and replicating the experiment to compare results. Experiments can be replicated by the same group or by compiling class data.
Content Limits	Rational numbers
Context	Context is required.

Sample Task Demands	Common Item Formats
Students will be required to, given a probability model, find the probability of a combination of events contained in the model. (a, b)	• Equation Response • Table Response
Students will be required to, given a situation that is clearly uniform (a), or given data (b), create a probability model.	
Students will be required to, given a situation that is clearly uniform (a), or given data (b), create a probability model and find the probability of a combination of events contained in the model.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies. If the agreement is not good, explain possible sources of the discrepancy. a. Identify a uniform probability model that assigns equal probability to all outcomes to determine probabilities of events. b. Identify a probability model (which may not be uniform) that observes frequencies in data generated from a chance process.	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies. If the agreement is not good, explain possible sources of the discrepancy. a. Use a uniform probability model that assigns equal probability to all outcomes to determine probabilities of events. b. Use a probability model (which may not be uniform) that observes frequencies in data generated from a chance process.
Proficient	**Highly Proficient**
Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies. If the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies. If the agreement is not good, explain possible sources of the discrepancy. a. Develop and explain a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected. b. Develop and explain a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?

The Number System

7.NS.A.1, 7.NS.A.1a, 7.NS.A.1b, 7.NS.A.1c, and 7.NS.A.1d

Content Standards	**7.NS.A.1** Add and subtract integers and other rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. **7.NS.A.1a** Describe situations in which opposite quantities combine to make 0. **7.NS.A.1b** Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world context. **7.NS.A.1c** Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world context. **7.NS.A.1d** Apply properties of operations as strategies to add and subtract rational numbers.
Explanations	Visual representations may be helpful as students begin this work; they become less necessary as students become more fluent with the operations.
Content Limits	Rational numbers: use integers, fractions, and decimals Limit decimals to the hundredths place Limit fractions to halves, fourths, and eighths.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to calculate the sum of a number and its opposite. (b)	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response
Students will be required to calculate the distance between two points on a number line. (b or c)	
Students will be required to identify the location of a point that is a specified distance from another point on a number line. (b or c)	
Students will be required to find the sum or difference of two rational numbers. (d)	
Students will be required to model each step of a multi-part addition/subtraction expression on number lines, and identify the value of the expression. (b or c)	
Students will be required to identify properties of values given a number line or calculation, using variables rather than actual numbers. (b or c)	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Add and subtract integers and other rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Identify opposite quantities. b. Identify a number and its opposite that have a sum of 0. c. Identify the distance between two rational numbers on the number line as the absolute value of their difference. d. Identify properties of operations as strategies to add and subtract rational numbers.	Add and subtract integers and other rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Identify situations in which opposite quantities combine to make 0. b. Recognize $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Identify a number and its opposite that have a sum of 0 (are additive inverses). c. Recognize subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference. d. Identify properties of operations as strategies to add and subtract rational numbers.
Proficient	**Highly Proficient**
Add and subtract integers and other rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world context. c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world context. d. Apply properties of operations as strategies to add and subtract rational numbers.	Add and subtract integers and other rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Interpret situations in which opposite quantities combine to make 0. b. Explain $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world context. c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world context. d. Apply properties of operations as strategies to add and subtract rational numbers.

7.NS.A.2, 7.NS.A.2a, 7.NS.A.2b, 7.NS.A.2c, and 7.NS.A.2d

Content Standards	**7.NS.A.2** Multiply and divide integers and other rational numbers. **7.NS.A.2a** Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world context. **7.NS.A.2b** Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world context. **7.NS.A.2c** Apply properties of operations as strategies to multiply and divide rational numbers. **7.NS.A.2d** Convert a rational number to decimal form using long division; know that the decimal form of a rational number terminates in 0's or eventually repeats.
Explanations	Multiplication and division of integers is an extension of multiplication and division of whole numbers.
Content Limits	Rational numbers 7.NS.2a and 2b require the incorporation of a negative value
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to convert a rational number to a decimal.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response
Students will be required to determine a product or quotient given an expression or real-world situation.	
Students will be required to identify properties of values given a number line or calculation, using variables rather than actual numbers.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Multiply and divide integers and other rational numbers. a. Identify that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Identify products of rational numbers. b. Identify that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. c. Multiply and divide rational numbers. d. Identify decimal form of a rational number.	Multiply and divide integers and other rational numbers. a. Recognize that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Identify products of rational numbers by describing real-world context. b. Recognize that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Identify quotients of rational numbers by describing real-world context. c. Use properties of operations as strategies to multiply and divide rational numbers. d. Identify decimal form of a rational number; know that the decimal form of a rational number terminates in 0's or eventually repeats.
Proficient	**Highly Proficient**
Multiply and divide integers and other rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world context. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world context. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to decimal form using long division; know that the decimal form of a rational number terminates in 0's or eventually repeats.	Multiply and divide integers and other rational numbers. a. Explain that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world context. b. Explain that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world context. c. Apply properties of operations as strategies to multiply and divide rational numbers in a real-world context. d. Convert a rational number to decimal form using long division; know that the decimal form of a rational number terminates in 0's or eventually repeats.

7.NS.A.3

Content Standards	Solve mathematical problems and problems in real-world context involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions where $a/b \div c/d$ when *a, b, c,* and *d* are all integers and *b, c*, and $d \neq 0$.
Explanations	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers except division by zero.
Content Limits	Rational numbers Complex fractions can be used, but should contain fractions with single-digit numerators and denominators
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to solve simple problems involving rational numbers given a scenario.	• Equation Response • Table Response
Students will be required to solve complex problems involving rational numbers given a scenario.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the solution of mathematical problems four operations with rational numbers.	Identify the solution of mathematical problems and problems in real-world context involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions where $a/b \div c/d$ when *a, b, c,*and *d* are all integers and *b, c,* and $d \neq 0$.
Proficient	**Highly Proficient**
Solve mathematical problems and problems in real-world context involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions where $a/b \div c/d$ when *a, b, c,* and *d* are all integers and *b, c*, and $d \neq 0$.	Solve mathematical problems and problems in real-world context involving the four operations with rational numbers and interpret the solution. Computations with rational numbers extend the rules for manipulating fractions to complex fractions where $a/b \div c/d$ when *a*,*b*,*c*,and *d* are all integers and *b*,*c*, and $d \neq 0$.

Ratio and Proportional Relationships

7.RP.A.1

Content Standards	Compute unit rates associated with ratios involving both simple and complex fractions, including ratios of quantities measured in like or different units.
Explanations	Analyze proportional relationships and use them to solve mathematical problems and problems in real-world context.
Content Limits	Rational numbers - some items may include one rational number and one whole number (other than 1), but the bulk of items from this standard should involve ratios expressed as fractions, including complex fractions Ratios can be expressed as fractions, with ":", or with words Units can be the same or different across the two quantities
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to find a unit rate for a given ratio from information within a situational context, table or mathematical problem.	• Equation response • Graphic Response • Multiple Choice Response • Multi-Select Response • Table Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify unit rates associated with ratios involving simple fractions, including ratios of quantities measured in like units.	Compute unit rates associated with ratios involving simple fractions, including ratios of quantities measured in like units.
Proficient	**Highly Proficient**
Compute unit rates associated with ratios involving both simple and complex fractions, including ratios of quantities measured in like or different units.	Interpret unit rates associated with ratios involving both simple and complex fractions, including ratios of quantities measured in like or different units.

7.RP.A.2, 7.RP.A.2a, 7.RP.A.2b, 7.RP.A.2c, and 7.RP.A.2d

Content Standards	**7.RP.A.2** Recognize and represent proportional relationships between quantities. **7.RP.A.2a** Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin). **7.RP.A.2b** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. **7.RP.A.2c** Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* **7.RP.A.2d** Explain what a point (*x, y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where *r* is the unit rate.
Explanations	Graphing proportional relationships represented in a table helps students recognize that the graph is a line through the origin (0,0) with a constant of proportionality equal to the slope of the line.
Content Limits	Rational numbers Ratios can be expressed as fractions, with ":", or with words Units can be the same or different across the two quantities Items should not require creating a graph of a proportional relationship (this is 8.EE.5), only interpreting given graphs
Context	Context is allowed.

<table>
<tr><th>Sample Task Demands</th><th>Common Item Formats</th></tr>
<tr><td>Students will be required to recognize a proportional relationship within a representation (description, graph, table, etc). (a)</td><td rowspan="6">• Equation response
• Multiple Choice Response
• Multi-Select Response
• Simulator Response</td></tr>
<tr><td>Students will be required to identify the unit rate in a proportional relationship, given a description, graph with (1, r) plotted, equation, or table of equivalent ratios. (b)</td></tr>
<tr><td>Students will be required to identify equivalent proportional relationship across representations. (c or d)</td></tr>
<tr><td>Students will be required to solve real-world problems involving a proportional relationship, given an equation or graph. (c or d)</td></tr>
<tr><td>Students will be required to create an equation to represent a proportional relationship; in some cases, also apply the equation to the situation. (c)</td></tr>
<tr><td>Students will be required to explain the points (0,0) and (1,r) and their significance in a graph of a proportional relationship, where r is the unit rate. (d)</td></tr>
</table>

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Recognize and represent proportional relationships between quantities. a. Identify two quantities in a proportional relationship. b. Identify the constant of proportionality (unit rate) in tables or graphs. c. Identify equations to represent proportional relationships. d. Identify a point (*x*, *y*) on the graph of a proportional relationship.	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship. b. Identify the constant of proportionality (unit rate) in tables, graphs, equation. c. Represent proportional relationships by equations. d. Identify what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where r is the unit rate.
Proficient	**Highly Proficient**
Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin). b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. For example, if total cost *t* is proportional to the number *n* of items purchased at a constant price *p*, the relationship between the total cost and the number of items can be expressed as $t = pn$. d. Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where *r* is the unit rate.	Recognize and represent proportional relationships between quantities. a. Explain whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin). b. Interpret the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. For example, if total cost *t* is proportional to the number *n* of items purchased at a constant price *p*, the relationship between the total cost and the number of items can be expressed as $t = pn$. d. Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where r is the unit rate.

7.RP.A.3

Content Standards	Use proportional relationships to solve multi-step ratio and percent problems (e.g., simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error).
Explanations	Students should be able to explain or show their work using a representation (numbers, words, pictures, physical objects, or equations) and verify that their answer is reasonable. Models help students to identify the parts of the problem and how the values are related. For percent increase and decrease, students identify the starting value, determine the difference, and compare the difference in the two values to the starting value.
Content Limits	Limit to rational numbers Units can be the same or different across the two quantities
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to calculate the solution for percent and ratio problems.	• Equation response • Graphic Response • Multiple Choice Response
Students will be required to create an expression that can be used to find a specified percent or percentage increase/decrease of a given whole.	
Students will be required to use percent increase or decrease to find two quantities given their relationship in a real world context.	
Students will be required to interpret a proportional pattern from percent increase/decrease problems as a graph or as an equation.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Use proportional relationships to solve one-step ratio and percent mathematical problems (e.g., simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error).	Use proportional relationships to solve one-step ratio and percent problems (e.g., simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error).
Proficient	**Highly Proficient**
Use proportional relationships to solve multi-step ratio and percent problems (e.g., simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error).	Interpret proportional relationships when solving multi-step ratio and percent problems (e.g., simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error).

Arizona's Statewide Achievement Assessment for English Language Arts and Mathematics

Mathematics Item Specifications

GRADE 8

Arizona Department of Education with American Institutes for Research - 2019

Table of Contents

Introduction

The Arizona Statewide Achievement Assessment for English Language Arts and Mathematics (AzM2) is Arizona's statewide achievement test. AzM2 assesses the Arizona English Language Arts Standards and Arizona Mathematics Standards adopted by the Arizona State Board of Education in December 2016. AzM2 will inform students, teachers, and parents about preparedness for college and careers upon graduating from high school. AzM2 tests are computer-based, meaning that they can better assess students' critical thinking skills and provide them with opportunities to demonstrate a deeper understanding of the materials. Computer-based testing also allows for the use of a variety of innovative items types.

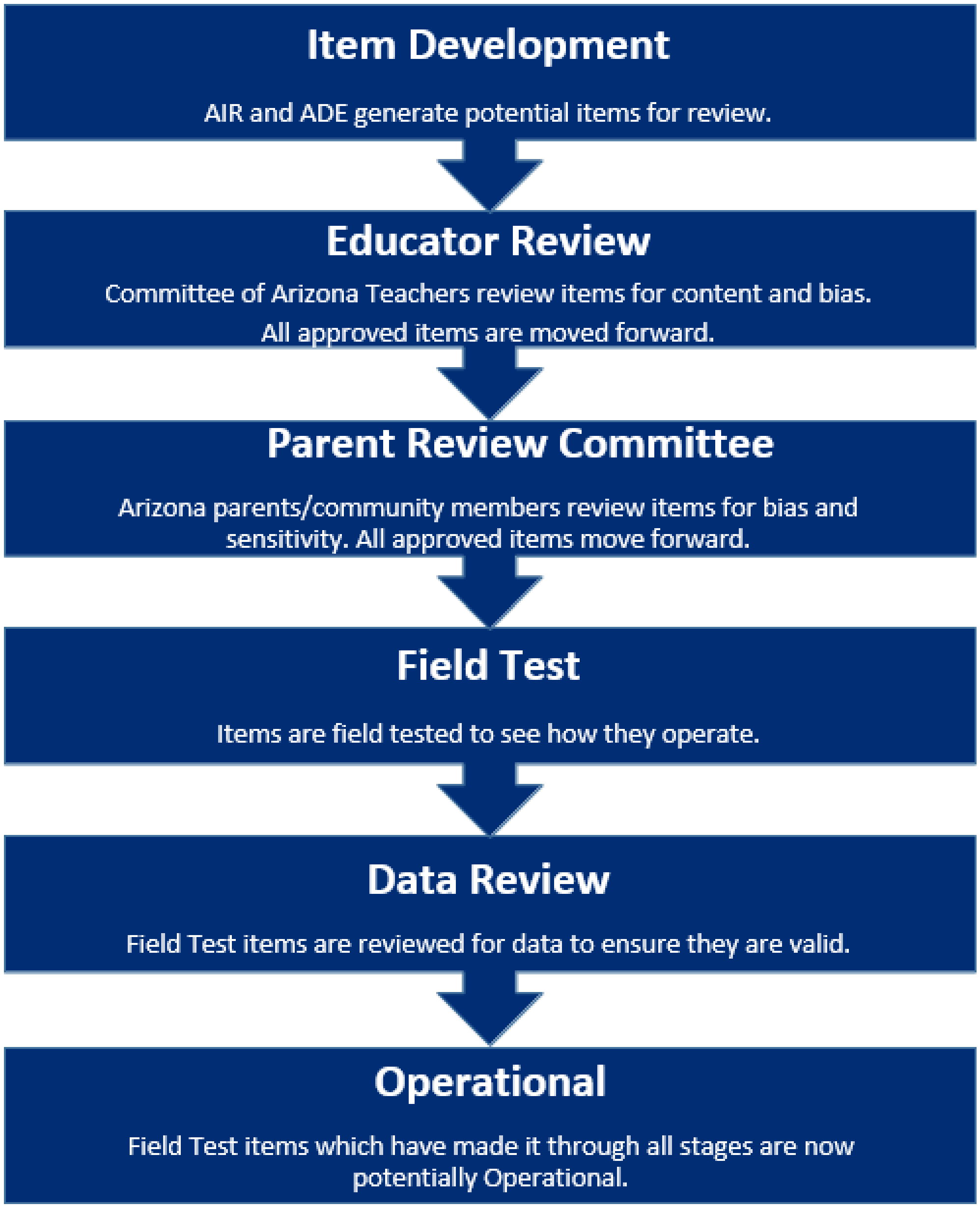
During the item-development process, all AzM2 items are written in accordance with the Item Specifications and are reviewed and approved by a committee of Arizona educators to confirm alignment and appropriateness for inclusion in the test. AzM2 items are generally representative of Arizona's geographic regions and culturally diverse population. Items are reviewed for the following kinds of bias: gender, racial, ethnic, linguistic, religious, geographic, and socioeconomic. Item reviews also include consideration of issues related to individuals with disabilities. Arizona community members also have an opportunity to review items for issues of potential concern to members of the community at large. Reviewers are asked to consider the variety of cultural, regional, philosophical, political, and religious backgrounds throughout Arizona, and then to determine whether the subject matter will be acceptable to Arizona students, families, and other members of Arizona communities.

This *AzM2 Item Specifications* is a resource document that defines the content and format of the test and test items for item writers and reviewers. Each *Item Specifications* document indicates the alignment of items with the Arizona Mathematics Standards. It also serves to provide all stakeholders with information about the scope and function of assessment items. This document can also serve to assist educators to understand how assessment items are developed in alignment with the standards for English language arts and math. These item specifications for AzM2 are intended to provide information regarding standards, item formats and response types. The descriptions of math blueprints and depth of knowledge in this document are meant to provide an overview of the test. Item specifications are meant for the purposes of assessment, not instruction. They are not intended to be tools for instruction or the basis for curricula. AzM2 has a test blueprint that was developed by Arizona and is different from any other state or consortium test blueprint.

For the math portion of AzM2, all of the test questions are aligned to the mathematic content standards for these subject areas. Any item specifications that are absent for standards listed in this document may be under development. This document does not endorse the exclusion of the instruction of any grade-level content standards. The test will ask questions that check a student's conceptual understanding of math as well as their procedural skills. These items have been written to be free from bias and sensitivity, and widely vary in their degree of difficulty.

Item Development Process

AzM2 items go through a rigorous review before they are operational. When an item is "operational" it means it is used to determine a student's score on the assessment. This is a description of the process every item must go through before it is operational on AzM2.



Sample tests are available online for the math portion of AzM2. For more information view the Guide to the Sample Tests at www.AzM2portal.org.

Test Construction Guidelines

The construction of the AzM2 assessment is guided by the depth and rigor of the Arizona College and Career Ready Standards. Items are created to address key components of the standards and assess a range of important skills. The AzM2 Blueprint provides an overview of the distribution of items on the AzM2 according to the standards. The standards for Math Practices are embedded within all AzM2 items. Further, the AzM2 blueprint outlines the Depth of Knowledge distribution of items.

Blueprint

Grade 8 AzM2 Blueprint 2016 Standards		
Reporting Category	**Min.**	**Max.**
Functions	21%	25%
Expressions & Equations	29%	33%
Geometry	17%	21%
Statistics & Probability & the Number System	19%	27%
Statistics and Probability	*4%*	*8%*
Number System	*15%*	*19%*

Depth of Knowledge (DOK)

DOK refers to the level of rigor or sophistication of the task in a given item, designed to reflect the complexity of the Arizona Mathematics Standards. Items at DOK level 1 focus on the recall of information, such as definitions, terms, and simple procedures. Items at DOK 2 require students to make decisions, solve problems, or recognize patterns; in general, they require a greater degree of engagement and cognitive processing than items at DOK 1. Items at DOK 3 feature higher-order cognitive tasks that assess students' capacities to approach abstract or complex problems.

Percentage of Points by Depth of Knowledge (DOK) Level			
Grade 8	DOK Level 1	DOK Level 2	DOK Level 3
	10% - 20%	60% - 70%	12% - 30%

For more information on DOK go to www.azed.gov/AzM2.

Calculators

Arizona Desmos Scientific Calculator is permitted for the paper-based and computer-based assessment for Grade 8 Math.

Item Formats

The AzM2 Assessments are composed of item formats that include traditional multiple-choice response items and technology-enhanced response items (TEI). TEIs are computer-delivered response items that require students to interact with test content to select, construct, and/or support their responses. TEIs are better able to assess a deeper level of understanding.

Currently, there are nine types of TEIs that may appear on the Math computer based assessment for AzM2:

- Editing Tasks (ET)
- Editing Task Choice (ETC)
- Equation Editor (EQ)
- Graphic Response Item Display (GRID)
- Hot Text (HT)
    - Selectable Hot Text
    - Drag-and-Drop Hot Text
- Matching Item (MI)
- Multi-Select (MS)
- Open Response
- Table Item (TI)

For paper-based assessments (including those for students with an IEP or 504 plan that specifies a paper-based accommodation), TEIs will be modified so that they can be scanned and scored electronically or hand-scored.

See the table below for a description of each TEI. In addition, for examples of each response item format described, see the AzM2 Training Tests at www.AzM2portal.org.

Item Format	Description
Editing Task (ET)	The student clicks on a highlighted word or phrase that may be incorrect, which reveals a text box. The directions in the text box direct the student to replace the highlighted word or phrase with the correct word or phrase. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Editing Task Choice (ETC)	The student clicks a highlighted word or phrase, which reveals a drop-down menu containing options for correcting an error as well as the highlighted word or phrase as it is shown in the sentence to indicate that no correction is needed. The student then selects the correct word or phrase from the drop-down menu. For paper-based assessments, the item is modified so that it can be scanned and scored electronically. The student fills in a circle to indicate the correct word or phrase.

Item Format	Description
Equation Editor (EQ)	The student is presented with a toolbar that includes a variety of mathematical symbols that can be used to create a response. Responses may be in the form of a number, variable, expression, or equation, as appropriate to the test item. For paper-based assessments, this item type may be replaced with a modified version of the item that can be scanned and scored electronically or replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Graphic Response Item Display (GRID)	The student selects numbers, words, phrases, or images and uses the drag-and-drop feature to place them into a graphic. This item type may also require the student to use the point, line, or arrow tools to create a response on a graph. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Hot Text (HT)	**Selectable Hot Text -** Excerpted sentences from the text are presented in this item type. When the student hovers over certain words, phrases, or sentences, the options highlight. This indicates that the text is selectable ("hot"). The student can then click on an option to select it. For paper- based assessments, a "selectable" hot text item is modified so that it can be scanned and scored electronically. In this version, the student fills in a circle to indicate a selection.
	Drag-and-Drop Hot Text - Certain numbers, words, phrases, or sentences may be designated "draggable" in this item type. When the student hovers over these areas, the text highlights. The student can then click on the option, hold down the mouse button, and drag it to a graphic or other format. For paper-based assessments, drag- and-drop hot text items will be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Matching Item (MI)	The student checks a box to indicate if information from a column header matches information from a row. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Multi-Select (MS)	The student is directed to select all of the correct answers from among a number of options. These items are different from multiple-choice items, which allow the student to select only one correct answer. These items appear in the online and paper-based assessments.
Open Response	The student uses the keyboard to enter a response into a text field. These items can usually be answered in a sentence or two. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.

Item Format	Description
Table Item (TI)	The student types numeric values into a given table. The student may complete the entire table or portions of the table depending on what is being asked. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.

Arizona Math Standards Grade 8

The Number System (NS)		
8.NS.A **Understand that there are irrational numbers, and approximate them using rational numbers.**	**8.NS.A.1**	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion. Know that numbers whose decimal expansions do not terminate in zeros or in a repeating sequence of fixed digits are called irrational.
	8.NS.A.2	Use rational approximations of irrational numbers to compare the size of irrational numbers. Locate them approximately on a number line diagram, and estimate their values.
	8.NS.A.3	Understand that given any two distinct **rational** numbers, $a < b$, there exist a rational number c and an irrational number d such that $a < c < b$ and $a < d < b$. Given any two distinct **irrational** numbers, $a < b$, there exist a rational number c and an irrational number d such that $a < c < b$ and $a < d < b$.
Expressions and Equations (EE)		
8.EE.A **Work with radicals and integer exponents.**	**8.EE.A.1**	Understand and apply the properties of integer exponents to generate equivalent numerical expressions.
	8.EE.A.2	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Know that $\sqrt{2}$ is irrational. a. Evaluate square roots of perfect squares less than or equal to 225. b. Evaluate cube roots of perfect cubes less than or equal to 1000.
	8.EE.A.3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and express how many times larger or smaller one is than the other.
	8.EE.A.4	Perform operations with numbers expressed in scientific notation including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities.
8.EE.B **Understand the connections between proportional relationships, lines, and linear equations.**	**8.EE.B.5**	Graph proportional relationships interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
	8.EE.B.6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. Derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at (0, b).

8.EE.C **Analyze and solve linear equations, inequalities, and pairs of simultaneous linear equations.**	**8.EE.C.7**	Fluently solve linear equations and inequalities in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solution. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations and inequalities with rational number coefficients, including solutions that require expanding expressions using the distributive property and collecting like terms.
	8.EE.C.8	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations including cases of no solution and infinite number of solutions. Solve simple cases by inspection. c. Solve mathematical problems and problems in real-world context leading to two linear equations in two variables.
Functions (F)		
8.F.A **Define, evaluate, and compare functions.**	**8.F.A.1**	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8.)
	8.F.A.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
	8.F.A.3	Interpret the equation $y = mx + b$ as defining a linear function whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length in not linear because its graph contains the points (1,1), (2,4), and (3,9) which are not on a straight line.*
8.F.B **Use functions to model relationships between quantities.**	**8.F.B.4**	Given a description of a situation, generate a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or a graph. Track how the values of the two quantities change together. Interpret the rate of change and initial value of a linear function in terms of the situation it models, its graph, or its table of values.
	8.F.B.5	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Geometry (G)		
8.G.A **Understand congruence and similarity.**	**8.G.A.1**	Verify experimentally the properties of rotations, reflections, and translations. Properties include: lines are taken to lines, line segments are taken to line segments of the same length, angles are taken to angles of the same measure, parallel lines are taken to parallel lines.
	8.G.A.2	Understand that a two-dimensional figure is congruent to another if one can be obtained from the other by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that demonstrates congruence.
	8.G.A.3	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
	8.G.A.4	Understand that a two-dimensional figure is similar to another if, and only if, one can be obtained from the other by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that demonstrates similarity.
	8.G.A.5	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*
8.G.B **Understand and apply the Pythagorean Theorem.**	**8.G.B.6**	Understand the Pythagorean Theorem and its converse.
	8.G.B.7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world context and mathematical problems in two and three dimensions.
	8.G.B.8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
8.G.C **Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**	**8.G.C.9**	Understand and use formulas for volumes of cones, cylinders and spheres and use them to solve real-world context and mathematical problems.

Statistics and Probability (SP)		
8.SP.A **Investigate patterns of association in bivariate data.**	**8.SP.A.1**	Construct and interpret scatter plots for bivariate measurement data to investigate and describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
	8.SP.A.2	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
	8.SP.A.3	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.
	8.SP.A.4	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.
8.SP.B **Investigate chance processes and develop, use, and evaluate probability models.**	**8.SP.B.5**	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using organized lists, tables, tree diagrams and other methods. Identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events.

Grade 8 Math Item Specifications

Expressions and Equations

8.EE.A.1

Content Standards	Understand and apply the properties of integer exponents to generate equivalent numerical expressions.
Explanations	Work with radicals and integer exponents.
Content Limits	Integer exponents Rational numbers for bases
Context	Context is not allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to identify equivalent numerical expressions using the properties of exponents.	• Equation Response • Graphic Response • Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to complete an equivalent expression using the properties of exponents.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Apply the properties of integer exponents to identify equivalent numerical expressions.	Apply the properties of integer exponents to generate equivalent numerical expressions.
Proficient	**Highly Proficient**
Understand and apply the properties of integer exponents to generate equivalent numerical expressions.	Understand and apply the properties of integer exponents to generate and interpret equivalent numerical expressions.

8.EE.A.2, 8.EE.A.2a, 8.EE.A.2b

Content Standards	**8.EE.A.2** Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Know that √2 is irrational. **8.EE.A.2a** Evaluate square roots of perfect squares less than or equal to 225. **8.EE.A.2b** Evaluate cube roots of perfect cubes less than or equal to 1000.
Explanations	Work with radicals and integer exponents.
Content Limits	Square roots and cube roots Rational and irrational numbers When evaluating roots, the base of a square root should be 100 or less and the base for a cube root should be 125 or less.
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify a square or cube root as the solution to a quadratic or cubic equation.	• Equation Response • Multiple Choice Response
Students will be required to find the value of a square or cube root.	
Students will be required to solve simple square or cube root equations.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Know that √2 is irrational. a. Identify square roots of perfect squares less than or equal to 100. b. Identify cube roots of perfect cubes less than or equal to 500.	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Know that √2 is irrational. a. Identify square roots of perfect squares less than or equal to 225. b. Identify cube roots of perfect cubes less than or equal to 1000.
Proficient	**Highly Proficient**
Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Know that √2 is irrational. a. Evaluate square roots of perfect squares less than or equal to 225. b. Evaluate cube roots of perfect cubes less than or equal to 1000.	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Know that √2 is irrational. a. Evaluate square roots less than or equal to 225. b. Evaluate cube roots less than or equal to 1000.

8.EE.A.3

Content Standards	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and express how many times larger or smaller one is than the other.
Explanations	Work with radicals and integer exponents.
Content Limits	None
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to convert between standard form and scientific notation.	• Equation Response • Multiple Choice Response
Students will be required to compare the magnitudes of different quantities given in scientific notation.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities.	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities.
Proficient	**Highly Proficient**
Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and express how many times larger or smaller one is than the other.	Use numbers expressed in the form of a single digit times an integer power of 10 to interpret very large or very small quantities, and express how many times larger or smaller one is than the other.

8.EE.A.4

Content Standards	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities.
Explanations	Students can convert decimal forms to scientific notation and apply rules of exponents to simplify expressions. In working with calculators or spreadsheets, it is important that students recognize scientific notation. Students should recognize that the output of 2.45E+23 is 2.45 x 1023 and 3.5E-4 is 3.5 x 10-4. Students enter scientific notation using E or EE (scientific notation), * (multiplication), and ^ (exponent) symbols.
Content Limits	For TD1, to distinguish from 8.EE.3, do not use single-digit leading terms
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to convert between standard form and scientific notation.	• Equation Response • Multiple Choice Response • Matching Item
Students will be required to perform operations with numbers expressed in scientific notation.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Perform operations with numbers expressed in scientific notation.	Perform operations with numbers expressed in scientific notation including problems where both decimal and scientific notation are used. Use scientific notation for measurements of very large or very small quantities.
Proficient	**Highly Proficient**
Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities.	Perform operations with numbers expressed in scientific notation including problems where both decimal and scientific notation are used. Use scientific notation to interpret for measurements of very large or very small quantities.

8.EE.B.5

Content Standards	Graph proportional relationships interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
Explanations	Using graphs of experiences that are familiar to students increases accessibility and supports understanding and interpretation of proportional relationship. Students are expected to both sketch and interpret graphs.
Content Limits	Rational numbers y-intercept is zero
Context	Context is required.

Sample Task Demands	Common Item Formats
Students will be required to calculate unit rate given a graph of a proportional relationship.	• Graphic Response • Multiple Choice Response • Multi-Select Response • Table Response
Students will be required to graph proportional relationships, including comparisons to other proportional relationships.	
Students will be required to compare two proportional relationships represented in two different ways.	
Students will be required to create a proportional relationship based on a comparison with another proportional relationship in a different representation.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Graph proportional relationships.	Graph proportional relationships interpreting the unit rate as the slope of the graph. Compare two different proportional relationships.
Proficient	**Highly Proficient**
Graph proportional relationships interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to	Graph proportional relationships interpreting the unit rate as the slope of the graph. Compare and explain two different proportional relationships represented in different ways.

8.EE.B.6

Content Standards	Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane. Derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at (0, *b*).
Explanations	Understand the connections between proportional relationships, lines, and linear equations.
Content Limits	None
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to given two points on a line, determine other points on the line.	• Equation Response • Graphic Response • Multiple Choice Response
Students will be required to given three points on a line described abstractly, determine a parameter for a fourth point on the line.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Use similar triangles to identify that the slope is the same between any two distinct points on a non-vertical line in the coordinate plane.	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. Use the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at (0, *b*).
Proficient	**Highly Proficient**
Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. Derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at (0, *b*).	Use similar triangles to prove why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. Derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at (0, *b*).

8.EE.B.7, 8.EE.B.7a, 8.EE.B.7b

Content Standards	**8.EE.B.7** Fluently solve linear equations and inequalities in one variable. **8.EE.B.7a** Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solution. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form *x = a, a = a, or a = b* results (where *a* and *b* are different numbers). **8.EE.B.7b** Solve linear equations and inequalities with rational number coefficients, including solutions that require expanding expressions using the distributive property and collecting like terms.
Explanations	As students transform linear equations in one variable into simpler forms, they discover the equations can have one solution, infinitely many solutions, or no solutions. When the equation has one solution, the variable has one value that makes the equation true as in 12 - 4*y* = 16. The only value for y that makes this equation true is -1. When the equation has infinitely many solutions, the equation is true for all real numbers as in 7*x* + 14 = 7 (*x*+2). As this equation is simplified, the variable terms cancel leaving 14 = 14 or 0 = 0. Since the expressions are equivalent, the value for the two sides of the equation will be the same regardless which real number is used for the substitution. When an equation has no solutions it is also called an inconsistent equation. This is the case when the two expressions are not equivalent as in 5*x* - 2 = 5(*x*+1). When simplifying this equation, students will find that the solution appears to be two numbers that are not equal or -2 = 1. In this case, regardless which real number is used for the substitution, the equation is not true and therefore has no solution.
Content Limits	Rational Numbers
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to determine the number of solutions of an equation where no simplification is required. (a)	• Equation Response • Multiple Choice Response • Matching Item Response • Multi-Select Response
Students will be required to determine the number of solutions of an equation where simplification is required.	
Students will be required to find the solution of an equation. (b)	
Students will be required to construct an equation given parameters including the solution or number of solutions. (a)	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Fluently solve linear equations and inequalities in one variable. a. Identify linear equations in one variable with one solution, infinitely many solutions, or no solution. b. Identify the solution to linear equations and inequalities with rational number coefficients.	Fluently solve linear equations and inequalities in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solution. b. Solve linear equations and inequalities with rational number coefficients.
Proficient	**Highly Proficient**
Fluently solve linear equations and inequalities in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solution. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations and inequalities with rational number coefficients, including solutions that require expanding expressions using the distributive property and collecting like terms.	Fluently solve linear equations and inequalities in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solution. Explain which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Explain how to solve linear equations and inequalities with rational number coefficients, including solutions that require expanding expressions using the distributive property and collecting like terms.

8.EE.B.8, 8.EE.B.8a, 8.EE.B.8b, 8.EE.B.8c

<table>
<tr><td>Content Standards</td><td colspan="2">8.EE.B.8 Analyze and solve pairs of simultaneous linear equations.

8.EE.B.8a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

8.EE.B.8b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations including cases of no solution and infinite number of solutions. Solve simple cases by inspection.

8.EE.B.8c Solve mathematical problems and problems in real-world context leading to two linear equations in two variables.</td></tr>
<tr><td>Explanations</td><td colspan="2">Systems of linear equations can also have one solution, infinitely many solutions or no solutions. Students will discover these cases as they graph systems of linear equations and solve them algebraically.

A system of linear equations whose graphs meet at one point (intersecting lines) has only one solution, the ordered pair representing the point of intersection. A system of linear equations whose graphs do not meet (parallel lines) has no solutions and the slopes of these lines are the same. A system of linear equations whose graphs are coincident (the same line) has infinitely many solutions, the set of ordered pairs representing all the points on the line.

By making connections between algebraic and graphical solutions and the context of the system of linear equations, students are able to make sense of their solutions. Students need opportunities to work with equations and context that include whole number and/or decimals/fractions.</td></tr>
<tr><td>Content Limits</td><td colspan="2">Rational Numbers

(8a) Should involve a graph</td></tr>
<tr><td>Context</td><td colspan="2">Context is subject to task demand.</td></tr>
<tr><th colspan="2">Sample Task Demands</th><th>Common Item Formats</th></tr>
<tr><td colspan="2">Students will be required to identify the integer solution of a system from a graph. (a) Context is not allowed.</td><td rowspan="5">Equation Response
Graphic Response
Multiple Choice Response
Matching Item Response
Multi-Select Response</td></tr>
<tr><td colspan="2">Students will be required to identify the number of solutions of a system by inspection given the two equations. (b) Context is not allowed.</td></tr>
<tr><td colspan="2">Students will be required to solve a system of two equations. (b) Context is not allowed.</td></tr>
<tr><td colspan="2">Students will be required to graph a system of equations and select an interval in which the x-or y-value of the solution lies. (b) Context is not allowed.</td></tr>
<tr><td colspan="2">Students will be required to solve a problem that can be modeled with a system of equations. (c) Context is required.</td></tr>
</table>

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Analyze and solve pairs of simultaneous linear equations. a. Identify the point of intersection for graphs of two linear equations in two variables. b. Identify solutions to simple systems of equations by inspection. c. Solve mathematical problems using two linear equations in two variables.	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs. b. Estimate solutions to systems of two linear equations in two variables by graphing the equations, including cases of no solution and infinite number of solutions. Solve simple cases by inspection. c. Solve mathematical problems and problems in real-world context using two linear equations in two variables.
Proficient	**Highly Proficient**
Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations, including cases of no solution and infinite number of solutions. Solve simple cases by inspection. c. Solve mathematical problems and problems in real-world contexts leading to two linear equations in two variables.	Analyze and solve pairs of simultaneous linear equations. a. Explain that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and solve solutions by graphing the equations including cases of no solution and infinite number of solutions. Solve simple cases by inspection. c. Solve mathematical problems and problems in real-world context by creating two linear equations in two variables.

Standards for Functions

8.F.A.1

Content Standards	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8.)
Explanations	Define, evaluate, and compare functions.
Content Limits	Function notation is not permitted Graphs should be discrete points and not continuous Distractors for Task Demand 3 should focus on misunderstandings of a function and not on incorrect computations
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify a function or a relation that is not a function, in table or graph form.	• Graphic Response • Multiple Choice Response • Matching Item Response • Table Response
Students will be required to create or complete a function or a relation that is not a function in table or graph form (item requires student to show both a function and a non-function).	
Students will be required to identify a graph of a function given a rule.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify a function rule that assigns to each input exactly one output. (Function notation is not required in Grade 8.)	Generate a function rule that assigns to each input exactly one output. Identify the graph of a function as the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8.)
Proficient	**Highly Proficient**
Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8.)	Explain that a function is a rule that assigns to each input exactly one output. Explain that the graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8.)

8.F.A.2

Content Standards	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
Explanations	Define, evaluate, and compare functions.
Content Limits	Function notation is not permitted Only linear functions Only two functions Examples of properties are rate of change, starting point (y-intercept), and values at specific inputs
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify correct statement(s) comparing properties of two functions presented using different representations.	• Multiple Choice Response • Matching Item Response
Students will be required to identify a linear function that has certain properties when compared with a given function.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify properties of two functions each represented in the same way (algebraically, graphically, numerically in tables, or by verbal descriptions).	Compare properties of two functions each represented in the same way (algebraically, graphically, numerically in tables, or by verbal descriptions).
Proficient	**Highly Proficient**
Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.	Interpret properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

8.F.A.3

Content Standards	Interpret the equation y = mx + b as defining a linear function whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4), and (3,9) which are not on a straight line.*
Explanations	Define, evaluate, and compare functions.
Content Limits	Function notation is not permitted
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to categorize functions represented as equations or graphs as linear or nonlinear.	• Multiple Choice Response • Matching Item Response
Students will be required to categorize functions represented as tables as linear or nonlinear.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify a linear function whose graph is a straight line.	Interpret the equation $y = mx + b$ as defining a linear function whose graph is a straight line.

Proficient	Highly Proficient
Interpret the equation $y = mx + b$ as defining a linear function whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear, because its graph contains the points (1, 1), (2, 4), and (3, 9), which are not on a straight line.	Interpret the equation $y = mx + b$ as defining a linear function whose graph is a straight line; give real-world examples of functions that are not linear.

8.F.B.4

Content Standards	Given a description of a situation, generate a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or a graph. Track how the values of the two quantities change together. Interpret the rate of change and initial value of a linear function in terms of the situation it models, its graph, or its table of values.
Explanations	Use functions to model relationships between quantities.
Content Limits	Function notation is not permitted Limit to linear functions Given equations should always have just the dependent variable on one side of the equation.
Context	Context is subject to task demand.

<table>
<tr><th>Sample Task Demands</th><th>Common Item Formats</th></tr>
<tr><td>Students will be required to determine the rate of change and/or initial value of a linear function from an equation. Context is allowed.</td><td rowspan="5">Equation Response
Graphic Response
Multiple Choice Response
Multi-Select Response
Proposition Response
Table Response</td></tr>
<tr><td>Students will be required to interpret the rate of change and initial value of a linear function in terms of its context. Context is required.</td></tr>
<tr><td>Students will be required to create a linear equation by interpreting a table, a graph, a description, or two ordered pairs of the function. Context is allowed.</td></tr>
<tr><td>Students will be required to determine the rate of change and/or initial value of a linear function from a table, a graph, a description, or two ordered pairs of the function. Context is allowed.</td></tr>
<tr><td>Students will be required to create a linear equation, graph, or table that has a different rate of change and/or initial value when compared with a given function. Context is allowed.</td></tr>
</table>

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Given a description of a situation, identify a function to model a linear relationship between two quantities.	Given a description of a situation, generate a function to model a linear relationship between two quantities. Identify the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or a graph.
Proficient	**Highly Proficient**
Given a description of a situation, generate a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or a graph. Track how the values of the two quantities change together. Interpret the rate of change and initial value of a linear function in terms of the situation it models, its graph, or its table of values.	Given a description of a situation, generate a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or a graph. Interpret how the values of the two quantities change together. Interpret the rate of change and initial value of a linear function in terms of the situation it models, its graph, or its table of values.

8.F.B.5

Content Standards	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
Explanations	Use functions to model relationships between quantities.
Content Limits	Linear and/or nonlinear relationships Graph descriptions traditionally move from left to right Graphs may or may not refer to quantitative measures as well as qualitative, i.e. the axes of graphs may or may not have scales Types of qualitative descriptions can include increasing/decreasing, linear/nonlinear, constant/variable, comparing rates (faster/slower), initial values that depend on the context and axes label, etc.
Context	Context is subject to task demand.

Sample Task Demands	**Common Item Formats**
Students will be required to identify a qualitative description given a graph, or a graph given a qualitative description, with no context. Context is not allowed.	• Graphic Response • Multiple Choice Response
Students will be required to identify a qualitative description given a graph, or a graph given a qualitative description, within a context. Context is allowed.	
Students will be required to construct the graph of a function that matches a given qualitative description. Context is required.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify a graph that exhibits the qualitative features of a function that has been described verbally.	Identify the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
Proficient	**Highly Proficient**
Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	Interpret the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Geometry

8.G.A.1

Content Standards	Verify experimentally the properties of rotations, reflections, and translations. Properties include: lines are taken to lines, line segments are taken to line segments of the same length, angles are taken to angles of the same measure, parallel lines are taken to parallel lines.
Explanations	Students need multiple opportunities to explore the transformation of figures so that they can appreciate that points stay the same distance apart and lines stay at the same angle after they have been rotated, reflected, and/or translated. Students are not expected to work formally with properties of dilations until high school.
Content Limits	The coordinate plane should not be used until 8.G.3. A pre-image and image should not include apostrophe-prime notation as this would give away the identification of similarity and congruence.
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify congruent properties based on a transformation(s).	• Equation Response • Multiple Choice Response • Multi-Select Response
Students will be required to solve a problem based on comparing part of a given shape to the corresponding part of its transformation.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the properties of rotations, reflections, and translations. Properties include: lines are taken to lines, line segments are taken to line segments of the same length, angles are taken to angles of the same measure, parallel lines are taken to parallel lines.	identify experimentally the properties of rotations, reflections, and translations. Properties include: lines are taken to lines, line segments are taken to line segments of the same length, angles are taken to angles of the same measure, parallel lines are taken to parallel lines.
Proficient	**Highly Proficient**
Verify experimentally the properties of rotations, reflections, and translations. Properties include: lines are taken to lines, line segments are taken to line segments of the same length, angles are taken to angles of the same measure, parallel lines are taken to parallel lines.	Prove the properties of rotations, reflections, and translations. Properties include: lines are taken to lines, line segments are taken to line segments of the same length, angles are taken to angles of the same measure, parallel lines are taken to parallel lines.

8.G.A.2

Content Standards	Understand that a two-dimensional figure is congruent to another if one can be obtained from the other by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that demonstrates congruence.
Explanations	Understand congruence and similarity.
Content Limits	The coordinate plane should not be used until 8.G.3. Simply stating "dilation" is not sufficient for identifying a transformation that does not maintain congruence, since dilation by a factor of 1 does maintain congruence
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify a transformation or set of transformations that maintain congruence.	• Multiple Choice Response • Multi-Select Response • Proposition Response
Students will be required to describe a transformation given two congruent figures.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Given two congruent figures, identify a sequence that demonstrates congruence.	Understand that a two-dimensional figure is congruent to another if one can be obtained from the other by a sequence of rotations, reflections, and translations; given two congruent figures, identify a sequence that demonstrates congruence.
Proficient	**Highly Proficient**
Understand that a two-dimensional figure is congruent to another if one can be obtained from the other by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that demonstrates congruence.	Prove that a two-dimensional figure is congruent to another if one can be obtained from the other by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that demonstrates congruence.

8.G.A.3

Content Standards	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
Explanations	Dilation: A dilation is a transformation that moves each point along a ray emanating from a fixed center, and multiplies distances from the center by a common scale factor. In dilated figures, the dilated figure is similar to its pre-image. Translation: A translation is a transformation of an object that moves the object so that every point of the object moves in the same direction as well as the same distance. In a translation, the translated object is congruent to its pre-image. Reflection: A reflection is a transformation that flips an object across a line of reflection (in a coordinate grid the line of reflection may be the x or y axis). In a rotation, the rotated object is congruent to its pre-image. When an object is reflected across the y axis, the reflected x coordinate is the opposite of the pre-image x coordinate. Rotation: A rotated figure is a figure that has been turned about a fixed point. This is called the center of rotation. A figure can be rotated up to 360˚. Rotated figures are congruent to their pre-image figures.
Content Limits	Limit coordinates to integer values of x and y Limit rotations to about the origin Limit dilations to about the centers of shapes, or about the vertices of shapes When a coordinate grid is given, all original figures and transformations, given or not given, should fit onto that coordinate grid.
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify the coordinates of a figure after a given transformation.	• Equation Response • Graphic Response • Multiple Choice Response • Table Response
Students will be required to given a figure and transformation, draw the image or pre-image.	
Students will be required to identify the transformation that has occurred given an image and a pre-image or coordinates.	
Students will be required to given a point (x, y), use coordinate rules to show how that point changes after a transformation or transformations.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the effect of dilations, translations, rotations, and reflections on two-dimensional figures.	Identify the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
Proficient	**Highly Proficient**
Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	Describe and interpret the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

8.G.A.4

Content Standards	Understand that a two-dimensional figure is similar to another if, and only if, one can be obtained from the other by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that demonstrates similarity.
Explanations	Understand congruence and similarity.
Content Limits	Items should not include the coordinate plane as the coordinate plane is needed in 8.G.3. Limited to polygons with up to 7 sides.
Context	Context is not allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to describe a transformation given two similar figures.	• Multiple Choice Response • Multi-Select Response

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Given two similar two-dimensional figures, identify a sequence that demonstrates similarity.	Understand that a two-dimensional figure is similar to another if, and only if, one can be obtained from the other by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, identify a sequence that demonstrates similarity.
Proficient	**Highly Proficient**
Understand that a two-dimensional figure is similar to another if, and only if, one can be obtained from the other by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that demonstrates similarity.	Explain that a two-dimensional figure is similar to another if, and only if, one can be obtained from the other by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that demonstrates similarity.

8.G.A.5

Content Standards	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*
Explanations	Students can informally prove relationships with transversals. Students can informally conclude that the sum of a triangle is 180o (the angle-sum theorem) by applying their understanding of lines and alternate interior angles.
Content Limits	Do not include shapes beyond triangles
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to use line-drawing tool to create angles of specified measure with respect to a given angle on a triangle.	• Equation Response • Graphic Response
Students will be required to use the AA criteria for similar triangles.	
Students will be required to create expressions that represent relationships between angles.	
Students will be required to drag/arrange text options to complete an argument/reasoning about angle measures of a triangle.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Use facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.	Identify facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.
Proficient	**Highly Proficient**
Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals explaining why this is so.	Prove arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

8.G.B.6

Content Standards	Understand the Pythagorean Theorem and its converse.
Explanations	Students should verify, using a model, that the sum of the squares of the legs is equal to the square of the hypotenuse in a right triangle. Students should also understand that if the sum of the squares of the 2 smaller legs of a triangle is equal to the square of the third leg, then the triangle is a right triangle.
Content Limits	For the converse, use only perfect roots
Context	Context is not allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to identify components of a sufficient/insufficient proof of the Pythagorean theorem.	• Graphic Response • HotText Response • Multiple Choice Response • Multi-Select Response • Proposition Response
Students will be required to explain or evaluate a proof of the Pythagorean theorem.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify examples of the application of the converse of the Pythagorean Theorem.	Apply the converse of the Pythagorean Theorem.
Proficient	**Highly Proficient**
Understand the Pythagorean Theorem and its converse.	Prove the converse of the Pythagorean Theorem.

8.G.B.7

Content Standards	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world context and mathematical problems in two and three dimensions.
Explanations	Through authentic experiences and exploration, students should use the Pythagorean Theorem to solve problems. Problems can include working in both two and three dimensions. Students should be familiar with the common Pythagorean triplets.
Content Limits	Given measures should be integers, though answers can be rational
Context	Context is subject to task demand.

Sample Task Demands	**Common Item Formats**
Students will be required to find missing side lengths in a right triangle. Context is not allowed.	• Equation Response • Multiple Choice Response • Multi-Select Response
Students will be required to solve simple real-world problems using the Pythagorean theorem. Context is required.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Apply the Pythagorean Theorem to determine the hypotenuse in right triangles in real-world context and mathematical problems in two dimensions.	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world context and mathematical problems in two dimensions.
Proficient	**Highly Proficient**
Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world contexts and mathematical problems in two and three dimensions.	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world context and mathematical problems in two and three dimensions and interpret the results.

8.G.B.8

Content Standards	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
Explanations	Understand and apply the Pythagorean Theorem.
Content Limits	Points on the coordinate grid should be where grid lines intersect
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to determine the distance between two points on a coordinate grid.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Use the Pythagorean Theorem to find the distance between two points in the first quadrant of a coordinate system.	Use the Pythagorean Theorem to find the distance between two points in a coordinate system.
Proficient	**Highly Proficient**
Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	Apply the Pythagorean Theorem to find the scaled distance between two points in a coordinate system.

8.G.C.9

Content Standards	Understand and use formulas for volumes of cones, cylinders and spheres and use them to solve real-world context and mathematical problems.
Explanations	Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.
Content Limits	Graphics of three-dimensional figures will be included in most items Dimensions are rational numbers Items should not require students to solve quadratic or cubic equations (i.e., find r given a volume) Rubrics should account for different estimations of pi (3.14, 22/7, the calculator button) if necessary
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to use formulas to determine the volume of a cylinder, cone, or sphere.	• Equation Response • Multiple Choice Response • Multi-Select Response
Students will be required to use formulas to determine the volume of composite objects composed of cylinders, cones, and/or spheres, or parts of these objects.	
Students will be required to compare the volumes/heights of cones and cylinders with the same base.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Apply formulas for volumes of cones, cylinders, and spheres.	Understand and use formulas for volumes of cones, cylinders, and spheres.
Proficient	**Highly Proficient**
Understand and use formulas for volumes of cones, cylinders, and spheres and use them to solve real-world context and mathematical problems.	Know and use formulas for volumes of cones, cylinders and spheres and use them to solve real-world context and mathematical problems.

Statistics and Probability & The Number System

8.NS.A.1

Content Standards	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion. Know that numbers whose decimal expansions do not terminate in zeros or in a repeating sequence of fixed digits are called irrational.
Explanations	Students can use graphic organizers to show the relationship between the subsets of the real number system.
Content Limits	All irrational numbers excluding e.
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify numbers that are irrational.	• Equation Response • Multiple Choice Response • Matching Item Response • Multi-Select Response • Proposition Response
Students will be required to convert a repeating decimal into a fraction.	
Students will be required to explain why a number is rational or irrational.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify irrational numbers.	Know that numbers that are not rational are called irrational. Identify a decimal expansion of irrational number.
Proficient	**Highly Proficient**
Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion. Know that numbers whose decimal expansions do not terminate in zeros or in a repeating sequence of fixed digits are called irrational.	Explain that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion. Explain that numbers whose decimal expansions do not terminate in zeros or in a repeating sequence of fixed digits are called irrational.

8.NS.A.2

Content Standards	Use rational approximations of irrational numbers to compare the size of irrational numbers. Locate them approximately on a number line diagram, and estimate their values.
Explanations	Students can approximate square roots by iterative processes.
Content Limits	All real numbers excluding e. Irrational expressions should only use one operation
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify the approximated value of an irrational number.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response
Students will be required to estimate values of expressions that include irrational values.	
Students will be required to plot irrational numbers on a number line.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Approximate irrational numbers on a number line diagram.	Use rational approximations of irrational numbers to compare the size of irrational numbers. Locate them approximately on a number line diagram.
Proficient	**Highly Proficient**
Use rational approximations of irrational numbers to compare the size of irrational numbers. Locate them approximately on a number line diagram, and estimate their values.	Use rational approximations of irrational numbers to compare the size of irrational numbers. Locate them approximately on a number line diagram, and calculate their values.

8.NS.A.3

Content Standards	Understand that given any two distinct **rational** numbers, $a < b$, there exist a rational number c and an irrational number d such that $a < c < b$ and $a < d < b$. Given any two distinct **irrational** numbers, $a < b$, there exist a rational number c and an irrational number d such that $a < c < b$ and $a < d < b$.
Explanations	Understand that there are irrational numbers, and approximate them using rational numbers.
Content Limits	
Context	No Context

Sample Task Demands	Common Item Formats
Students will be required to recognize that there are rational and irrational numbers is between two rational or irrational numbers.	• Multiple Choice Response • Equation Response • Editing Task Choice
Students will be required to identify a rational or irrational number that has a value between two rational or irrational numbers.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Understand that given any two distinct rational numbers, $a < b$, identify a rational number c and an irrational number d such that $a < c < b$ and $a < d < b$.	Understand that given any two distinct rational numbers, $a < b$, identify a rational number c and an irrational number d such that $a < c < b$ and $a < d < b$. Given any two distinct irrational numbers, $a < b$, identify a rational number c and an irrational number d such that $a < c < b$ and $a < d < b$.
Proficient	**Highly Proficient**
Understand that given any two distinct rational numbers, $a < b$, there exist a rational number c and an irrational number d such that $a < c < b$ and $a < d < b$. Given any two distinct irrational numbers, $a < b$, there exists a rational number c and an irrational number d, such that $a < c < b$ and $a < d < b$.	Explain that given any two distinct rational numbers, $a < b$, there exist a rational number c and an irrational number d such that $a < c < b$ and $a < d < b$. Given any two distinct irrational numbers, $a < b$, there exist a rational number c and an irrational number d such that $a < c < b$ and $a < d < b$.

8.SP.A.1

Content Standards	Construct and interpret scatter plots for bivariate measurement data to investigate and describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
Explanations	Students build on their previous knowledge of scatter plots examine relationships between variables. They analyze scatterplots to determine positive and negative associations, the degree of association, and type of association. Students examine outliers to determine if data points are valid or represent a recording or measurement error.
Content Limits	Items at this standard should not require the student to perform calculations using values of data represented on a scatter plot. This will be reserved for High School statistics standards, when the appropriate technology is available. This standard should focus more on recognizing patterns of association.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify patterns of association (clusters, outliers, positive/negative association, linear/nonlinear association) for a scatter plot.	• Graphic Response • Multiple Choice Response • Multi-Select Response
Students will be required to interpret patterns of association found in scatter plots in terms of a given context.	
Students will be required to construct a scatter plot using given data points and interpret patterns therein.	
Students will be required to construct scatter plots given a verbal description of the association.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Construct scatter plots for bivariate measurement data.	Construct scatter plots for bivariate measurement data to investigate and describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
Proficient	**Highly Proficient**
Construct and interpret scatter plots for bivariate measurement data to investigate and describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	Construct and interpret scatter plots for bivariate measurement data to investigate and interpret patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

8.SP.A.2

Content Standards	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
Explanations	Investigate patterns of association in bivariate data.
Content Limits	Rational numbers, trend/association – not based on numbers, only based on visual strength, linear association only For items where student identify/construct a line of best fit, a correct line should not go through the origin - it is a common misconception that lines of best fit must go through the origin, so scatterplots should be given so that a line that goes through the origin is clearly incorrect. For items where the student judges the closeness of the data, the line of best fit should be correct for that data - the student is just judging how close those points are to the line.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to identify an approximate line of best fit for a given scatter plot.	• Graphic Response • Multiple Choice Response • Multi-Select Response
Students will be required to construct an approximate line of best fit.	
Students will be required to compare the accuracy of a model by how closely the data follows the line of best fit for several models.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
For scatter plots that suggest a linear association, informally fit a straight line.	Identify straight lines used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
Proficient	**Highly Proficient**
Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

8.SP.A.3

Content Standards	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.
Explanations	Investigate patterns of association in bivariate data.
Content Limits	Rational numbers Limit to linear equations Students should not be required to create an equation of a line of best fit; if a scatterplot/line of best fit is given, the parameter(s) of interest should also be given.
Context	Context is required.

