Solving Systems of Equations
An ADE Mathematics Lesson
Days 28-35

Author	ADE Content Specialists
Grade Level	9th grade
Duration	Eight days

Aligns To

Mathematics HS:
Strand 3: Patterns, Algebra, and Functions
Concept 2: Functions and Relationships
PO 4. Use equations, graphs, tables, descriptions, or sets of ordered pairs to express a relationship between two variables.
PO 5. Recognize and solve problems that can be modeled using a system of two equations in two variables.
Concept 3: Algebraic Representations
PO 1. Create and explain the need for equivalent forms of an equation or expression.
PO 7. Solve systems of two linear equations in two variables.
Concept 4: Analysis of Change
PO 1. Determine the slope and intercepts of the graph of a linear function, interpreting slope as a constant rate of change.

Strand 4: Geometry and Measurement
Concept 3: Coordinate Geometry
PO 5. Graph a linear equation or linear inequality in two variables.
PO 7. Determine the solution to a system of linear equations in two variables from the graphs of the equations.

Strand 5: Structure and Logic
Concept 2: Logic, Reasoning, Problem Solving, and Proof
PO 1. Analyze a problem situation, determine the question(s) to be answered, organize given information, determine how to represent the problem, and identify implicit and explicit assumptions that have been made.
PO 2. Solve problems by formulating one or more strategies, applying the strategies, verifying the solution(s), and communicating the reasoning used to obtain the solution(s).
PO 3. Evaluate a solution for reasonableness and interpret the meaning of the solution in the context of the original problem.

Connects To

Mathematics HS:
Strand 3: Patterns, