Simplifying Radical Expressions
An ADE Mathematics Lesson
Days 36-40

Author	ADE Content Specialists
Grade Level	9th grade
Duration	Five days

Aligns To	Connects To
Mathematics HS: **Strand 1: Number and Operations** **Concept 1: Number Sense** **PO 1.** Justify with examples the relation between the number system being used (natural numbers, whole numbers, integers, rational numbers and irrational numbers) and the question of whether or not an equation has a solution in that number system. **Concept 2: Numerical Operations** **PO 1.** Solve word problems involving absolute value, powers, roots, and scientific notation. **PO 3.** Calculate powers and roots of rational and irrational numbers. **Concept 3: Estimation** **PO 1.** Determine rational approximations of irrational numbers. **PO 4.** Estimate the location of the rational or irrational numbers on a number line. **Strand 3: Patterns, Algebra, and Functions** **Concept 3: Algebraic Representations** **PO 8.** Simplify and evaluate polynomials, rational expressions, expressions containing absolute value, and radicals. **Strand 5: Structure and Logic** **Concept 2: Logic, Reasoning, Problem Solving, and Proof** **PO 2.** Solve problems by formulating one or more strategies, applying the strategies, verifying the solution(s), and communicating the reasoning used to obtain the solution(s).	**Mathematics HS:** **Strand 1: Number and Operations** **Concept 2: Number Sense** **PO 2.** Solve word problems involving absolute value, powers, roots, and scientific notation. **Concept 3: Estimation** **PO 2.** Use estimation to determine the reasonableness of a solution. **PO 3.** Determine when an estimate is more appropriate than an exact answer.

Overview
Many algebraic expressions contain radicals. These radicals may contain numbers and/or variables. Some of these radicals can be simplified prior to simplifying the entire algebraic expression. The value of some radicals needs to be estimated in order to solve problems that are in a contextual format.

Purpose

Working with radical expressions allows you to work with both rational and irrational numbers when they are simplifying algebraic expressions. This lesson provides the foundation for solving algebraic equations that contain radical expressions.

Materials

- Radical expressions worksheets
- Number lines worksheets

Objectives

Students will:

- Simplify numerical radical expressions.
- Estimate the square root of a number to the nearest tenth.
- Solve contextual problems that involve square roots.
- Determine the number system(s) (natural numbers, whole numbers, integers, rational, and irrational numbers) to which the terms in a problem belong.
- Predict whether or not the solution to a problem will be from a different number system(s) than terms in the original problem, justify the prediction, and prove whether or not the prediction was correct.
- Simplify radical expressions that contain both numbers and variables.
- Multiply radical expressions.

Lesson Components

Prerequisite Skills: This lesson builds on skills from earlier grades of simplifying algebraic expressions that did not contain radical expressions. Prior experience with the various number sets including rational, irrational, natural numbers, whole numbers, and integers is helpful. You will need to set up and read Venn Diagrams. You will also need to remember operations with integers.

Vocabulary: *radical, radical expression, radicand, index, square root, cube root, principal square root, natural numbers, whole numbers, integers, rational numbers, irrational numbers.*

Session 1 (1 day)

1. Simplify radicals that contain only numbers in their radicands.
2. Estimates the value of a square root to the nearest tenth.
3. Identify a decimal as representing either a rational or an irrational number and locate the numbers on a number line.

Session 2 (1 day)

1. Simplify radicals that contain variables in the radicand.

Session 3 (2 days)

1. Create a Venn diagram or other graphic organizer that shows the relationship between natural numbers, whole numbers, integers, rational numbers, irrational numbers and real numbers.
2. Identify the number system(s) for terms in a problem, and predict to which number system(s) the solution will belong.
3. Justify their prediction and prove whether or not the prediction was correct.

Session 4 (1 day)

1. Solve problems that contain radicals in contextual situations.

Assessment

There is one assessment for each session to help you identify errors before moving on to the next session.

Radical Expressions
Session 1

A mathematical expression that contains a **radical** (a symbol used to refer to the root of a number or term), $\sqrt{\ }$, is called a **radical expression**. There are rules to simplifying these expressions. Radical expressions include variables and/or numbers under the radical sign. The number, variable, or algebraic expression under the radical sign is called the **radicand**.

Example : In the expression $\sqrt{49}$ **,** 49 is the radicand. In $\sqrt{211}$, 211 is the radicand.

A radical expression can take many forms such as $\sqrt{25}$ or $\sqrt[3]{27}$ or $\sqrt[4]{81}$. In these expressions, 25, 27, and 81 are radicands. The 3 in the second radical expression and the 4 in the third radical expressions are referred to as the **index** of the radical expression. When there is no index written we assume that the index is 2 and we are finding the **square root** of an expression.

$\sqrt{25} = \sqrt[2]{25}$ but we do not write the 2. The index is 2. $\sqrt{25} = 5$ because $5 \bullet 5 = 25$.

If the index is 3, we are trying to find the **cube root** of a number or of an algebraic expression. The index tells us how many repeat factors are needed.

A root is the inverse of an exponent. The inverse of an exponent of 2 is a square root. The inverse of a cube root is raising to the exponent 3. That is important because it let's you know how to simplify a term and remove it from the radical. $\sqrt{9}$ can be rewritten as $\sqrt{3 \bullet 3}$ or $\sqrt{3^2}$.

Since the square root of 3 raised to the second power is being taken, the exponent and the root cancel. That leaves just 3.

$\sqrt[3]{8}$ = 2 because we can factor 8 into $2 \bullet 2 \bullet 2$. Consider $\sqrt[3]{8} = \sqrt[3]{2^3}$. We have three 2's under the radical sign when the index is three so the inverse property allows us to simplify the term to 2. Then $\sqrt[3]{8}$ = 2.

We will only work with square roots in this lesson but it is important that you understand the meaning of the index of a radical expression. You may work with cube roots in the extension of this lesson.

Example 1: Simplify $\sqrt{49}$.

Solution: We can factor 49 to $7 \bullet 7$. Then we can replace the radicand 49 with 7^2 and then simplify. $\sqrt{49}$ = $\sqrt{7^2}$. Therefore, according to the inverse property $\sqrt{49}$ = 7. Every pair of the same numbers or the same variables under the radical sign can be simplified to the number/variable. Note that we can say -7$\bullet$-7 =49 or 7$\bullet$7=49. Therefore, $\sqrt{49}$ = 7 or $\sqrt{49}$ = -7. We can write $\sqrt{49}$ = $\pm$7 as a short way to express both answers. The positive square root of a number is also called the **<u>principal square root</u>**.

The square root of a fraction equals the square root of its numerator divided by the square root of its denominator.

Example 2: Find $\pm\sqrt{\frac{100}{81}}$.

Solution: $\sqrt{100}$ = $\pm$10 $\pm\sqrt{81}$. Therefore $\pm\sqrt{\frac{100}{81}} = \pm\frac{10}{9}$.

(Note that we only write one $\pm$ sign as it would be repetitive to write it more than once.)

Simplify:

1. $\sqrt{225}$

2. $-\sqrt{49}$

3. $\sqrt{121}$

4. $\pm\sqrt{\frac{121}{25}}$

5. $-\sqrt{\frac{49}{169}}$

Example 3: Find $\sqrt{75}$.

Method 1 *(Factor the radicand completely)***:** We can factor 75 as $5 \bullet 5 \bullet 3$.

Then, $\sqrt{75} = \sqrt{5 \bullet 5 \bullet 3}$. Therefore $\sqrt{75} = 5\sqrt{3}$.

Method 2 *(Factor out perfect squares)***:** We can factor 75 as $25 \bullet 3$.

Then, $\sqrt{75} = \sqrt{25 \bullet 3}$. The square root of 25 is 5. Therefore $\sqrt{75} = 5\sqrt{3}$.

Simplify:

1. $\sqrt{200}$

2. $\sqrt{27}$

3. $\sqrt{120}$

Write the missing perfect squares from 1 to 225 in the table below.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1						49								225

To estimate the square root of a rational number, determine between which two perfect squares it falls. Then make an estimate based on the proximity of the rational number to these two numbers.

For example, find $\sqrt{200}$ Since 200 falls between 14^2 = 196 and 15^2 = 225, we know that the square root of 200 must lie between 14 and 15. It is probably closer to 14 since 196 is much closer to 200 than to 225. Therefore, a good estimate of $\sqrt{200}$ would be 14.1 or 14.2. <u>Either estimate is acceptable</u>.

Note that the difference between 196 and 225 is 29. We may then consider 29 ÷ 10 = 2.9. Since 2.9 is close to 3, we may increase our estimate by .1 for every 3 units. Therefore, 14.2 may be a slightly closer estimate to the actual square root of 200.

Estimate each square root to the nearest tenth and explain your reasoning.

1. $\sqrt{140}$

2. $\sqrt{62}$

3. $\sqrt{192}$

4. $\sqrt{200}$

5. $\sqrt{50}$

The square root of a number that is a perfect square always represents a rational number. The square root of a number that does not represent a perfect square is always an irrational number.

Examples:

1. $\sqrt{36}$ is a rational number because 36 is a perfect square. $\sqrt{36}$ can be rewritten as 6.
2. $\sqrt{5}$ is an irrational number because 5 is not a perfect square.

Radical Expressions
Assessment 1

Simplify the following radical expressions showing your work.

1. $\sqrt{169}$

2. $-\sqrt{81}$

3. $\pm\sqrt{\dfrac{144}{49}}$

4. $\sqrt{300}$

Complete the following table with the missing perfect squares.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1				25							144			

Which set contains an irrational number?

a. $\{\sqrt{4n^2}$, 0.23, $\dfrac{15}{1}\}$

b. {21, 0.3, $\dfrac{\sqrt{121}}{2\sqrt{1}}$}

c. {$\dfrac{5}{8}$, 8, $\sqrt{26}$}

d. {0.222…, $\sqrt{9}$, 15}

Radical Expressions
Session 2

We can simplify radical expressions that contain variables by following the same process as we did for radical expressions that contain only numbers. Factor the expression completely (or find perfect squares). For every pair of a number or variable under the radical, they become one when simplified. If a pair does not exist, the number or variable must remain in the radicand.

Example 1: Simplify $\sqrt{x^2y^4}$.

We can represent this in an equivalent form by $\sqrt{x \bullet x \bullet y \bullet y \bullet y \bullet y}$.

We take out one pair of x's and two pairs of y's from the radicand .

$\sqrt{(x \bullet x) \bullet (y \bullet y) \bullet (y \bullet y)}$

= $x \bullet y \bullet y$

= xy^2

Example 2: Simplify $\sqrt{36a^3b^4c^2}$

We can represent this by $\sqrt{6 \bullet 6 \bullet a \bullet a \bullet a \bullet b \bullet b \bullet b \bullet b \bullet c \bullet c}$.

Look at all the pairs in the radicand.

$\sqrt{(6 \bullet 6) \bullet (a \bullet a) \bullet a \bullet (b \bullet b) \bullet (b \bullet b) \bullet (c \bullet c)}$

Place one number or variable outside the radical for each pair in the radicand, leaving only those numbers or variables in the radicand that are not pairs.