Sample Task Demands	Common Item Formats
Students will be required to interpret the slope and intercept of a line of best fit, with slope and/or intercept parameter identified, in terms of the context.	• Equation Response • Multiple Choice Response • Multi-Select Response
Students will be required to interpret the slope and intercept of a modeling equation in terms of the context.	
Students will be required to solve problems about the slope and intercept of a line of best fit in terms of the context.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify properties of the equation of a linear model to solve problems in the context of bivariate measurement data.	Use the equation of a linear model to solve problems in the context of bivariate measurement data, identifying the slope and intercept.
Proficient	**Highly Proficient**
Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.	Create an equation for a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

8.SP.A.4

Content Standards	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.
Explanations	Investigate patterns of association in bivariate data.
Content Limits	Relate questions to grand total of survey Categorical variables Two columns (plus category and total) and two rows (plus category and total)
Context	Context is required.

Sample Task Demands	Common Item Formats
Students will be required to interpret and/or compare values in a two-way frequency table.	• Equation Response • Multiple Choice Response • Table Response
Students will be required to complete a two-way table based on given frequencies or relative frequencies.	
Students will be required to relate a two-way relative frequency table to whether there is an association between two variables.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Construct a two-way table summarizing data on two categorical variables collected from the same subjects.	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.
Proficient	**Highly Proficient**
Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.	Explain patterns of association seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

8.SP.B.5, 8.SP.B.5a, 8.SP.B.5b, 8.SP.B.5c

Content Standards	**8.SP.B.5** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. **8.SP.B.5a** Understand that the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. **8.SP.B.5b** Represent sample spaces for compound events using organized lists, tables, tree diagrams and other methods. Identify the outcomes in the sample space which compose the event. **8.SP.B.5c** Design and use a simulation to generate frequencies for compound events.
Explanations	Investigate chance processes and develop, use, and evaluate probability models.
Content Limits	
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Identify the sample space for a compound event given an experimental design or a context.	• Equation Response • Multiple Choice Response • Table Response
Determine the probability of a compound event.	
Use simulations to determine the probability of compound events.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Identify the probability of a compound event. b. Identify sample spaces for compound events using organized lists, tables, tree diagrams and other methods. c. Use a simulation to identify frequencies for compound events.	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Identify the probability of a compound event as the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using organized lists, tables, tree diagrams, and other methods. c. Use a simulation to generate frequencies for compound events.

Proficient	**Highly Proficient**
Find probabilities of compound events using organized lists, tables, tree diagrams, and simulations. a. Understand that the probability of a compound event is the fraction of outcomes in the sample space in which the compound event occurs. b. Represent sample spaces for compound events using organized lists, tables, tree diagrams, and other methods. Identify the outcomes in the sample that composes the event. c. Design and use a simulation to generate frequencies for compound events.	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Explain why the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using organized lists, tables, tree diagrams, and other methods. Identify and interpret the outcomes in the sample space that composes the event. c. Design and use a simulation to generate frequencies for compound events and interpret in context.

AzM2 | Arizona's Statewide Achievement Assessment for English Language Arts and Mathematics

Mathematics Item Specifications

GRADE 10

Arizona Department of Education with American Institutes for Research - 2019

Table of Contents

Introduction

AzM2 is Arizona's statewide achievement test. AzM2 assesses the Arizona English Language Arts Standards and Arizona Mathematics Standards adopted by the Arizona State Board of Education in December 2016. AzM2 will inform students, teachers, and parents about preparedness for college and careers upon graduating from high school. AzM2 tests are computer-based, meaning that they can better assess students' critical thinking skills and provide them with opportunities to demonstrate a deeper understanding of the materials. Computer-based testing also allows for the use of a variety of innovative items types.

During the item-development process, all AzM2 items are written in accordance with the Item Specifications and are reviewed and approved by a committee of Arizona educators to confirm alignment and appropriateness for inclusion in the test. AzM2 items are generally representative of Arizona's geographic regions and culturally diverse population. Items are reviewed for the following kinds of bias: gender, racial, ethnic, linguistic, religious, geographic, and socioeconomic. Item reviews also include consideration of issues related to individuals with disabilities. Arizona community members also have an opportunity to review items for issues of potential concern to members of the community at large. Reviewers are asked to consider the variety of cultural, regional, philosophical, political, and religious backgrounds throughout Arizona, and then to determine whether the subject matter will be acceptable to Arizona students, families, and other members of Arizona communities.

This *AzM2 Item Specifications* is a resource document that defines the content and format of the test and test items for item writers and reviewers. Each *Item Specifications* document indicates the alignment of items with the Arizona Mathematics Standards. It also serves to provide all stakeholders with information about the scope and function of assessment items. This document can also serve to assist educators to understand how assessment items are developed in alignment with the standards for English language arts and math. These item specifications for AzM2 are intended to provide information regarding standards, item formats and response types. The descriptions of blueprints, and depth of knowledge in this document are meant to provide an overview of the test. Item specifications are meant for the purposes of assessment, not instruction. They are not intended to be tools for instruction or the basis for curricula. AzM2 has a test blueprint that was developed by Arizona and is different from any other state or consortium test blueprint.

For the math portion of AzM2, all test questions are aligned to the mathematic content standards for these subject areas. Any item specifications that are absent for standards listed in this document may be under development. This document does not endorse the exclusion of the instruction of any grade-level content standards. The test will ask questions that check a student's conceptual understanding of math as well as their procedural skills. These items have been written to be free from bias and sensitivity, and widely vary in their degree of difficulty.

Item Development Process

AzM2 items go through a rigorous review before they are operational. When an item is "operational" it means it is used to determine a student's score on the assessment. This is a description of the process every item must go through before it is operational on AzM2.

Item Development

AIR and ADE generate potential items for review.

↓

Educator Review

Committee of Arizona Teachers review items for content and bias. All approved items are moved forward.

↓

Parent Review Committee

Arizona parents/community members review items for bias and sensitivity. All approved items move forward.

↓

Field Test

Items are field tested to see how they operate.

↓

Data Review

Field Test items are reviewed for data to ensure they are valid.

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Operational

Field Test items which have made it through all stages are now potentially Operational.

Sample tests are available online for the math portion of AzM2. For more information view the Guide to the Sample Tests at www.AzM2portal.org.

Test Construction Guidelines

The construction of the AzM2 assessment is guided by the depth and rigor of the Arizona College and Career Ready Standards. Items are created to address key components of the standards and assess a range of important skills. The AzM2 Blueprint provides an overview of the distribution of items on the AzM2 according to the standards. The standards for Math Practices are embedded within all AzM2 items. Further, the AzM2 blueprint outlines the Depth of Knowledge distribution of items.

Blueprint

Grade 10 AzM2 Blueprint 2016 Standards		
Reporting Category	**Min.**	**Max.**
Algebra	20%	28%
Functions	16%	20%
Statistics and Quantitative Reasoning	16%	20%
Congruence & Geometric Properties with Equations	18%	22%
Similarity, Right Triangles and Trigonometry & Circles and Geometric Measurement	18%	22%

Within a test, approximately 70% of the assessment will be on major content within that grade or course.

Percentage of Points by Depth of Knowledge Level	
DOK 1	10% - 20%
DOK 2	53% - 63%
DOK 3	19% - 37%

Depth of Knowledge (DOK)

DOK refers to the level of rigor or sophistication of the task in a given item, designed to reflect the complexity of the Arizona Mathematics Standards. Items at DOK level 1 focus on the recall of information, such as definitions, terms, and simple procedures. Items at DOK 2 require students to make decisions, solve problems, or recognize patterns; in general, they require a greater degree of engagement and cognitive processing than items at DOK 1. Items at DOK 3 feature higher-order cognitive tasks that assess students' capacities to approach abstract or complex problems.

Calculators

Arizona Desmos Graphing Calculator is permitted for both the paper-based and computer-based assessment for High School Math.

Item Formats

The AzM2 Assessments are composed of item formats that include traditional multiple-choice response items and technology-enhanced response items (TEI). TEIs are computer-delivered response items that require students to interact with test content to select, construct, and/or support their responses. TEIs are better able to assess a deeper level of understanding.

Currently, there are nine types of TEIs that may appear on the Math computer-based assessment for AzM2:

- Editing Tasks (ET)
- Editing Task Choice (ETC)
- Equation Editor (EQ)
- Graphic Response Item Display (GRID)
- Hot Text (HT)
    - Selectable Hot Text
    - Drag-and-Drop Hot Text
- Matching Item (MI)
- Multi-Select (MS)
- Open Response
- Table Item (TI)

For paper-based assessments (including those for students with an IEP or 504 plan that specifies a paper-based accommodation), TEIs will be modified so that they can be scanned and scored electronically or hand-scored.

See the table below for a description of each TEI. In addition, for examples of each response item format described, see the AzM2 Training Tests at www.AzM2portal.org.

Item Format	Description
Editing Task (ET)	The student clicks on a highlighted word or phrase that may be incorrect, which reveals a text box. The directions in the text box direct the student to replace the highlighted word or phrase with the correct word or phrase. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Editing Task Choice (ETC)	The student clicks a highlighted word or phrase, which reveals a drop-down menu containing options for correcting an error as well as the highlighted word or phrase as it is shown in the sentence to indicate that no correction is needed. The student then selects the correct word or phrase from the drop-down menu. For paper-based assessments, the item is modified so that it can be scanned and scored electronically. The student fills in a circle to indicate the correct word or phrase.

Item Format	Description
Equation Editor (EQ)	The student is presented with a toolbar that includes a variety of mathematical symbols that can be used to create a response. Responses may be in the form of a number, variable, expression, or equation, as appropriate to the test item. For paper-based assessments, this item type may be replaced with a modified version of the item that can be scanned and scored electronically or replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Graphic Response Item Display (GRID)	The student selects numbers, words, phrases, or images and uses the drag-and-drop feature to place them into a graphic. This item type may also require the student to use the point, line, or arrow tools to create a response on a graph. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Hot Text (HT)	**Selectable Hot Text -** Excerpted sentences from the text are presented in this item type. When the student hovers over certain words, phrases, or sentences, the options highlight. This indicates that the text is selectable ("hot"). The student can then click on an option to select it. For paper- based assessments, a "selectable" hot text item is modified so that it can be scanned and scored electronically. In this version, the student fills in a circle to indicate a selection.
	Drag-and-Drop Hot Text - Certain numbers, words, phrases, or sentences may be designated "draggable" in this item type. When the student hovers over these areas, the text highlights. The student can then click on the option, hold down the mouse button, and drag it to a graphic or other format. For paper-based assessments, drag-and-drop hot text items will be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Matching Item (MI)	The student checks a box to indicate if information from a column header matches information from a row. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Multi-Select (MS)	The student is directed to select all of the correct answers from among a number of options. These items are different from multiple-choice items, which allow the student to select only one correct answer. These items appear in the online and paper-based assessments.
Open Response	The student uses the keyboard to enter a response into a text field. These items can usually be answered in a sentence or two. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.

Item Format	Description
Table Item (TI)	The student types numeric values into a given table. The student may complete the entire table or portions of the table depending on what is being asked. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.

Arizona Mathematics Standards Algebra I

Number and Quantity - N		
The Real Number System (N-RN)		
A1.N-RN.B **Use properties of rational and irrational numbers.**	**A1.N-RN.B.3**	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
Quantities (N-Q)		
A1.N-Q.A **Reason quantitatively and use units to solve problems.**	**A1.N-Q.A.1**	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays, include utilizing real-world context.
	A1.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling. Include problem-solving opportunities utilizing real-world context.
	A1.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities utilizing real-world context.
Algebra - A		
Seeing Structure in Expressions (A-SSE)		
A1.A-SSE.A **Interpret the structure of expressions.**	**A1.A-SSE.A.1**	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret expressions by viewing one or more of their parts as a single entity.
	A1.A-SSE.A.2	Use structure to identify ways to rewrite numerical and polynomial expressions. Focus on polynomial multiplication and factoring patterns.
A1.A-SSE.B **Write expressions in equivalent forms to solve problems.**	**A1.A-SSE.B.3**	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines.

		b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
Arithmetic with Polynomials and Rational Expressions (A-APR)		
A1.A-APR.A **Perform arithmetic operations on polynomials.**	**A1.A-APR.A.1**	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
A1.A-APR.B **Understand the relationship between zeros and factors of polynomials.**	**A1.A-APR.B.3**	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. Focus on quadratic and cubic polynomials in which linear and quadratic factors are available.
Creating Equations (A-CED)		
A1.A-CED.A **Create equations that describe numbers or relationships.**	**A1.A-CED.A.1**	Create equations and inequalities in one variable and use them to solve problems. Include problem-solving opportunities utilizing real-world context. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).
	A1.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
	A1.A-CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
	A1.A-CED.A.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*
Reasoning with Equations and Inequalities (A-REI)		
A1.A-REI.A **Understand solving equations as a process of reasoning and explain the reasoning.**	**A1.A-REI.A.1**	Explain each step in solving linear and quadratic equations as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
A1.REI.B **Solve equations and inequalities in one variable.**	**A1.A-REI.B.3**	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
	A1.A-REI.B.4	Solve quadratic equations in one variable.

		a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - k)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Focus on solutions for quadratic equations that have real roots. Include cases that recognize when a quadratic equation has no real solutions.
A1.A-REI.C **Solve systems of equations.**	**A1.A-REI.C.5**	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
	A1.A-REI.C.6	Solve systems of linear equations exactly and approximately, focusing on pairs of linear equations in two variables. Include problem solving opportunities utilizing real-world context.
A1.A-REI.D **Represent and solve equations and inequalities graphically.**	**A1.A-REI.D.10**	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve, which could be a line.
	A1.A-REI.D.11	Explain why the x-coordinates of the points where the graphs of the equations $y=f(x)$ and $y=g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately (e.g., using technology to graph the functions, make tables of values, or find successive approximations). Focus on cases where $f(x)$ and/or $g(x)$ are linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).
	A1.A-REI.D.12	Graph the solutions to a linear inequality in two variables as a half-plane, excluding the boundary in the case of a strict inequality, and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
Functions - F		
Interpreting Functions (F-IF)		
A1.F-IF.A **Understand the concept of a function and use function notation.**	**A1.F-IF.A.1**	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then $f(x)$ denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation $y = f(x)$.

	A1.F-IF.A.2	Evaluate a function for inputs in the domain, and interpret statements that use function notation in terms of a context.
	A1.F-IF.A.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
A1.F-IF.B **Interpret functions that arise in applications in terms of the context**	**A1.F-IF.B.4**	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Include problem-solving opportunities utilizing real-world context. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).
	A1.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
A1.F-IF.B (cont.)	**A1.F-IF.B.6**	Calculate and interpret the average rate of change of a continuous function (presented symbolically or as a table) on a closed interval. Estimate the rate of change from a graph. Include problem-solving opportunities utilizing real-world context. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).
A1.F-IF.C **Analyze functions using different representations.**	**A1.F-IF.C.7**	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).
	A1.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square of a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
	A1.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

		Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).
Building Functions (F-BF)		
A1.F-BF.A **Build a function that models a relationship between two quantities.**	**A1.F-BF.A.1**	Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from real-world context. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).
A1.F-BF.B **Build new functions from existing functions.**	**A1.F-BF.B.3**	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, and $f(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).
Linear, Quadratic, and Exponential Models (F-LE)		
A1.F-LE.A **Construct and compare linear, quadratic, and exponential models and solve problems.**	**A1.F-LE.A.1**	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
A1.F-LE.A (cont.)	**A1.F-LE.A.2**	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or input/output pairs.
	A1.F-LE.A.3	Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.
A1.F-LE.B **Interpret expressions for functions in terms of the situation they model.**	**A1.F-LE.B.5**	Interpret the parameters in a linear or exponential function with integer exponents utilizing real world context.

Statistics and Probability - S		
Summarize, represent, and interpret data on a single count or measurement variable. (S-ID)		
A1.S-ID.A **Summarize, represent, and interpret data on a single count or measurement variable.**	**A1.S-ID.A.1**	Represent real-value data with plots for the purpose of comparing two or more data sets.
	A1.S-ID.A.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
	A1.S-ID.A.3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of outliers if present.
A1.S-ID.B **Summarize, represent, and interpret data on two categorical and quantitative variables.**	**A1.S-ID.B.5**	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data, including joint, marginal, and conditional relative frequencies. Recognize possible associations and trends in the data.
	A1.S-ID.B.6	Represent data on two quantitative variables on a scatter plot and describe how the quantities are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Focus on linear models. b. Informally assess the fit of a function by plotting and analyzing residuals.
A1.S-ID.C **Interpret linear models.**	**A1.S-ID.C.7**	Interpret the slope as a rate of change and the constant term of a linear model in the context of the data.
	A1.S-ID.C.8	Compute and interpret the correlation coefficient of a linear relationship.
	A1.S-ID.C.9	Distinguish between correlation and causation.
Conditional Probability and the rules of Probability (S-CP)		
A1.S-CP.A **Understand independence and conditional probability and use them to interpret data.**	**A1.S-CP.A.1**	Describe events as subsets of a sample space using characteristics of the outcomes, or as unions, intersections, or complements of other events.
	A1.S-CP.A.2	Use the Multiplication Rule for independent events to understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities and use this characterization to determine if they are independent.

A1.N-RN.B.3

Content Standards	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
Explanations	Since every difference is a sum and every quotient is a product, this includes differences and quotients as well. Explaining why the four operations on rational numbers produce rational numbers can be a review of students understanding of fractions and negative numbers. Explaining why the sum of a rational and an irrational number is irrational, or why the product is irrational, includes reasoning about the inverse relationship between addition and subtraction (or between multiplication and addition).
Content Limits	This standard is aligned to Algebra I only. For products, can include [irrational number] x 0 as rational.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to given sums/products of numbers, identify which are rational and which are irrational.	• Multiple Choice Response • Multi-Select Response
Students will be required to justify why the sums/products of two rational numbers, two irrational numbers, and one irrational and one rational numbers are necessarily rational or irrational.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient

Recognize that the sum or product of two rational numbers is rational.	Recognize that the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational.
Proficient	**Highly Proficient**
Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	Generalize and develops rules for the sum or product of two rational numbers being rational; the sum of a rational number and an irrational number being irrational; and the product of a nonzero rational number and an irrational number being irrational.