$\sqrt{36a^3b^4c^2}$

= $6 \bullet a \bullet b \bullet b \bullet c\sqrt{a}$

= $6ab^2c\sqrt{a}$

Example 3: Simplify $\sqrt{27c^3d^5}$

= $\sqrt{3 \bullet 3 \bullet 3 \bullet c \bullet c \bullet c \bullet d \bullet d \bullet d \bullet d \bullet d}$

= $\sqrt{(3 \bullet 3) \bullet 3 \bullet (c \bullet c) \bullet c \bullet (d \bullet d) \bullet (d \bullet d) \bullet d}$

= $3 \bullet c \bullet d \bullet d\sqrt{3 \bullet c \bullet d}$ = $3cd^2\sqrt{3cd}$

Simplify the following radial expressions showing all your work:

1. $\sqrt{25x^2}$

2. $\sqrt{121a^2b^2}$

3. $\sqrt{40x^3y^4}$

4. $\sqrt{50a^3b^5}$

5. $\sqrt{200xy^2z^3}$

6. $\sqrt{24c^3d^2e}$

7. $\sqrt{60r^3s^2}t^4$

Radical Expressions
Assessment 2

Simplify the following radical expressions showing your work.

1. $\sqrt{16x^2}$

2. $\sqrt{144a^4b^2}$

3. $\sqrt{50c^2d^3}$

4. $\sqrt{24x^3yz^5}$

Radical Expressions
Session 3

We can determine to which number set a solution belongs. All real numbers are either rational or irrational. You have worked with natural numbers, whole numbers, rational numbers, irrational numbers, and integers in previous grades. Let's review these number sets.

The set of **natural numbers** is the set of counting numbers and can be represented by N. **N = {1, 2, 3, 4, 5, 6, …}.**

The set of **whole numbers** includes the set of natural numbers and 0. It can be represented by W. **W = {0, 1, 2, 3, 4, …}.**

The set of **integers** is the set of real numbers consisting of the whole numbers and their opposites. It can be represented by Z. **Z = {… -2, -1, 0, 1, 2, ….}.**

The set of **rational numbers** is the set of real numbers that can be expressed as a quotient of two integers. We can represent the set **of rational numbers by Q.**

The set of **irrational numbers** is the set of real numbers that cannot be expressed as a ratio of two integers. We can represent the set of **irrational numbers by I.**

The set of **real numbers** is the set of rational and irrational numbers. We can represent the set of **real numbers by R**.

Review:

Using a Venn diagram or other graphic organizer, show the relationship between the different number sets to include natural numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers.

Sometimes equations have terms that belong to a number system(s) but the solution belongs to a different number system(s). We can look at equations to try and predict to which number system(s) the solution will belong.

Example 1: The square dog run (fenced area for housing a dog) has a perimeter of 24 ft. Miriam wants to split the dog run diagonally so that she can separate her two dogs. What length of fence will she need to install?

Solution:

24 is a rational, whole, integer, and natural number. The side length of the square is 6 since there are four equal side lengths in a square and the perimeter (24) divided by 4 is 6. When you draw a line diagonally across a square you get two right triangles. See Figure 1 below. You can use the Pythagorean Theorem to find the length of the fence (f) Miriam needs to split the square diagonally. By using the Pythagorean Theorem, you will take the square root of a number. Therefore, the solution may be irrational if the number you take the square root of is not a perfect square. Using the Pythagorean Theorem you find that $6^2+6^2=f^2$. $36+36= f^2$. $72= f^2$. To find out what f equals you need to use the inverse property. The inverse of raising a term to the exponent 2 is finding the square root. Taking the square root of both sides you get

$$\sqrt{72} = \sqrt{f^2}$$

$$\sqrt{3 \bullet 3 \bullet 2 \bullet 2 \bullet 2} = \sqrt{f \bullet f}$$

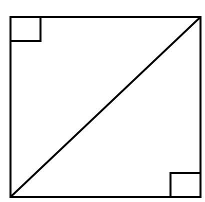
$$\sqrt{(3 \bullet 3) \bullet 2 \bullet (2 \bullet 2)} = \sqrt{(f \bullet f)}$$

$$3 \bullet 2\sqrt{2} = f$$

$$6\sqrt{2} = f$$

The solution is an **irrational number** like we predicted, even though the numbers in the problem were all **rational numbers**. The reason the number system changed was that we had to take the **square root** of a number. Miriam will have to estimate the fence she needs is about 7 and a half feet long.

Figure 1



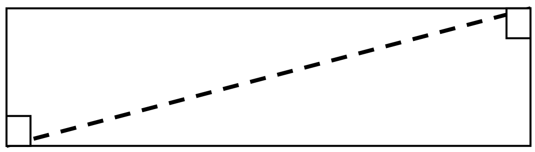
Example 2: Riley was asked to calculate the batting average for Chris, the best hitter on the baseball team. Chris was at bat 17 times this year. He had 6 hits. Will the number system(s) of the solution be the same as the number system(s) of the terms in the problem? Explain your reasoning and prove you are correct.

Solution:

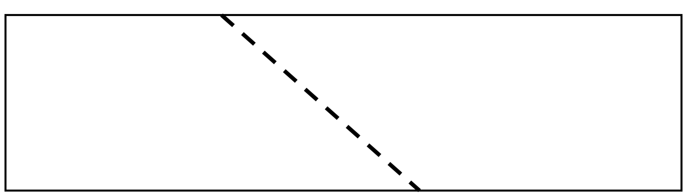
6 and 17 are **rational, integer, whole, and natural numbers**. To calculate batting average the number of "hits" is divided by the number of "at bats". 17 will not divide evenly into 6. Therefore, the solution will not be a whole, natural, or integer. Since a **ratio** can be made with 6 out of 17, the solution will be rational. 6 divided by 17 is .29411764705…Therefore, the solution is a **rational number**.

Predict whether or not the solutions to the problems below will belong to the same number system(s) as the terms in the original problem. Justify your reasoning and prove whether or not your prediction was correct.

1. A rectangular rug with a width of 4 feet and a length of 6 feet is being cut diagonally to fit in a corner of a room. What will the length of the diagonal be?



2. A rectangular table 4 feet wide and 16 feet long is being cut into two trapezoidal tables. In order to cut the tables to fit the room correctly, the length is cut diagonally so that the small base is 6. What would the length of the larger base be? What would the length of the cut be? (Hint: make the trapezoid into a triangle and a rectangle. Use a theorem to find the length of the cut.)



3. Reyna is making a budget and needs to calculate the average electric bill for her apartment. The table below includes the cost of electricity each month this year. What is her average monthly electric bill?

Jan	Feb	Mar	Apr	May	June	July	Aug	Sep	Oct	Nov	Dec
85.12	88.13	77.56	71.00	79.25	94.91	101.00	98.89	98.80	89.25	84.22	86.78

4. Gene, an eleven-year-old, asked his teacher how old she was. Instead of answering, the teacher gave Gene a logic problem that can reveal the teacher's age. How old is his teacher?

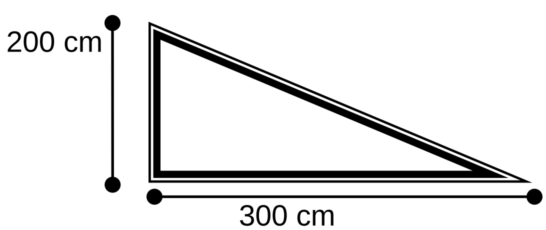
> I am more than twice your age but less than three times your age.
>
> When you were beginning winter break, I was one fortieth of a millennium.
>
> I am now just over a quarter of a century.

Radical Expressions
Assessment 3

Predict whether or not the solution to each problem will belong to the same number system(s) as the terms in the original problem. Justify your prediction. Solve the problem and prove whether or not the prediction was correct.

1. Giovana is balancing her bank account. She withdrew money from the ATM seven times this pay period. She withdrew $20.00 three times, $40.00 twice, $80.00 once, and $140.00 once. She used her own bank's ATM three times and did not have to pay any fees for those withdrawls. She used other banks four times and paid the following fees: $2.50, $3.00, $2.50, and $1.50. She had a beginning balance of $367.50. What is her final balance?