Quantities (N-Q)

A1.N-Q.A.1

Content Standards	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays, include utilizing real-world context.
Explanations	Include word problems where quantities are given in different units, which must be converted to make sense of the problem. Graphical representations and data displays include, but are not limited to: line graphs, circle graphs, histograms, multi-line graphs, scatterplots, and multi-bar graphs.
Content Limits	Rational numbers Linear equations and graph Exponential equations and graphs Customary and metric units of measure
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Given a solution, students will determine the correct units based on the context.	• Equation Response • Multiple Choice Response • Multiple Select Response • Editing Task Choice
Students will use dimensional analysis to convert one unit to another in order provide a solution within a real-world situation.	
Students will convert between different units in order to determine the solution for a real-world problem.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**

Identify units for the solution of multi-step problems; Identify units consistently in formulas; Identify the scale and the origin in graphs and data displays, include utilizing real-world context.	Choose units for the solution of multi-step problems; choose units consistently in formulas; choose the scale and the origin in graphs and data displays, include utilizing real-world context.
Proficient	**Highly Proficient**
Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays, include utilizing real-world context.	Use units as a way to understand problems and to justify the solution of multi-step problems; choose and interpret units consistently in formulas; interpret and explain the scale and the origin in graphs and data displays, include utilizing real-world context.

A1.N-Q.A.2

Content Standards	Define appropriate quantities for the purpose of descriptive modeling. Include problem-solving opportunities utilizing real-world context.
Explanations	Reason quantitatively and use units to solve problems.
Content Limits	Linear and exponential models.
Context	Context is required.

Sample Task Demands	**Common Item Formats**
Students will be required to use quantities appropriate to the context to solve problems.	• Equation Response • Editing Task Choice • Multiple Choice Response

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify appropriate quantities for the purpose of descriptive modeling.	Define appropriate quantities for the purpose of descriptive modeling.
Proficient	**Highly Proficient**
Define appropriate quantities for the purpose of descriptive modeling. Include problem-solving opportunities utilizing real-world context.	Define and use appropriate quantities for the purpose of descriptive modeling. Include problem-solving opportunities utilizing real-world context.

A1.N-Q.A.3

Content Standards	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities utilizing real-world context.
Explanations	The margin of error and tolerance limit varies according to the measure, tool used, and context.
Content Limits	
Context	Context is required.

Sample Task Demands	Common Item Formats
Students will be required to choose a level of accuracy when reporting quantities in a real-world context.	• Editing Task Choice • Equation Response • Multiple Choice Response • Multi-Select Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify a level of accuracy on measurement when reporting quantities utilizing real-world context.	Identify a level of accuracy appropriate to limitations on measurement when reporting quantities utilizing real-world context.
Proficient	**Highly Proficient**
Choose a level of accuracy appropriate to limitations on measurement when reporting quantities utilizing real-world context.	Compare the levels of accuracy appropriate to limitations on measurement when reporting quantities utilizing real-world context.

Seeing Structure in Expressions (A-SSE)

A1.A-SSE.A.1, A1.A-SSE.A.1a, A1.A-SSE.A.1b

Content Standards	A1.A-SSE.A.1 Interpret expressions that represent a quantity in terms of its context. A1.A-SSE.A.1a Interpret parts of an expression, such as terms, factors, and coefficients. A1.A-SSE.A.1b Interpret expressions by viewing one or more of their parts as a single entity.
Explanations	Students should understand the vocabulary for the parts that make up the whole expression and be able to identify those parts and interpret their meaning in terms of a context.
Content Limits	This standard is aligned to Algebra I only. Focus on factors and coefficients of simpler expressions for A-SSE.A.1a. Focus on termos of complicated expressions for A-SSE.A.1b.
Context	Context is required.

Sample Task Demands	**Common Item Formats**
Students will be required to select the meaning for part of a given expression.	• Multiple Choice Response
Students will be required to identify what part of a given expression has a given meaning.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Interpret expressions that represent a quantity in terms of its context. a. Identify parts of an expression, such as terms, factors, and coefficients. b. Match expressions by viewing one or more of their parts as a single entity.	Interpret expressions that represent a quantity in terms of its context. a. Define parts of an expression, such as terms, factors, and coefficients. b. Use expressions by viewing one or more of their parts as a single entity.
Proficient	**Highly Proficient**
Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret expressions by viewing one or more of their parts as a single entity.	Interpret expressions that represent a quantity in terms of its context. a. Differentiate parts of an expression, such as terms, factors, and coefficients. b. Make observations about expressions by viewing one or more of their parts as a single entity.

A1.A-SSE.A.2

Content Standards	Use structure to identify ways to rewrite numerical and polynomial expressions. Focus on polynomial multiplication and factoring patterns.
Explanations	Students should extract the greatest common factor (whether a constant, a variable, or a combination of each). If the remaining expression is quadratic, students should factor the expression further.
Content Limits	Numerical expressions and polynomial expression in one variable The given expression must be in a form that allows students to use the structure to identify an equivalent expression - not simply using properties of operations.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify an equivalent expression.	• Equation Response • Multiple Choice Response • Multi-Select Response
Students will be required to construct a new equivalent expression from a given expression.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify equivalent numerical and polynomial expressions. Focus on polynomial multiplication patterns.	Identify ways to rewrite equivalent numerical and polynomial expressions. Focus on polynomial multiplication and factoring patterns.
Proficient	**Highly Proficient**

Use structure to identify ways to rewrite numerical and polynomial expressions. Focus on polynomial multiplication and factoring patterns.	Assess ways to rewrite numerical and polynomial expressions. Focus on polynomial multiplication and factoring patterns.

A1.A-SSE.B.3, A1.A-SSE.B.3a, A1.A-SSE.B.3b

Content Standards	A1.A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. A1.A-SSE.B.3a Factor a quadratic expression to reveal the zeros of the function it defines. A1.A-SSE.B.3b Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
Explanations	Students will use the properties of operations to create equivalent expressions.
Content Limits	This standard is aligned to Algebra I only. Quadratic expressions The item must require factoring as the solution method for A-SSE.B.3a. The item must require completing the square as a solution method for A-SSE.B.3b.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify the zeros of a function given in factored form.	• Equation Response • Multiple Choice Response
Students will be required to identify the factored form of a quadratic expression.	

Content Standards	A1.A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. A1.A-SSE.B.3a Factor a quadratic expression to reveal the zeros of the function it defines. A1.A-SSE.B.3b Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	
Students will be required to identify the factored form of a quadratic expression and the zeroes of the function it defines.		
Students will be required to identify the maximum or minimum of a quadratic expression in vertex form.		
Students will be required to identify the vertex form of a quadratic expression.		
Students will be required to identify the vertex form of a quadratic expression and the max/min of the function it defines.		

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Identify a factored quadratic expression that reveals the zeros of the function it defines. b. Identify a quadratic expression that reveals the maximum or minimum value of the function it defines.	a. Use a factored quadratic expression that reveals the zeros of the function it defines. b. Use a quadratic expression that reveals the maximum or minimum value of the function it defines.
Proficient	**Highly Proficient**
Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Explain conditions for the zeros of a quadratic function. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines and use it to solve problems

Arithmetic with Polynomials & Rational Expressions (A-APR)

A1.A-APR.A.1

Content Standards	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
Explanations	Perform arithmetic operations on polynomials.
Content Limits	This standard is aligned to Algebra I only.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to calculate the sum, difference or product of polynomials.	• Multiple Choice Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Add and subtract polynomials.	Add, subtract, and multiply polynomials.
Proficient	**Highly Proficient**
Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	Explain that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

A1.A-APR.B.3

Content Standards	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. Focus on quadratic and cubic polynomials in which linear and quadratic factors are available.
Explanations	Understand the relationship between zeros and factors of polynomials.
Content Limits	Quadratic and cubic polynomials in which linear and quadratic factors are available
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify the zeroes of a polynomial.	• Equation Response • Multiple Choice Response • Multi-Select Response
Students will be required to given a polynomial, determine its graph.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify zeros of polynomials when suitable factorizations are available. Focus on quadratic and cubic polynomials in which linear and quadratic factors are available.	Use the zeros of polynomials to construct a rough graph of the function defined by the polynomial. Focus on quadratic and cubic polynomials in which linear and quadratic factors are available.
Proficient	**Highly Proficient**
Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the

Content Standards	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. Focus on quadratic and cubic polynomials in which linear and quadratic factors are available.

Focus on quadratic and cubic polynomials in which linear and quadratic factors are available.	polynomial. Focus cubic polynomials in which quadratic factors are available.

Creating Equations (A-CED)

A1.A-CED.A.1

Content Standards	Create equations and inequalities in one variable and use them to solve problems. Include problem-solving opportunities utilizing real-world context. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).
Explanations	Equations can represent real world and mathematical problems. Include equations and inequalities that arise when comparing the values of two different functions, such as one describing linear growth and one describing exponential growth.
Content Limits	Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).
Context	Context is subject to task demand.

Sample Task Demands	Common Item Formats
Students will be required to identify the solution for a given equation or inequality. Context is not allowed.	• Equation Response • Multiple Choice Response
Students will be required to construct an equation or inequality to model a context. Context is required.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify equations and inequalities in one variable that can be used to solve problems. Include problem-solving opportunities utilizing real-world context. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).	Use equations and inequalities in one variable to solve problems. Include problem-solving opportunities utilizing real-world context. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).

Content Standards	Create equations and inequalities in one variable and use them to solve problems. Include problem-solving opportunities utilizing real-world context. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).

Proficient	**Highly Proficient**
Create equations and inequalities in one variable and use them to solve problems. Include problem-solving opportunities utilizing real-world context. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).	Analyze equations and inequalities in one variable and use them to solve problems. Include problem-solving opportunities utilizing real-world context. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).

A1.A-CED.A.2

Content Standards	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
Explanations	Create equations that describe numbers or relationships.
Content Limits	This standard is aligned to Algebra I only. Students must be required to construct an equation and/or graph given equations.
Context	Context is subject to task demand.

Sample Task Demands	**Common Item Formats**
Students will be required to identify the solution for an equation. Context is not allowed.	• Equation Response • Graphic Response • Multiple Choice Response
Students will be required to construct a graphical representation of an equation. Context is not allowed.	
Students will be required to construct an equation to represent a context. Context is required.	
Students will be required to construct an equation and identify a solution. Context is required.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**

Content Standards	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
Identify equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	Use equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
Proficient	**Highly Proficient**
Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	Analyze equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A1.A-CED.A.3

Content Standards	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
Explanations	Create equations that describe numbers or relationships.
Content Limits	This standard is aligned to Algebra I only.
Context	Context is required.

Sample Task Demands	Common Item Formats
Students will be required to given a constraint or set of constraints, identify possible solutions.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response
Students will be required to construct a graphical representation of a constraint or set of constraints.	
Students will be required to create or identify a constraint or set of constraints given a context.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify constraints of equations or inequalities, and of systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.	Apply constraints of equations or inequalities, and of systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
Proficient	**Highly Proficient**

Content Standards	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.	Justify constraints of equations or inequalities, and by systems of equations and/or inequalities, and justify solutions as viable or non-viable options in a modeling context.

A1.A-CED.A.4

Content Standards	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*
Explanations	Create equations that describe numbers or relationships.
Content Limits	This standard is aligned to Algebra I only. The student must be provided an equation. Generally, if the equation to be created is very complex, consider using multiple choice response rather than equation response.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to given an equation, identify or create a form of that equation solved for a specific variable.	• Equation Response • Multiple Choice Response
Students will be required to given an equation, describe how one quantity changes when another changes (ex. Given $V = IR$, how does I change if R is doubled and V remains constant?).	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify formulas that highlight a quantity of interest, using the same reasoning as in solving equations.	Apply formulas that highlight a quantity of interest, using the same reasoning as in solving equations.
Proficient	**Highly Proficient**

Content Standards	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*
Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*	Rearrange and apply formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Reasoning with Equations and Inequalities (A-REI)

A1.A-REI.A.1

Content Standards	Explain each step in solving linear and quadratic equations as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
Explanations	Properties of operations can be used to change expressions on either side of the equation to equivalent expressions. In addition, adding the same term to both sides of an equation or multiplying both sides by a non-zero constant produces an equation with the same solutions. Other operations, such as squaring both sides, may produce equations that have extraneous solutions.
Content Limits	Linear and quadratic equations
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to justify a next step in a solution process (i.e., "commutative property", etc.).	• Equation Response • Graphic Response • Multiple Choice Response
Students will be required to identify a correct next step in a solution process.	
Students will be required to given a series of steps in an attempt to solve an equation identify the error(s) and the correct solution.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**

Identify each step in solving linear and quadratic equations as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution.	Carry out each step in solving linear and quadratic equations as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Identify a viable argument to justify a solution method.
Proficient	**Highly Proficient**
Explain each step in solving linear and quadratic equations as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	Critique each step in solving linear and quadratic equations as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A1.A-REI.B.3

Content Standards	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
Explanations	Solve equations and inequalities in one variable.
Content Limits	This standard is aligned to Algebra I only. Equations must be given to the student.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to solve equations or inequalities from context or no context.	• Equation Response • Graphic Response • Multiple Choice Response
Students will be required to graph the solution of an inequality on a number line.	
Students will be required to analyze and solve equations or inequalities with unknown constant coefficients.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Solve one-step and two-step linear equations and inequalities in one variable, including equations with coefficients represented by letters.	Solve two- step linear equations and inequalities in one variable, including equations with coefficients represented by letters.
Proficient	**Highly Proficient**
Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	Compare different methods to solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

A1.A-REI.B.4, A1.A-REI.B.4a, A1.A-REI.B.4b

Content Standards	A1.A-REI.B.4 Solve quadratic equations in one variable. A1.A-REI.B.4a Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x-k)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. A1.A-REI.B.4b Solve quadratic equations by inspection (e.g., $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Focus on solutions for quadratic equations that have real roots. Include cases that recognize when a quadratic equation has no real solutions.
Explanations	Students should solve by factoring, completing the square, and using the quadratic formula. The zero product property is used to explain why the factors are set equal to zero. Students should relate the value of the discriminant to the type of root to expect. A natural extension would be to relate the type of solutions to $ax^2 + bx + c = 0$ to the behavior of the graph of $y = ax^2 + bx + c$.
Content Limits	This standard is aligned to Algebra I only. Quadratics with real solutions.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to create equivalent quadratic equations in the form $(x-p)^2 = q$.	• Equation Response • Multiple Choice Response • Multi-Select
Students will be required to solve quadratic equations.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Solve quadratic equations in one variable. a. Identify the quadratic formula. b. Solve quadratic equations by inspection (e.g., $x^2 = 49$), taking square roots, as appropriate to the initial form of the equation. Focus on solutions for quadratic equations that have real roots. Include cases that recognize when a quadratic equation has no real solutions.	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - k)^2 = q$ that has the same solutions where $q = 0$. Use the quadratic formula. b. Solve quadratic equations by inspection (e.g., $x^2 = 49$), taking square roots, the quadratic formula and factoring, as appropriate to the initial form of the equation. Focus on solutions for quadratic equations that have real roots. Include cases that recognize when a quadratic equation has no real solutions.
Proficient	**Highly Proficient**
Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - k)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Focus on solutions for quadratic equations that have real roots. Include cases that recognize when a quadratic equation has no real solutions.	Solve quadratic equations in one variable. a. Derive the quadratic formula. b. Determine whether to solve quadratic equations by inspection (e.g., $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Focus on solutions for quadratic equations that have real roots. Include cases that recognize when a quadratic equation has no real solutions.

A1.A-REI.C.5

Content Standards	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
Explanations	Solve systems of equations.
Content Limits	This standard is aligned to Algebra I only. Linear systems.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to given a system of equations, identify another system that has the same solutions (based on the process described in the standard).	• Multiple Choice Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Understand that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	Explain that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
Proficient	**Highly Proficient**
Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	Given two systems of two equations in two variables, verify that they have the same solutions by replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A1.A-REI.C.6

Content Standards	Solve systems of linear equations exactly and approximately, focusing on pairs of linear equations in two variables. Include problem solving opportunities utilizing real-world context.
Explanations	The system solution methods can include but are not limited to graphical, elimination/linear combination, substitution, and modeling. Systems can be written algebraically or can be represented in context.
Content Limits	Linear systems with exact solutions and limited calculations. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution)
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to given the graph of a system of equations, identify a possible solution.	• Equation Response • Graphic Response • Multiple Choice Response
Students will be required to solve a system of equations.	
Students will be required to graph a system of equations and identify an approximate solution.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Solve systems of linear equations approximately, focusing on pairs of linear equations in two variables.	Solve systems of linear equations approximately, focusing on pairs of linear equations in two variables. Include problem solving opportunities utilizing real-world context.

Content Standards	Solve systems of linear equations exactly and approximately, focusing on pairs of linear equations in two variables. Include problem solving opportunities utilizing real-world context.

Proficient	**Highly Proficient**
Solve systems of linear equations exactly and approximately, focusing on pairs of linear equations in two variables. Include problem solving opportunities utilizing real-world context.	Analyzes a system of linear equations exactly and approximately, focusing on pairs of linear equations in two variables. Include problem solving opportunities utilizing real-world context.

A1.A-REI.D.10

Content Standards	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve, which could be a line.
Explanations	Represent and solve equations and inequalities graphically.
Content Limits	This standard is aligned to Algebra I only. Linear and exponential equations
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify coordinates of points that lie on the graph of a given equation.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response
Students will be required to plot points that are solutions to a given equation.	
Students will be required to identify other possible solutions to a given equation, type of equation, and/or the graph of a solution to the equation.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the graph of an equation in two variables.	Identify a solution given the graph of an equation in two variables.
Proficient	**Highly Proficient**

Content Standards	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve, which could be a line.
Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve, which could be a line.	Explain that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve, which could be a line.

A1.A-REI.D.11

Content Standards	Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately (e.g., using technology to graph the functions, make tables of values, or find successive approximations). Focus on cases where $f(x)$ and/or $g(x)$ are linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).
Explanations	Students need to understand that numerical solution methods (data in a table used to approximate an algebraic function) and graphical solution methods may produce approximate solutions, and algebraic solution methods produce precise solutions that can be represented graphically or numerically.
Content Limits	Focus on cases where $f(x)$ and/or $g(x)$ are linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step). Note that this standard is not about systems, but about the solution(s) to $f(x) = g(x)$; thus, solutions should be values of x.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to identify the solution(s) to $f(x) = g(x)$, given the graph of the two functions.	• Equation Response • Graphic Response • Multiple Choice Response • Proposition Response
Students will be required to identify the solutions to $f(x) = g(x)$.	
Students will be required to identify a possible $g(x)$, given $f(x)$ and the value(s) of x where $f(x) = g(x)$.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect as the solutions of the equation $f(x) = g(x)$. Focus on cases where $f(x)$ and/or $g(x)$ are linear.	Identify the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect as the solutions of the equation $f(x) = g(x)$; find the solutions approximately (e.g., using technology to graph the functions, make tables of values, or find successive approximations). Focus on cases where $f(x)$ and/or $g(x)$ are linear and exponential functions.
Proficient	**Highly Proficient**
Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately (e.g., using technology to graph the functions, make tables of values, or find successive approximations). Focus on cases where $f(x)$ and/or $g(x)$are linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).	Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions exactly (e.g., using technology to graph the functions, make tables of values, or find successive approximations). Focus on cases where $f(x)$ and/or $g(x)$ are linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).

A1.A-REI.D.12

Content Standards	Graph the solutions to a linear inequality in two variables as a half-plane, excluding the boundary in the case of a strict inequality, and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
Explanations	Represent and solve equations and inequalities graphically.
Content Limits	This standard is aligned to Algebra I only.
Context	Context is allowed.