2. The physics class built a ramp. The class is measuring the time it takes for toy cars to roll down the ramp. The ramp is 200 centimeters tall at the highest point and has a horizontal length of 300 centimeters. What is the slope of the ramp?



Radical Expressions
Session 4

Many problems involve radicals. Study the examples. Then solve the problems that follow showing your work. Leave your answer in simplest radical form.

Example 1: The measure of the area of a square is 132 square centimeters. Find the length of a side of this square, rounded to the nearest tenth.

Solution:

Recall that the formula for the area of a square is $A = s^2$.

Since the area is given, substitute that number for A in the formula. $132 = s^2$

To find the measure of the side of the square, we must solve for s. Recall that the square root is the inverse of an exponent of 2. Therefore, $\sqrt{s^2} = \sqrt{132}$.

132 = 2 • 2 • 33 and $s^2 = s \bullet s$

Therefore $\sqrt{s \bullet s}$ = $\sqrt{2 \bullet 2 \bullet 33}$

s = $2\sqrt{33}$

$\sqrt{33}$ is between the perfect squares $\sqrt{25}$ & $\sqrt{36}$. That means it is between 5 and 6, and is closer to 6. A good estimate would be 5.7 because it is almost ¾ of the way between 5 and 6.

The solution can be estimated as 2•5.7=11.4.

The side is approximately 11.4 cm.

Example 2: If two sides of a right triangle are 4 and 6, find the hypotenuse.

Remember that the **Pythagorean Theorem** tells us how to find the hypotenuse of a right triangle given the two sides of the right triangle. If c is the hypotenuse and a and b are the sides of the right triangle, then $c^2 = a^2 + b^2$

Solution:

$$c^2 = a^2 + b^2$$

$$c^2 = 4^2 + 6^2$$

$$c^2 = 52$$

$$c^2 = 2 \bullet 2 \bullet 13$$

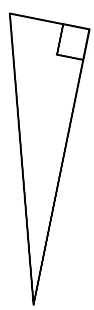
$$\sqrt{c^2} = \sqrt{2 \bullet 2 \bullet 13}$$

$$\sqrt{(c \bullet c)} = \sqrt{(2 \bullet 2) \bullet 13}$$

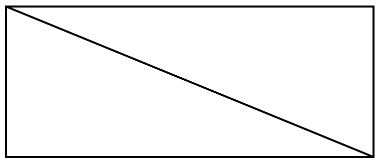
c = $2\sqrt{13}$

Find the solutions to the following problems. It may be helpful to label the diagram first. Show your work and explain your answer. Leave your answers in simplest radical form.

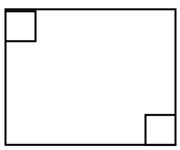
1. If two sides of a right triangle are 3 and 15, find the hypotenuse.



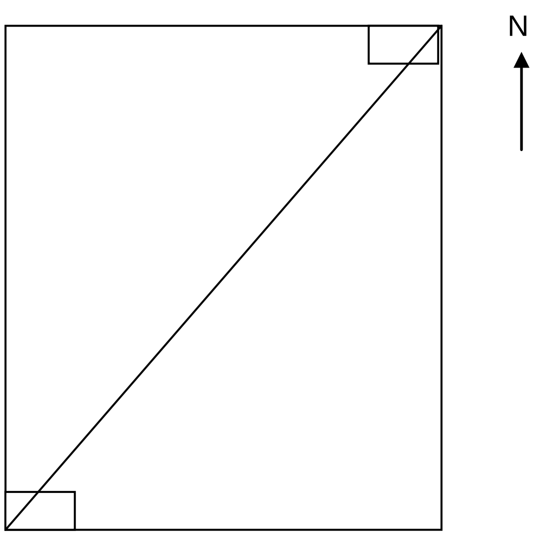
2. The diagonal of a rectangle is 12 cm and the width is 7 cm. Find the area of the rectangle.