<table>
<tr><th>Sample Task Demands</th><th>Common Item Formats</th></tr>
<tr><td>Students will be required to select the solution region for a system of inequalities.</td><td rowspan="4">Graphic Response
Multiple Choice Response</td></tr>
<tr><td>Students will be required to graph the boundary for a non-strict inequality and drag a symbol to show the solution set.</td></tr>
<tr><td>Students will be required to graph the boundaries for a system of non-strict inequalities and drag a symbol to show the solution set.</td></tr>
<tr><td>Students will be required to identify the graph and solution set for a system of non-strict inequalities.</td></tr>
</table>

Performance Level Descriptors

Minimally Proficient	Partially Proficient

Content Standards	Graph the solutions to a linear inequality in two variables as a half-plane, excluding the boundary in the case of a strict inequality, and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	

Identify a solution to a linear inequality in two variables as a half-plane, excluding the boundary in the case of a strict inequality.	Graph the solutions to a linear inequality in two variables as a half-plane, excluding the boundary in the case of a strict inequality.
Proficient	**Highly Proficient**
Graph the solutions to a linear inequality in two variables as a half-plane, excluding the boundary in the case of a strict inequality, and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	Create a system of linear inequalities given a graph of the solution set.

Functions- Interpreting Functions (F-IF)

A1.F-IF.A.1

Content Standards	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
Explanations	The domain of a function given by an algebraic expression, unless otherwise specified, is the largest possible domain.
Content Limits	This standard is aligned to Algebra I only.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to recognize functions.	• Multiple Choice Response • Matching Item Response

Content Standards	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).	
Students will be required to create or complete examples of functions and nonfunctions.	• Multi-Select Response • Proposition Response • Table Response	
Students will be required to explain why a relation is or is not a function.		

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Understand that the graph of *f* is the graph of the equation *y* = *f*(*x*).	Understand that if f is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).
Proficient	**Highly Proficient**
Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).	Create a function or non-function based on understanding that a function from the domain to the range assigns to each element of the domain exactly one element of the range.

A1.F-IF.A.2

Content Standards	Evaluate a function for inputs in the domain, and interpret statements that use function notation in terms of a context.
Explanations	The domain of a function given by an algebraic expression, unless otherwise specified, is the largest possible domain.
Content Limits	This standard is aligned to Algebra I only. Linear, quadratic, and exponential functions
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to recognize and identify input or output values from the table of a function.	• Equation Response • Graphic Response • HotText Response • Multiple Choice Response • Table Response
Students will be required to recognize correct uses of function notation.	
Students will be required to complete a table of input and output values for a given function.	
Students will be required to interpret statements that use function notation in terms of a context.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Evaluate a function for an input in the domain.	Evaluate a function for inputs in the domain.
Proficient	**Highly Proficient**

Content Standards	Evaluate a function for inputs in the domain, and interpret statements that use function notation in terms of a context.
Evaluate a function for inputs in the domain, and interpret statements that use function notation in terms of a context.	Evaluate a function for inputs in the domain, and apply statements that use function notation in terms of a context.

A1.F-IF.A.3

Content Standards	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
Explanations	Understand the concept of a function and use function notation.
Content Limits	Linear or exponential Limit sequence representations to rational values
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to construct a function to model a sequence.	• Equation Response

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify sequences or functions defined recursively, whose domain is a subset of the integers.	Use sequences or functions defined recursively, whose domain is a subset of the integers.
Proficient	**Highly Proficient**
Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	Create a function defined recursively.

A1.F-IF.B.4

Content Standards	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Include problem-solving opportunities utilizing real-world context. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).
Explanations	Students may be given graphs to interpret or produce graphs given an expression or table for the function, by hand or using technology.
Content Limits	Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step). Key features may also include domain and range
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify an interval on a graph where the function is increasing or decreasing.	• Equation Response • Graphic Response • Multiple Choice Response
Students will be required to identify intercepts of a function.	
Students will be required to construct the graph of a linear function with a given verbal description for the intercept and/or slope.	
Students will be required to identify key features, such as relative maximums and minimums, symmetries, and end behavior, of graphs and tables in terms of the quantities.	

Students will be required to create a linear function with the same slope but different y-intercept.	
Students will be required to create an exponential function that grows at a different rate than a given one.	
Students will be required to describe the meaning of key features of a function.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
For a function that models a relationship between two quantities, identify key features of graphs and tables in terms of the quantities. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums. Focus on linear and exponential and functions.	For a function that models a relationship between two quantities, identify key features of graphs and tables in terms of the quantities. Include problem-solving opportunities utilizing real-world context. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums. Focus on linear and exponential and functions.
Proficient	**Highly Proficient**
For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Include problem-solving opportunities utilizing real-world context. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).	For a function that models a relationship between two quantities, explain key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Include problem-solving opportunities utilizing real-world context. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).

A1.F-IF.B.5

Content Standards	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
Explanations	Students may explain orally, or in written format, the existing relationships.
Content Limits	This standard is aligned to Algebra I only.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to create a graph with a given domain.	• Equation Response • Graphic Response • Multiple Choice Response
Students will be required to determine the domain of the given graph of a function.	
Students will be required to determine the domain of a given function based on context.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify the domain of a function from its graph.	Identify the domain of a function from its graph and, where applicable, relate it to the quantitative relationship it describes.
Proficient	**Highly Proficient**
Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes in a real-world context.

A1.F-IF.B.6

Content Standards	Calculate and interpret the average rate of change of a continuous function (presented symbolically or as a table) on a closed interval. Estimate the rate of change from a graph. Include problem-solving opportunities utilizing real-world context. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).
Explanations	The average rate of change of a function $y = f(x)$ over an interval $[a,b]$ is $\Delta y/\Delta x=(f(b)-f(a))/(b-a)$ In addition to finding average rates of change from functions given symbolically, graphically, or in a table, Students may collect data from experiments or simulations (ex. falling ball, velocity of a car, etc.) and find average rates of change for the function modeling the situation.
Content Limits	Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to estimate the average rate of change of the graph of a given function over a given interval.	• Equation Response • Multiple Choice Response
Students will be required to calculate the average rate of change of a function expressed symbolically or as a table over a given interval.	
Students will be required to interpret the rate of change in context.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Estimate the rate of change from a graph. Focus on linear and exponential functions.	Calculate the average rate of change of a continuous function (presented symbolically or as a table) on a closed interval. Estimate the rate of change from a graph. Include problem-solving opportunities utilizing real-world context. Focus on linear and exponential functions.
Proficient	**Highly Proficient**
Calculate and interpret the average rate of change of a continuous function (presented symbolically or as a table) on a closed interval. Estimate the rate of change from a graph. Include problem-solving opportunities utilizing real-world context.	Analyze the average rate of change of a continuous function (presented symbolically or as a table) on a closed interval. Estimate the rate of change from a graph. Include problem-solving opportunities utilizing real-world context.

Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).	Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).

A1.F-IF.C.7

Content Standards	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).
Explanations	Analyze functions using different representations.
Content Limits	Linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step)
Context	Context is not required.

Sample Task Demands	Common Item Formats
Graph a linear function	• Editing Task Choice • Equation Response • Multiple Choice Response • Multi-Select Response
Identify key features of a piecewise function	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify key features of linear and exponential functions shown on a graph.	Identify key features functions shown on a graph. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).
Proficient	**Highly Proficient**
Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).	Graph more than one function expressed symbolically, and compare key features of the graphs. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).

A1.F-IF.C.8, A1.F-IF.C.8a

Content Standards	A1.F-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. A1.F-IF.C.8a Use the process of factoring and completing the square of a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
Explanations	Analyze functions using different representations.
Content Limits	Functions in one form must be given to students, who are then expected to write these functions in different forms.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to create an equivalent function in a specific form that reveals characteristics of the function defined by that expression.	• Equation Response • Multiple Choice Response • Proposition Response
Students will be required to interpret parameters of a function in terms of the context.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring a quadratic function to show zeros.	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square of a quadratic function to show zeros, extreme values, and symmetry of the graph.
Proficient	**Highly Proficient**

Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square of a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Determine an appropriate method to rewrite a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

A1.F-IF.C.9

Content Standards	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).
Explanations	Analyze functions using different representations.
Content Limits	Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to compare numeric values representing properties of two functions.	• Equation Response • Graphic Response • Multiple Choice Response
Students will be required to compare two functions qualitatively.	
Students will be required to construct a graph of a function for which a given comparison with another function is true.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify properties of two functions each represented in a different way (graphically or numerically in tables). Focus on linear and exponential functions.	Define properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Focus on linear, quadratic, and exponential functions.
Proficient	**Highly Proficient**

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).	Analyze two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).

Functions- Building Functions (F-BF)

A1.F-BF.A.1

Content Standards	Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from real-world context. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).
Explanations	Students will analyze a given problem to determine the function expressed by identifying patterns in the function's rate of change. They will specify intervals of increase, decrease, constancy, and, if possible, relate them to the function's description in words or graphically.
Content Limits	Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to perform arithmetic operations to write one function that models a context for another.	• Equation Response • Multiple Choice Response
Students will be required to create a multi-faceted function to model a context.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify a function that describes a relationship between two quantities. Identify an explicit expression, steps for calculation from real-world context. Focus on linear and exponential functions.	Identify a function that describes a relationship between two quantities. Identify an explicit expression, a recursive process, or steps for calculation from real-world context. Focus on linear, quadratic and exponential functions.

Content Standards	Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from real-world context. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).

Proficient	**Highly Proficient**
Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from real-world context. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).	Write a function that describes a relationship between two quantities. Compare the explicit expression to the recursive process. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).

A1.F-BF.B.3

Content Standards	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, and $f(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).
Explanations	Students will apply transformations to functions and recognize functions as even and odd.
Content Limits	Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to show the effects of a transformation by translating a graph.	• Equation Response • Graphic Response • Multiple Choice Response
Students will be required to determine the value of k from two related functions or graphs.	
Students will be required to create a function to model a transformation of a given graph.	
Students will be required to describe the effects of k on a transformation of a function.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the effect on the graph of replacing f(x) by f(x) + k, and f(x+k) for specific positive values of k. Illustrate the effects on the graph. Focus on linear and exponential functions.	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), and f(x+k) for specific positive values of k; identify the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph. Focus on linear, quadratic, and exponential functions.
Proficient	**Highly Proficient**
Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), and f(x+k) for specific values of k (both positive and negative); find the value of k given	Explain the effect on the graph of replacing f(x) by f(x) + k, k f(x), and f(x+k) for specific values of k (both positive and negative rational numbers); determine

Content Standards	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, and $f(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).
the graphs. Experiment with cases and illustrate an explanation of the effects on the graph. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).	the value of k given the graphs. Experiment with cases and explain an explanation of the effects on the graph. Focus on linear, quadratic, exponential and piecewise-defined functions (limited to absolute value and step).

Functions- Linear, Quadratic, and Exponential Models (F-LE)

A1.F-LE.A.1, A1.F-LE.A.1a, A1.F-LE.A.1b, A1.F-LE.A.1c

Content Standards	A1.F-LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. A1.F-LE.A.1a Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. A1.F-LE.A.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. A1.F-LE.A.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
Explanations	Students can investigate functions and graphs modeling different situations involving simple and compound interest. Students can compare interest rates with different periods of compounding (monthly, daily) and compare them with the corresponding annual percentage rate. Spreadsheets and applets can be used to explore and model different interest rates and loan terms.
Content Limits	This standard is aligned to Algebra I only.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to create a value or expression to show how a function grows over equal intervals.	• Equation Response • Graphic Response • Multiple Choice Response
Students will be required to identify situations that represent linear growth.	
Students will be required to identify situations that represent exponential growth.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Recognize that linear functions grow by equal differences over equal intervals. b. Identify situations in which one quantity changes at a constant rate per unit interval relative to another. c. Identify situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Recognize that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Identify situations in which one quantity changes at a constant rate per unit interval relative to another as a situation that can be modeled with a linear function. c. Identify situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another as a situation that can be modeled with an exponential function.
Proficient	**Highly Proficient**
Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Explain why linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Create situations in which one quantity changes at a constant rate per unit interval relative to another. c. Create situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

A1.F-LE.A.2

Content Standards	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or input/output pairs.
Explanations	Construct and compare linear and exponential models and solve problems.
Content Limits	Constructing linear and exponential functions in simple context (not multi-step)
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to create an equation of a linear function passing through two given points.	• Equation Response
Students will be required to create an equation of a linear function given a graph of that function.	
Students will be required to create an equation of a linear function given a description of that function.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify linear functions, including arithmetic sequences, given a graph, a description of a relationship, or input/output pairs.	Identify linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or input/output pairs.
Proficient	**Highly Proficient**

Content Standards	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or input/output pairs.
Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or input/output pairs.	Explain how linear and exponential functions, can model arithmetic and geometric sequences.

A1.F-LE.A.3

Content Standards	Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.
Explanations	Construct and compare linear, quadratic, and exponential models and solve problems.
Content Limits	This standard is aligned to Algebra I only.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to compare two or more functions for values over various intervals given graphs or other representations of the functions.	• Equation Response • Multi-Select Response
Students will be required to solve problems based on the fact that exponential functions grow/decay faster than linear or quadratic functions.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify graphs and tables that have a quantity increasing linearly, exponentially, or quadratically.	Compare graphs and tables that have quantities increasing linearly, exponentially, and quadratically.
Proficient	**Highly Proficient**
Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.	Explain why a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.

A1.F-LE.B.5

Content Standards	Interpret the parameters in a linear or exponential function with integer exponents utilizing real-world context.
Explanations	Interpret expressions for functions in terms of the situation they model.
Content Limits	Exponential functions limited to those with domains in the integers
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to interpret the meaning of a parameter of a function.	• Multi-Select Response
Students will be required to interpret the meaning of a parameter in a function that combines linear and exponential terms.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify the parameters in a linear function with integer exponents utilizing real world context.	Identify the parameters in a linear or exponential function with integer exponents utilizing real world context.
Proficient	**Highly Proficient**
Interpret the parameters in a linear or exponential function with integer exponents utilizing real world context.	Define the parameters while creating a linear or exponential function with integer exponents utilizing real world context.

Statistics and Probability- Summarize, represent, and interpret data on a single count or measurement variable (S-ID)

A1.S-ID.A.1

Content Standards	Represent real-value data with plots for the purpose of comparing two or more data sets.
Explanations	Summarize, represent, and interpret data on a single count or measurement variable.
Content Limits	This standard is aligned to Algebra I only. The amount of data to be plotted should be reasonable.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to construct a data display.	• Graphic Response • Multiple Choice Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Match real-value data with dot plots, histograms, and box plots.	Represent real-value data with dot plots, histograms, and box plots.
Proficient	**Highly Proficient**
Represent real-value data with plots for the purpose of comparing two or more data sets.	Represent real-value data with the most appropriate plots and analyze the similarities and differences between two or more data sets.

A1.S-ID.A.2

Content Standards	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
Explanations	Summarize, represent, and interpret data on a single count or measurement variable.
Content Limits	This standard is aligned to Algebra I only.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify data distributions that share commonalities (i.e., same spread, interquartile range, median, and mean) through inspection.	• Equation Response • Multiple Choice Response • Multi-Select Response
Students will be required to distinguish between different spreads to compare the mean and medians of the data set.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the center (median, mean) and spread (interquartile range) of two or more different data sets.	Compare the center (median, mean) or spread (interquartile range, standard deviation) of two or more different data sets.
Proficient	**Highly Proficient**
	Use statistics appropriate to the shape of the data distribution to analyze and explain the similarities and differences between the center (median,

Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	mean) and spread (interquartile range, standard deviation) of two or more different data sets.

A1.S-ID.A.3

Content Standards	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of outliers if present.
Explanations	Summarize, represent, and interpret data on a single count or measurement variable.
Content Limits	This standard is aligned to Algebra I only.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to construct a graph given information about the shape, center, and spread.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response
Students will be required to compare different distributions in order to draw conclusions about the effects of an extreme outlier on different spreads	
Students will be required to make inferences about the spread of distributions to draw conclusions about the given context. (i.e., what does a skewed distribution of test scores tell us about the test questions).	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify differences in shape, center, and spread in the context of the data sets.	Compare informally differences in shape, center, and spread in the context of the data sets, accounting for possible effects of outliers if present.
Proficient	**Highly Proficient**

Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of outliers if present.	Interpret and explain differences in shape, center, and spread in the context of the data sets, make observations about the effects different outlier would have.

A1.S-ID.B.5

Content Standards	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data, including joint, marginal, and conditional relative frequencies. Recognize possible associations and trends in the data.
Explanations	Summarize, represent, and interpret data on two categorical and quantitative variables.
Content Limits	This standard is aligned to Algebra I only. Bivariate data Positive rational numbers
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to construct a contingency table in order to show the relationships between variables.	• Equation Response • Multiple Choice Response • Table Resposne
Students will be required to interpret tables to calculate marginal and joint frequencies within the context.	
Students will be required to identify patterns in a distribution in order to answer questions pertaining to the data set and context.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
For categorical data summarized for two categories in two-way frequency tables, identify relative frequencies in the context of the data.	Complete a partially filled in frequency table to summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data, including joint, and conditional relative frequencies.
Proficient	**Highly Proficient**

Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data, including joint, marginal, and conditional relative frequencies. Recognize possible associations and trends in the data.	Summarize categorical data for two categories in two-way frequency tables. Interpret and explain relative frequencies in the context of the data, including joint, marginal, and conditional relative frequencies. Explain possible associations and trends in the data.

A1.S-ID.B.6, **A1.S-ID.B.6a**, A1.S-ID.B.6b

Content Standards	A1.S-ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the quantities are related. A1.S-ID.B.6a Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Focus on linear models. A1.S-ID.B.6b Informally assess the fit of a function by plotting and analyzing residuals.
Explanations	The residual in a regression model is the difference between the observed and the predicted *y* for some *x* (y the dependent variable and *x* the independent variable). So if we have a model $y = ax + b$ and a data point (xi, yi), the residual is for this point is $ri = yi - (axi + b)$. Students may use spreadsheets, graphing calculators, and statistical software to represent data, describe how the variables are related, fit functions to data, perform regressions, and calculate residuals.
Content Limits	Rational numbers; Bivariate data; Linear, quadratic, and exponential models
Context	Context is not allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to select a function that best represents the data given a set of data. (a)	• Equation Response • Graphic Response • Multiple Choice Response
Students will be required to plot and analyze residuals on a number line. (b)	
Students will be required to create a linear function that best represents the data given a scatter plot. (c)	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Represent data on two quantitative variables on a scatter plot, and describe how the quantities are related. a. Identify a linear function that best fits the data represented in a scatter plot. b. Informally assess the fit of a function when given a residual plot.	Represent data on two quantitative variables on a scatter plot, and describe how the quantities are related. a. Identify a linear function that best fits the data represented in a scatter plot; use functions fitted to data to identify the solutions to problems in the context of the data. Focus on linear models. b. Plot the residuals of a function.
Proficient	**Highly Proficient**
Represent data on two quantitative variables on a scatter plot, and describe how the quantities are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Focus on linear models. b. Informally assess the fit of a function by plotting and analyzing residuals.	Represent data on two quantitative variables on a scatter plot, and describe how the quantities are related. a. Compare the fit of different functions to the data, including exponential functions with domains in the integers; use functions fitted to data to solve problems in the context of the data. b. Informally assess the fit of different functions by plotting and analyzing their residuals.