3. The measure of the area of a square is 98 square centimeters. Find the length of a side of this square.



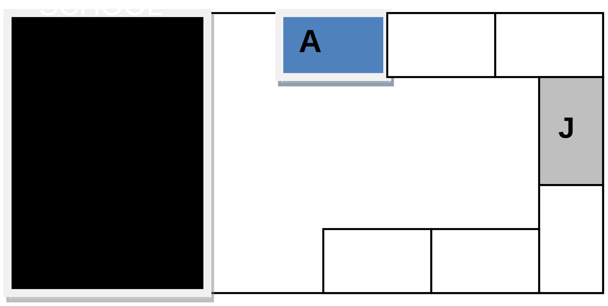
4. Emily is walking home from school. She decides to cut across a rectangular park by walking on a diagonal path that cuts through the park. If the north side of the park is 30 feet and the west side is 40 feet, how long is the path? How much less does Emily walk by using the diagonal path than by walking along the sides of the park?



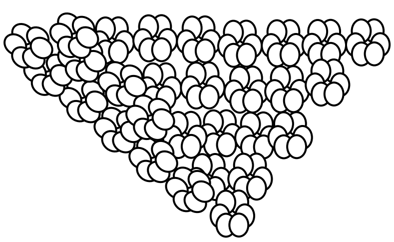
Radical Expressions
Assessment 4

Solve the following problems showing your work. It will be helpful to label the diagram. Leave answers in simplest radical form unless otherwise specified. Make sure to answer all questions.

1. Anthony is going to Jason's, house for dinner. Rather than walking the two blocks to his friend's house, Anthony will cut a diagonal path through the school playground. The distance from Anthony's house to Jason's street is 84 feet. The distance from that intersection to Jason's house is 60 feet. How far will Anthony travel along the diagonal path (round your answer to the nearest tenth)? How many feet will he save by taking the shortcut rather than walking along the sidewalks to Jason's house?



2. Maria is planting a flower garden. Her garden will be shaped like a right triangle. She is going to plant different types of flowers along each edge. The two shorter edges are 15 feet and 15 feet. How long is the other edge? What is the total amount of fencing that Maria needs to buy to place a fence along each edge?



Extensions

1. Find the product of two radicals and simplify their answer.
2. Find the cube roots of algebraic expressions.

Radical Expressions
Extension

To find the product of two radical expressions, multiply what is outside the radical of one radical by what is outside the other radical. Multiply the radicand of one radical by the radicand of the other radical. Then simplify your answer if needed.

The radical is a special grouping symbol. You can combine radicands that are being multiplied, but you cannot combine radicands with terms outside of the radical.

Example 1:

$4\sqrt{3} \bullet \sqrt{7}$ can be simplified to $4\sqrt{3 \bullet 7}$

$= 4\sqrt{21}$

Example 2:

Find $3\sqrt{2} \bullet 4\sqrt{3}$

$3\sqrt{2} \bullet 4\sqrt{3}$

$= 3 \bullet 4\sqrt{2 \bullet 3}$

$= 12\sqrt{6}$

Example 3:

Find $5\sqrt{10} \bullet 7\sqrt{10}$

$= 5 \bullet 7\sqrt{10 \bullet 10}$

$= 35 \bullet 10 = 350$

Example 4:

Find $(-3\sqrt{3})(2\sqrt{15})$

$= -3 \bullet 2\sqrt{3 \bullet 15}$

$= -6\sqrt{3 \bullet 3 \bullet 5}$

$= -6 \bullet 3\sqrt{5}$

$= -18\sqrt{5}$

Example 5:

Find $(2\sqrt{5})(7\sqrt{30})$

$= 2 \bullet 7\sqrt{5 \bullet 30}$

$= 14\sqrt{5 \bullet 2 \bullet 3 \bullet 5}$

$= 14 \bullet 5\sqrt{2 \bullet 3}$

$= 70\sqrt{6}$

Find the product of each of the following set of radical expressions showing your work.

1. $(3\sqrt{5})(7\sqrt{11})$

2. $2\sqrt{6} \bullet 4\sqrt{6}$

3. $(-5\sqrt{5})(8\sqrt{10})$

4. $(12\sqrt{3})(-2\sqrt{27})$

5. $11\sqrt{7} \bullet 3\sqrt{14}$

6. $(-10\sqrt{5})(-4\sqrt{50})$

We can find the cube root of a radical expression in the same way as we found the square root of a radical expression. Instead of finding a pair of numbers under the radical, we find a group of three numbers or variables that are the same. Then we can take each group out and it becomes one of those numbers.

Example 1:

Simplify $\sqrt[3]{a^3b^3}$

$$\sqrt[3]{a^3b^3} = \sqrt[3]{a \bullet a \bullet a \bullet b \bullet b \bullet b}$$

Because the index is 3 (we are trying to find the cube root), we look for groups of three numbers or variables under the radical.

$$\sqrt[3]{a \bullet a \bullet a \bullet b \bullet b \bullet b}$$

$$= \sqrt[3]{(a \bullet a \bullet a) \bullet (b \bullet b \bullet b)}$$

$$= ab$$

Example 2:

Simplify $\sqrt[3]{x^4y^5}$

$$\sqrt[3]{x^4y^5} =$$

$$\sqrt[3]{x \bullet x \bullet x \bullet x \bullet y \bullet y \bullet y \bullet y \bullet y}$$

$$= \sqrt[3]{(x \bullet x \bullet x) \bullet x \bullet (y \bullet y \bullet y) \bullet y \bullet y}$$

$$= x \bullet y \quad \sqrt[3]{x \bullet y \bullet y}$$

$$= xy \ \sqrt[3]{xy^2}$$

(Remember to write the index in your answer.)

Example 3:

Simplify $\sqrt[3]{27a^4}$

$$\sqrt[3]{27a^4} = \sqrt[3]{3 \bullet 3 \bullet 3 \bullet a \bullet a \bullet a \bullet a}$$

$$= \sqrt[3]{3 \bullet 3 \bullet 3 \bullet a \bullet a \bullet a \bullet a}$$

$$= \sqrt[3]{(3 \bullet 3 \bullet 3) \bullet (a \bullet a \bullet a) \bullet a}$$

$$= 3 \bullet a \bullet \sqrt[3]{a}$$

$$= 3a \sqrt[3]{a}$$

Example 4:

Simplify $7 \sqrt[3]{200m^5n}$

$$7 \sqrt[3]{200m^5n} = 7 \bullet \sqrt[3]{2 \bullet 10 \bullet 10 \bullet m \bullet m \bullet m \bullet m \bullet m \bullet n}$$

$$= 7 \bullet \sqrt[3]{2 \bullet 2 \bullet 2 \bullet 5 \bullet 5 \bullet m \bullet m \bullet m \bullet m \bullet m \bullet n}$$

$$= 7 \bullet \sqrt[3]{(2 \bullet 2 \bullet 2) \bullet 5 \bullet 5 \bullet (m \bullet m \bullet m) \bullet m \bullet m \bullet n}$$

$$= 7 \bullet 2 \bullet m \bullet \sqrt[3]{5 \bullet 5 \bullet m \bullet m \bullet n}$$

$$= 14m \sqrt[3]{25m^2n}$$

Simplify the following radical expressions showing your work.

1. $\sqrt[3]{64}$

2. $\sqrt[3]{27x^3y^3}$

3. $\sqrt[3]{16a^4}$

4. $\sqrt[3]{24s^7}$

5. $\sqrt[3]{c^2d^3e^4}$

6. $\sqrt[3]{81x^3yz^5}$

Sources

2008 AZ Mathematics Standards
2000 NCTM Principles and Standards, p. 292-294
2008 The Final Report of the National Mathematics Advisory Panel, p. 16