A1.S-ID.C.7

Content Standards	Interpret the slope as a rate of change and the constant term of a linear model in the context of the data.
Explanations	Interpret linear models.
Content Limits	This standard is aligned to Algebra I only. A linear model should be provided The model should not fit exactly a set of data, if given
Context	Context is required.

Sample Task Demands	Common Item Formats
Students will be required to interpret the rate of change and/or constant term of a linear model to identify valid conclusions.	• Equation Response • Multiple Choice Response • Multi-Select Response
Students will be required to identify the value in a linear model that represents a given interpretation.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Match the slope and the constant term of a linear model with their meaning in the context of the data.	Identify the slope of a linear model as a rate of change in the context of the data, and identify the constant term of a linear model in the context of the data.
Proficient	**Highly Proficient**

Content Standards	Interpret the slope as a rate of change and the constant term of a linear model in the context of the data.
Interpret the slope as a rate of change and the constant term of a linear model in the context of the data.	Define the meaning of the slope as a rate of change in the context of the data, and define the constant term of a linear model in the context of the data.

A1.S-ID.C.8

Content Standards	Compute and interpret the correlation coefficient of a linear relationship.
Explanations	Interpret linear models.
Content Limits	This standard is aligned to Algebra I only. Items should focus on interpreting a given correlation coefficient
Context	Context is required.

Sample Task Demands	Common Item Formats
Students will be required to interpret the correlation coefficient of a linear fit.	• Equation Response • Multiple Choice Response
Students will be required to identify another correlation coefficient that satisfies a given condition given a correlation coefficient (i.e., a coefficient that shows a better positive correlation than 0.7).	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Select the correlation coefficient of a linear relationship represented with a scatter plot where the correlation coefficient can be easily estimated.	Identity the correlation coefficient of a linear relationship.
Proficient	**Highly Proficient**
Compute and interpret the correlation coefficient of a linear relationship.	Explain the meaning of different correlation coefficients for linear relationships.

A1.S-ID.C.9

Content Standards	Distinguish between correlation and causation.
Explanations	Some data leads observers to believe that there is a cause and effect relationship when a strong relationship is observed. Students should be careful not to assume that correlation implies causation. The determination that one thing causes another requires a controlled randomized experiment.
Content Limits	This standard is aligned to Algebra I only. Bivariate, linear data Items should focus on the fact that causation cannot be determined from correlation, rather than asking the student to decide which relationships are causal and which are not.
Context	Context is required.

Sample Task Demands	Common Item Formats
Students will be required to distinguish information that a correlation coefficient provides (fit, trend) to information it does not (causation).	• Multiple Choice Response • Multi-Select Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Define correlation and causation.	Identify examples of correlation and causation.
Proficient	**Highly Proficient**
Distinguish between correlation and causation.	Supports or refutes claims of causation, distinguishing between correlation and causation.

Statistics and Probability- Conditional Probability and the Rules of Probability(S-CP)

A1.S-CP.A.1

Content Standards	Describe events as subsets of a sample space using characteristics of the outcomes, or as unions, intersections, or complements of other events.
Explanations	Intersection: The intersection of two sets *A* and *B* is the set of elements that are common to both set *A* and set *B*. It is denoted by $A \cap B$ and is read '*A* intersection *B*.' Union: The union of two sets *A* and *B* is the set of elements, which are in *A* or in *B* or in both. It is denoted by $A \cup B$ and is read '*A* union *B*.' Complement: The complement of the set $A \cup B$ is the set of elements that are members of the universal set U but are not in $A \cup B$. It is denoted by '$(A \cup B)$''
Content Limits	This standard is aligned to Algebra I only. Positive rational numbers
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify events as outcomes of a trial.	• Multiple Choice Response • Multi-Select Response
Students will be required to identify multiple events as subsets of the sample space, including unions, intersections, and complements.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify an event as a subset of a sample space.	

Content Standards	Describe events as subsets of a sample space using characteristics of the outcomes, or as unions, intersections, or complements of other events.
	Identify events as subsets of a sample space using characteristics of the outcomes, or as unions, intersections, or complements of other events, as shown in a visual model.
Proficient	**Highly Proficient**
Describe events as subsets of a sample space using characteristics of the outcomes, or as unions, intersections, or complements of other events.	Using complex representations, explain how specific events are subsets of a sample space using characteristics of the outcomes, or as unions, intersections, or complements of other events.

A1.S-CP.A.2

Content Standards	Use the Multiplication Rule for independent events to understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
Explanations	Understand independence and conditional probability and use them to interpret data.
Content Limits	Positive rational numbers
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify independent events given their probabilities.	• Equation Response • Multiple Choice Response • Multi-Select Response
Students will be required to determine the probability of the other event given that two events are independent and the probability of one event.	
Students will be required to interpret two events in terms of independence given the probabilities of the two events.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Use the Multiplication Rule for independent events to calculate the probability of 2 independent events.	Use the Multiplication Rule for independent events to determine if two events *A* and *B* are independent, given the probability of *A*, the probability of *B*, and the probability of *A* and *B* occurring together.

Content Standards	Use the Multiplication Rule for independent events to understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	
Proficient		**Highly Proficient**
Use the Multiplication Rule for independent events to understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities and use this characterization to determine if they are independent.		Use the Multiplication Rule for independent events to understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities and use this characterization to determine if several events in a sample space are dependent or independent.

Number and Quantity - N		
Quantities (N-Q)		
G.N-Q.A **Reason quantitatively and use units to solve problems.**	**G.N-Q.A.1**	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays, include utilizing real-world context.
	G.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling. Include problem-solving opportunities utilizing real-world context.
	G.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities utilizing real-world context.
Geometry - G		
Congruence (G-CO)		
G.G-CO.A **Experiment with transformations in the plane.**	**G.G-CO.A.1**	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
	G.G-CO.A.2	Represent and describe transformations in the plane as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not.
	G.G-CO.A.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
	G.G-CO.A.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
	G.G-CO.A.5	Given a geometric figure and a rotation, reflection, or translation draw the transformed figure. Specify a sequence of transformations that will carry a given figure onto another.
G.G-CO.B **Understand congruence in terms of rigid motions.**	**G.G-CO.B.6**	Use geometric definitions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

	G.G-CO.B.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
	G.G-CO.B.8	Explain how the criteria for triangle congruence (ASA, AAS, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
G.G-CO.C **Prove geometric theorems.**	**G.G-CO.C.9**	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
	G.G-CO.C.10	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangle are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
	G.G-CO.C.11	Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and rectangles are parallelograms with congruent diagonals.
G.G-CO.D **Make geometric constructions.**	**G.G-CO.D.12**	Make formal geometric constructions with a variety of tools and methods. Constructions include: copying segments; copying angles; bisecting segments; bisecting angles; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
	G.G-CO.D.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle; with a variety of tools and methods.
Similarity, Right Triangles, and Trigonometry (G-SRT)		
G.G-SRT.A **Understand similarity in terms of similarity transformations.**	**G.G-SRT.A.1**	Verify experimentally the properties of dilations given by a center and a scale factor: a. Dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
	G.G-SRT.A.2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for

		triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
	G.G-SRT.A.3	Use the properties of similarity transformations to establish the AA, SAS, and SSS criterion for two triangles to be similar.
G.G-SRT.B **Prove theorems involving similarity.**	**G.G-SRT.B.4**	Prove theorems about triangles. Theorems include: an interior line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
	G.G-SRT.B.5	Use congruence and similarity criteria to prove relationships in geometric figures and solve problems utilizing real-world context.
G.G-SRT.C **Define trigonometric ratios and solve problems involving right triangles.**	**G.G-SRT.C.6**	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
	G.G-SRT.C.7	Explain and use the relationship between the sine and cosine of complementary angles.
	G.G-SRT.C.8	Use trigonometric ratios (including inverse trigonometric ratios) and the Pythagorean Theorem to find unknown measurements in right triangles utilizing real-world context.
Circles (G-C)		
G.G-C.A **Understand and apply theorems about circles.**	**G.G-C.A.1**	Prove that all circles are similar.
	G.G-C.A.2	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
	G.G-C.A.3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
G.G-C.B **Find arc lengths and areas of sectors of circles.**	**G.G-C.B.5**	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Convert between degrees and radians.
Expressing Geometric Properties with Equations (G-GPE)		
G.G-GPE.A	**G.G-GPE.A.1**	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Translate between the geometric description and the equation for a conic section.		
G.G-GPE.B **Use coordinates to prove geometric theorems algebraically.**	**G.G-GPE.B.4**	Use coordinates to algebraically prove or disprove geometric relationships. Relationships include: proving or disproving geometric figures given specific points in the coordinate plane; and proving or disproving if a specific point lies on a given circle.
	G.G-GPE.B.5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems, including finding the equation of a line parallel or perpendicular to a given line that passes through a given point.
	G.G-GPE.B.6	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
	G.G-GPE.B.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles.
Geometric Measurement and Dimension (G-GMD)		
G.G-GMD.A **Explain volume formulas and use them to solve problems.**	**G.G-GMD.A.1**	Analyze and verify the formulas for the volume of a cylinder, pyramid, and cone.
	G.G-GMD.A.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems utilizing real-world context.
G.G-GMD.B **Visualize relationships between two-dimensional and three-dimensional objects.**	**G.G-GMD.B.4**	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
Modeling with Geometry (G-MG)		
G.G-MG-A **Apply geometric concepts in modeling situations.**	**G.G-MG.A.1**	Use geometric shapes, their measures, and their properties to describe objects utilizing real-world context.
	G.G-MG.A.2	Apply concepts of density based on area and volume in modeling situations utilizing real-world context.
	G.G-MG.A.3	Apply geometric methods to solve design problems utilizing real-world context.

Geometry Item Specifications

Congruence (G-CO)

G.G-CO.A.1

Content Standards	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
Explanations	Experiment with transformations in the plane.
Content Limits	This standard is aligned to Geometry only. Item writers should take care that the key does not stand out compared to the distractor options.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to select a definition for a geometric object.	• Multiple Choice Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	Informally define angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
Proficient	**Highly Proficient**
Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	Create precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G.G-CO.A.2

Content Standards	Represent and describe transformations in the plane as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not.
Explanations	Experiment with transformations in the plane.
Content Limits	This standard is aligned to Geometry only.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify a correct transformation given a starting shape and an ending shape.	• Graphic Response • Multiple Choice Response
Students will be required to construct a transformation given a starting shape and a sequence of steps.	
Students will be required to explain the difference between two transformations or a transformation and a stretch in terms of preservation of properties.	
Students will be required to given a transformation, describe a rule that maps the coordinates of a starting shape to an ending shape.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify transformations in the plane as functions that take points in the plane as inputs and give other points as outputs.	Interpret transformations in the plane as functions that take points in the plane as inputs and give other points as outputs. Identify transformations that preserve distance and angle to those that do not.
Proficient	**Highly Proficient**
Represent and describe transformations in the plane as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not.	Create and rewrite transformations in the plane as functions that take points in the plane as inputs and give other points as outputs. Evaluate and compare transformations that preserve distance and angle to those that do not.

G.G-CO.A.3

Content Standards	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
Explanations	Experiment with transformations in the plane.
Content Limits	This standard is aligned to Geometry only. Shapes should be given on a coordinate grid
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to describe rotations and/or reflections that carry a figure onto itself.	• Multiple Choice Response • Multi-Select Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Given a rectangle, parallelogram, trapezoid, or regular polygon, identify a rotation or reflection that could carry it onto itself.	Given a rectangle, parallelogram, trapezoid, or regular polygon, identify the rotations and reflections that carry it onto itself.
Proficient	**Highly Proficient**
Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	Given a rectangle, parallelogram, trapezoid, or regular polygon, create and justify the rotations and reflections that carry it onto itself.

G.G-CO.A.4

Content Standards	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
Explanations	Students may observe patterns and develop definitions of rotations, reflections, and translations.
Content Limits	This standard is aligned to Geometry only. Items should focus on formal definitions of these concepts, i.e. what makes a definition complete or incomplete. Simply recognizing a description of a rotation compared with ones for reflections or rotations is a middle-school skill.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to describe definitions for a given transformation.	• Multiple Choice Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	Interpret definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
Proficient	**Highly Proficient**
Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	Create and evaluate definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G.G-CO.A.5

Content Standards	Given a geometric figure and a rotation, reflection, or translation draw the transformed figure. Specify a sequence of transformations that will carry a given figure onto another.
Explanations	Experiment with transformations in the plane.
Content Limits	This standard is aligned to Geometry only. Two-dimensional figures
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to recognize and identify transformations of a given figure.	• Graphic Response • Multiple Choice Response
Students will be required to construct a transformation of a figure from given information.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Given a geometric figure and a rotation, reflection, or translation, identify the transformed figure.	Given a geometric figure and a rotation, reflection, or translation, describe the transformed figure. Identify a sequence of transformations that will carry a given figure onto another.
Proficient	**Highly Proficient**
Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure. Specify a sequence of transformations that will carry a given figure onto another.	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure. Specify sequences of transformations that will carry a given figure onto another.

G.G-CO.B.6

Content Standards	Use geometric definitions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
Explanations	A rigid motion is a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are assumed to preserve distances and angle measures.
Content Limits	This standard is aligned to Geometry only. Two-dimensional figures
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to describe rigid motions involved in a given transformation in terms of size and orientation.	• Multiple Choice Response • Multi-Select Response
Students will be required to describe how rigid motions can be used to show congruence.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Use geometric definitions of rigid motions to transform a figure; given two figures, use the definition of congruence in terms of rigid motions to identify if they are congruent.	Use geometric definitions of rigid motions to transform a figure or to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to identify if they are congruent.
Proficient	**Highly Proficient**
Use geometric definitions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	Use geometric definitions of rigid motions to transform figures and to predict and describe the effect of a sequence of rigid motions on a given figure; given two figures, use the definition of congruence in terms of rigid motions to describe if and why they are congruent.

G.G-CO.B.7

Content Standards	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
Explanations	A rigid motion is a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are assumed to preserve distances and angle measures. Two triangles are said to be congruent if one can be exactly superimposed on the other by a rigid motion, and the congruence theorems specify the conditions under which this can occur.
Content Limits	This standard is aligned to Geometry only.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to show/explain that if two triangles are congruent, their corresponding sides and angles are congruent.	• Graphic Response • Multiple Choice Response
Students will be required to show/explain that if two triangles' corresponding sides and angles are congruent, then the figures are congruent.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Use the definition of congruence in terms of rigid motions to understand that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	Use the definition of congruence in terms of rigid motions to identify that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
Proficient	**Highly Proficient**
Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	Use the definition of congruence in terms of rigid motions to justify that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

G.G-CO.B.8

Content Standards	Explain how the criteria for triangle congruence (ASA, AAS, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
Explanations	Understand congruence in terms of rigid motions.
Content Limits	This standard is aligned to Geometry only.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to explain how, given that rigid motions preserve congruence, the criteria ASA, SAS, and/or SSS are true.	• HotText Response • Multiple Choice Response • Proposition Response

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Understand how the criteria for triangle congruence (ASA, AAS, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	Show how the criteria for triangle congruence (ASA, AAS, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
Proficient	**Highly Proficient**
Explain how the criteria for triangle congruence (ASA, AAS, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	Justify how the criteria for triangle congruence (ASA, AAS, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

G.G-CO.C.9

Content Standards	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
Explanations	Prove geometric theorems.
Content Limits	This standard is aligned to Geometry only. Theorems are not limited to only those in the "include" list, however they must be about lines and angles.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to complete a proof.	• HotText Response • Multiple Choice Response • Proposition Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.	Interpret theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
Proficient	**Highly Proficient**
Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.	Construct and evaluate proofs for theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

G.G-CO.C.10

Content Standards	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of an isosceles triangle are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
Explanations	Prove geometric theorems.
Content Limits	This standard is aligned to Geometry only. Theorems are not limited to only those in the "include" list, however they must be about triangles.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to complete a proof.	• HotText Response • Multiple Choice Response • Proposition Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of an isosceles triangle are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.	Interpret theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of an isosceles triangle are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
Proficient	**Highly Proficient**
Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of an isosceles triangle are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.	Construct and evaluate proofs for theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of an isosceles triangle are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

G.G-CO.C.11

Content Standards	Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and rectangles are parallelograms with congruent diagonals.
Explanations	Prove geometric theorems.
Content Limits	This standard is aligned to Geometry only. Theorems are not limited to only those in the "include" list, however they must be about parallelograms
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to complete a proof.	• HotText Response • Multiple Choice Response • Proposition Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and rectangles are parallelograms with congruent diagonals.	Interpret theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and rectangles are parallelograms with congruent diagonals.
Proficient	**Highly Proficient**
Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and rectangles are parallelograms with congruent diagonals.	Construct and evaluate proofs for theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and rectangles are parallelograms with congruent diagonals.

G.G-CO.D.12

Content Standards	Make formal geometric constructions with a variety of tools and methods. Constructions include: copying segments; copying angles; bisecting segments; bisecting angles; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
Explanations	Make geometric constructions.
Content Limits	This standard is aligned to Geometry only.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to draw a shape within a construction framework (item must require or at least refer student to use common construction techniques).	• Graphic Response • HotText Response • Multiple Choice Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify formal geometric constructions with a variety of tools and methods. Constructions include: copying segments; copying angles; bisecting segments; bisecting angles; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.	Complete formal geometric constructions with a variety of tools and methods. Constructions include: copying segments; copying angles; bisecting segments; bisecting angles; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
Proficient	**Highly Proficient**
Make formal geometric constructions with a variety of tools and methods. Constructions include: copying segments; copying angles; bisecting segments; bisecting angles; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.	Critique formal geometric constructions with a variety of tools and methods. Constructions include: copying segments; copying angles; bisecting segments; bisecting angles; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

G.G-CO.D.13

Content Standards	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle; with a variety of tools and methods.
Explanations	Make geometric constructions.
Content Limits	This standard is aligned to Geometry only.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to construct a figure or show the vertices of the figure inscribed in a circle.	• Graphic Response • Multiple Choice Response
Students will be required to explain the reason(s) points on a circle are vertices of a figure.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify steps needed to construct an equilateral triangle, a square, or a regular hexagon inscribed in a circle.	Identify steps needed to construct an equilateral triangle, a square, or a regular hexagon inscribed in a circle with a variety of tools and methods.
Proficient	**Highly Proficient**
Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle with a variety of tools and methods.	Make observations about a constructed equilateral triangle, square, and regular hexagon inscribed in a circle with a variety of tools and methods.

Similarity, Right Triangles, and Trigonometry (G-SRT)

G.G-SRT.A.1, G.G-SRT.A.1a, G.G-SRT.A.1b

Content Standards	G.G-SRT.A.1 Verify experimentally the properties of dilations given by a center and a scale factor: G.G-SRT.A.1a Dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. G.G-SRT.A.1b The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
Explanations	Dilation is a transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor. Students may observe patterns and verify experimentally the properties of dilations.
Content Limits	This standard is aligned to Geometry only. For 1a, limited to polygons with an emphasis on line segments and right triangles Items should include centers of dilation on a line segment, and not just in the middle of a figure For 1b, limit figures to points, triangles, or rectangles
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify the slope of a given side of a dilation, based on the slope of the corresponding side of the original figure.	• Equation Response • Graphic Response • Multiple Choice Response
Students will be required to understand a given scale factor and construct a dilation.	
Students will be required to describe and relate properties of dilations.	
Students will be required to find the length of one side of a dilated figure, given the original figure and a scale factor.	
Students will be required to describe how a scale factor relates to side lengths, and use this relationship to solve problems.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the properties of dilations given by a center and a scale factor: a. Dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	Interpret examples demonstrating the properties of dilations given by a center and a scale factor: a. Dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
Proficient	**Highly Proficient**
Verify experimentally the properties of dilations given by a center and a scale factor: a. Dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	Explain quantitatively the properties of dilations given by a center and a scale factor: a. Dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

G.G-SRT.A.2

Content Standards	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
Explanations	A similarity transformation is a rigid motion followed by dilation.
Content Limits	This standard is aligned to Geometry only. Figures should be given on a coordinate plane Items for task demand 3 should assess knowledge of this line of reasoning - "Two shapes are similar if one can be obtained using reflections, rotations, translations, and/or dilations on the other. All of these transformations maintain angle measure and ratios of side lengths. Therefore, similar figures have equal corresponding angle measures and corresponding pairs of sides are proportional."
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to identify similar figures.	• Equation Response • Multiple Choice Response
Students will be required to identify a transformation or series of transformations, including dilations (including scale factors of dilations), that show that two figures are similar.	
Students will be required to describe the connection between similarity and transformations.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; identify the meaning of similarity for triangles as the equality of all corresponding pairs of angles or the proportionality of all corresponding pairs of sides.	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; qualitatively describe the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
Proficient	**Highly Proficient**
Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; make observations using similarity transformations on the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

G.G-SRT.A.3

Content Standards	Use the properties of similarity transformations to establish the AA, SAS, and SSS criterion for two triangles to be similar.
Explanations	Understand similarity in terms of similarity transformations.
Content Limits	This standard is aligned to Geometry only.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required toidentify similar triangles based on AA.	• Multiple Choice Response
Students will be required to informally describe why the AA criterion is true.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Use the properties of similarity transformations to identify the AA, SAS, and SSS criterion for two triangles to be similar.	Use the properties of similarity transformations to interpret the AA, SAS, and SSS criterion for two triangles to be similar.
Proficient	**Highly Proficient**
Use the properties of similarity transformations to establish the AA, SAS, and SSS criterion for two triangles to be similar.	Use the properties of similarity transformations to develop definitions for the AA, SAS, and SSS criterion for two triangles to be similar.

G.G-SRT.B.4

Content Standards	Prove theorems about triangles. Theorems include: an interior line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
Explanations	Prove theorems involving similarity
Content Limits	This standard is aligned to Geometry only. Theorems about triangles are restricted to the following: Prove that a line constructed parallel to one side of a triangle intersecting the other two sides of the triangle divides the intersected side proportionally. Prove that a line that divides two sides of a triangle proportionally is parallel to the third side. Prove that if three sides of one triangle are proportional to the corresponding sides of another triangle, the triangles are similar. Prove the Pythagorean Theorem using similarity.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to complete a proof.	• HotText Response • Multiple Choice Response • Proposition Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify theorems about triangles. Theorems include: an interior line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.	Interpret theorems about triangles. Theorems include: an interior line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
Proficient	**Highly Proficient**
Prove theorems about triangles. Theorems include: an interior line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.	Construct and evaluate proofs of theorems about triangles. Theorems include: an interior line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

G.G-SRT.B.5

Content Standards	Use congruence and similarity criteria to prove relationships in geometric figures and solve problems utilizing real-world context.
Explanations	Similarity postulates include SSS, SAS, and AA. Congruence postulates include SSS, SAS, ASA, AAS, and H-L.
Content Limits	This standard is aligned to Geometry only. Items use SSS, SAS, ASA, and/or AAS for congruence Items use AA, SAS (ratios) and/or SSS (ratios) for similarity
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to solve a problem that uses congruence and/or similarity criteria.	• Equation Response • HotText Response • Multiple Choice Response
Students will be required to construct, analyze, and/or critique a proof that uses congruence and/or similarity criteria to shows a relationship between two figures.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Use congruence and similarity criteria to interpret problems.	Use congruence and similarity criteria to identify relationships in geometric figures and solve problems utilizing real-world context.
Proficient	**Highly Proficient**
Use congruence and similarity criteria to prove relationships in geometric figures and solve problems utilizing real-world context.	Use congruence and similarity criteria to construct and evaluate proofs for relationships in geometric figures and solve complex problems utilizing real-world context.

G.G-SRT.C.6

Content Standards	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
Explanations	Define trigonometric ratios and solve problems involving right triangles.
Content Limits	This standard is aligned to Geometry only. The trigonometric ratios are limited to sine, cosine, and tangent.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to define the trigonometric ratios: sine, cosine, and tangent.	• Equation Response • Multiple Choice Response • Matching Item Response
Students will be required to identify the sine, cosine, and/or tangent ratio of a given triangle.	
Students will be required to use the trigonometric ratios to find the length of an unknown side.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	Specify that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
Proficient	**Highly Proficient**
Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	Explain that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

G.G-SRT.C.7

Content Standards	Explain and use the relationship between the sine and cosine of complementary angles.
Explanations	Define trigonometric ratios and solve problems involving right triangles.
Content Limits	This standard is aligned to Geometry only.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify the relationship between the sine and cosine of acute angles in a right triangle: the sine of an angle is equal to the cosine of its complement and vice versa.	• Equation Response • Multiple Choice Response
Students will be required to use the sine and cosine functions to find the measure of an unknown angle given the measure of its complementary angle.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the relationship between the sine and cosine of complementary angles.	Interpret and use the relationship between the sine and cosine of complementary angles.
Proficient	**Highly Proficient**
Explain and use the relationship between the sine and cosine of complementary angles.	Prove the relationship between the sine and cosine of complementary angles.

G.G-SRT.C.8

Content Standards	Use trigonometric ratios (including inverse trigonometric ratios) and the Pythagorean Theorem to find unknown measurements in right triangles utilizing real-world context.
Explanations	Define trigonometric ratios and solve problems involving right triangles.
Content Limits	This standard is aligned to Geometry only. Items at this standard must require the student to solve real-life problems (e.g., use Pythagorean to find distance traveled on a map), and not simply find side lengths or angles of given triangles
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to use the Pythagorean Theorem and/or trigonometric ratios to solve problems involving right triangles.	• Equation Response • Graphic Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Use trigonometric ratios and the Pythagorean Theorem to identify unknown measurements in right triangles.	Use trigonometric ratios (including inverse trigonometric ratios) and the Pythagorean Theorem to find unknown measurements in right triangles.
Proficient	**Highly Proficient**
Use trigonometric ratios (including inverse trigonometric ratios) and the Pythagorean Theorem to find unknown measurements in right triangles utilizing real-world context.	Use trigonometric ratios (including inverse trigonometric ratios) and the Pythagorean Theorem to describe a solution process to find unknown measurements in right triangles utilizing real-world context.

Circles (G-C)

G.G-C.A.1

Content Standards	Prove that all circles are similar.
Explanations	Understand and apply theorems about circles.
Content Limits	This standard is aligned to Geometry only. Aside from items that ask the student to find the ratio of dilation between circles, items should focus on the fact that any circle can be obtained by a translation and dilation of any other circle - thus, they are similar (this is related to many of the SRT standards).
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to use transformations between two or more circles to show similarity.	• Equation Response • Graphic Response • Multiple Choice Response • Matching Item Response
Students will be required to show that the ratios of the circumference to the diameter of any circle are the same.	
Students will be required to graph the resulting circle from a transformed circle.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Recognize that all circles are similar.	Explain qualitatively that all circles are similar.
Proficient	**Highly Proficient**
Prove that all circles are similar.	Construct and evaluate proofs that all circles are similar.

G.G-C.A.2

Content Standards	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
Explanations	Understand and apply theorems about circles.
Content Limits	This standard is aligned to Geometry only.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to describe the relationship between inscribed angles, radius, and chords of a circle.	• Equation Response • Graphic Response • Multiple Choice Response • Proposition Response
Students will be required to find measures of central, inscribed and circumscribed angles.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Use relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	Find relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
Proficient	**Highly Proficient**
Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	Prove relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

G.G-C.A.3

Content Standards	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
Explanations	Understand and apply theorems about circles.
Content Limits	This standard is aligned to Geometry only.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to construct an inscribed/circumscribed circle of a triangle.	• Graphic Response • HotText Response • Multiple Choice Response • Proposition Response
Students will be required to explain the validity of proofs using properties of angles for a quadrilateral inscribed in a circle.	
Students will be required to complete a two-column proof proving properties of angles for a quadrilateral inscribed in a circle.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify inscribed and circumscribed circles of a triangle.	Construct the inscribed and circumscribed circles of a triangle, and use properties of angles for a quadrilateral inscribed in a circle.
Proficient	**Highly Proficient**
Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	Evaluate constructions of inscribed and circumscribed circles of a triangle, and prove unique relationships between the angles for a quadrilateral inscribed in a circle.

G.G-C.B.5

Content Standards	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Convert between degrees and radians.
Explanations	Find arc lengths and areas of sectors of circles.
Content Limits	This standard is aligned to Geometry only. Emphasize the similarity of all circles. Note that by similarity of sectors with the same central angle, arc lengths are proportional to the radius. Use this as a basis for introducing radian as a unit of measure. It is not intended that it be applied to Use radian measures for all angles
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to understand that sectors with different arcs have arc lengths that are proportional.	• Equation Response • Multiple Choice Response
Students will be required to understand that sectors with the same arc of two different circles are proportional.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify that the length of the arc intercepted by an angle is proportional to the radius and that the radian measure of the angle is the constant of proportionality; define the formula for the area of a sector. Identify the relationship between degrees and radians.	Solves problems using the fact that the length of the arc intercepted by an angle is proportional to the radius and that the radian measure of the angle is the constant of proportionality; solve problems using the formula for the area of a sector. Convert between degrees and radians.
Proficient	**Highly Proficient**
Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Convert between degrees and radians.	Prove using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; prove the formula for the area of a sector. Derive the formula to convert between degrees and radians.

Expressing Geometric Properties with Equations (G-GPE)

G.G-GPE.A.1

Content Standards	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
Explanations	Translate between the geometric description and the equation for a conic section.
Content Limits	This standard is aligned to Geometry only. All four quadrants of the coordinate plane, whole number coordinates, and a perfect square radius.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to construct an equation of a circle given information about the center and radius.	• Equation Response • Multiple Choice Response
Students will be required to find the center and/or radius of a circle given an equation not in standard form.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the center and radius of a circle given by an equation of the form $(x-h)^2 + (y-k)^2 = r^2$.	Create the equation of a circle of given center and radius; find the center and radius of a circle given by an equation of the form $(x-h)^2 + (y-k)^2 = r^2$.
Proficient	**Highly Proficient**
Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	Explain the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

G.G-GPE.B.4

Content Standards	Use coordinates to algebraically prove or disprove geometric relationships. Relationships include: proving or disproving geometric figures given specific points in the coordinate plane; and proving or disproving if a specific point lies on a given circle.
Explanations	Use coordinates to prove geometric theorems algebraically.
Content Limits	This standard is aligned to Geometry only. All four quadrants, may use radical values
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to rearrange statements to form a proof.	• Equation Response • Hot Text Response • Multiple Choice Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Use coordinates to identify geometric relationships. Relationships include: proving or disproving geometric figures given specific points in the coordinate plane; and proving or disproving if a specific point lies on a given circle.	Use coordinates to algebraically solve problems involving geometric relationships. Relationships include: proving or disproving geometric figures given specific points in the coordinate plane; and proving or disproving if a specific point lies on a given circle.
Proficient	**Highly Proficient**
Use coordinates to algebraically prove or disprove geometric relationships. Relationships include: proving or disproving geometric figures given specific points in the coordinate plane; and proving or disproving if a specific point lies on a given circle.	Use coordinates to algebraically justify statements about geometric relationships. Relationships include: proving or disproving geometric figures given specific points in the coordinate plane; and proving or disproving if a specific point lies on a given circle.

G.G-GPE.B.5

Content Standards	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems, including finding the equation of a line parallel or perpendicular to a given line that passes through a given point.
Explanations	Lines can be horizontal, vertical, or neither.
Content Limits	This standard is aligned to Geometry only. All four quadrants of the coordinate plane; coordinates are restricted to whole numbers.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to construct an equation of a line parallel or perpendicular to another line and containing a specific point.	• Equation Response • Multiple Choice Response • Proposition Response
Students will be required to solve a problem using slope criteria for parallel and perpendicular lines.	
Students will be required to describe aspects of why parallel lines have the same slope and why perpendicular lines have slopes that are negative reciprocals.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Use the slope criteria for parallel or perpendicular lines to solve simple geometric problems, including finding the equation of a line parallel or perpendicular to a given line.	Use the slope criteria for parallel and perpendicular lines to solve simple geometric problems, including finding the equation of a line parallel or perpendicular to a given line that passes through a given point.
Proficient	**Highly Proficient**
Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems, including finding the equation of a line parallel or perpendicular to a given line that passes through a given point.	Prove and explain the slope criteria for parallel and perpendicular lines and use them to solve geometric problems, including finding the equation of a line parallel or perpendicular to a given line that passes through a given point.

G.G-GPE.B.6

Content Standards	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
Explanations	Use coordinates to prove geometric theorems algebraically.
Content Limits	This standard is aligned to Geometry only. Rational numbers
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify the ratio a point divides a line segment into.	• Equation Response • Multiple Choice Response
Students will be required to identify points on a line segment that partition it based on a given ratio.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the point on a directed horizontal or vertical line segment between two given points that partitions the segment in a given ratio, given visual representation.	Identify the point on a directed line segment between two given points that partitions the segment in a given ratio, given visual representation.
Proficient	**Highly Proficient**
Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	Construct a line segment that partitions the segment in a given ratio.

G.G-GPE.B.7

Content Standards	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles.
Explanations	Use coordinates to prove geometric theorems algebraically.
Content Limits	This standard is aligned to Geometry only. At least part of the computation must require the distance formula. Coordinates of all points must be given.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify the perimeter of a polygon.	• Equation Response
Students will be required to identify the area of a triangle or rectangle.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Use coordinates to compute perimeters and areas of right triangles and rectangles.	Use coordinates to compute perimeters of regular polygons and areas of right triangles and rectangles.
Proficient	**Highly Proficient**
Use coordinates to compute perimeters of polygons and areas of triangles and rectangles.	Use coordinates to justify perimeters of polygons and areas of triangles and rectangles.

Geometric Measurement and Dimensions (G-GMD)

G.G-GMD.A.1

Content Standards	Analyze and verify the formulas for the volume of a cylinder, pyramid, and cone.
Explanations	Cavalieri's principle is if two solids have the same height and the same cross-sectional area at every level, then they have the same volume.
Content Limits	This standard is aligned to Geometry only.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to complete an informal argument.	• HotText Response • Multiple Choice Response • Proposition Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the formulas for the volume of a cylinder, pyramid, and cone.	Informally describe the formulas for the volume of a cylinder, pyramid, and cone.
Proficient	**Highly Proficient**
Analyze and verify the formulas for the volume of a cylinder, pyramid, and cone.	Create and interpret the relationships between the formulas for the volume of a cylinder, pyramid, and cone.

G.G-GMD.A.3

Content Standards	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems utilizing real-world context.
Explanations	Missing measures can include but are not limited to slant height, altitude, height, diagonal of a prism, edge length, and radius.
Content Limits	This standard is aligned to Geometry only. Focus should be on solving problems, not simply finding the volume of given figures.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to solve problems based on the volume of cylinders, pyramids, cones, or spheres.	• Equation Response
Students will be required to solve problems based on the volumes of compositions or parts of cylinders, pyramids, cones, or spheres.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Substitute given measures into volume formulas for cylinders, pyramids, cones, and spheres to solve simple problems.	Use volume formulas for cylinders, pyramids, cones, and spheres to solve simple problems.
Proficient	**Highly Proficient**
Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems utilizing real-world context.	Compare volume formulas for cylinders, pyramids, cones, and spheres.

G.G-GMD.B.4

Content Standards	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
Explanations	Visualize relationships between two-dimensional and three-dimensional objects.
Content Limits	This standard is aligned to Geometry only. The focus for the first part of the standard should be on diagonal (not horizontal or vertical) cross-sections.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify cross-sections of three-dimensional objects to two-dimensional shapes.	• Multiple Choice Response • Matching Item Response
Students will be required to identify the three-dimensional object generated by a rotation of a given two-dimensional object.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the shapes of two-dimensional horizontal or vertical cross-sections of three-dimensional objects.	Identify three-dimensional objects generated by rotations of two-dimensional objects about a line of symmetry.
Proficient	**Highly Proficient**
Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	Describe or create the shapes of two-dimensional cross-sections of three-dimensional objects, and describe three-dimensional objects generated by rotations of two-dimensional objects.

Modeling with Geometry (G-MG)

G.G-MG.A.1

Content Standards	Use geometric shapes, their measures, and their properties to describe objects utilizing real-world context.
Explanations	Apply geometric concepts in modeling situations.
Content Limits	This standard is aligned to Geometry only.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to explain how a real-life object can be modeled by three-dimensional geometric objects.	• Equation Response • Multiple Choice Response
Students will be required to construct an equation that models an object and can be used to find its unknown measure (i.e., the object's volume, area).	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Use simple geometric shapes to qualitatively describe objects utilizing real-world context.	Use geometric shapes and their properties to qualitatively describe objects utilizing real-world context.
Proficient	**Highly Proficient**
Use geometric shapes, their measures, and their properties to describe objects utilizing real-world context.	Use geometric shapes, their measures, and their properties to model complex objects utilizing real-world context.

G.G-MG.A.2

Content Standards	Apply concepts of density based on area and volume in modeling situations utilizing real-world context.
Explanations	Apply geometric concepts in modeling situations.
Content Limits	This standard is aligned to Geometry only. Only some of these items should deal with density of an object, etc. Others should deal with broader applications of the word density, like wolves per square mile.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to calculate a density.	• Equation Response • Multiple Choice Response
Students will be required to draw conclusions based on a density.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Calculate density based on area and volume.	Calculate density based on area and volume in modeling situations utilizing real-world context.
Proficient	**Highly Proficient**
Apply concepts of density based on area and volume in modeling situations utilizing real-world context.	Apply concepts of density based on area and volume in comparative modeling situations utilizing real-world context.

G.G-MG.A.3

Content Standards	Apply geometric methods to solve design problems utilizing real-world context.
Explanations	Apply geometric concepts in modeling situations.
Content Limits	This standard is aligned to Geometry only.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to satisfy a constraint given parameters in a geometric context.	• Equation Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify relevant geometric models to solve design problems utilizing real-world context.	Apply geometric methods to identify solutions for design problems utilizing real-world context.
Proficient	**Highly Proficient**
Apply concepts of density based on area and volume in modeling situations utilizing real-world context.	Apply geometric methods to create composite structures as solutions for design problems utilizing real-world context.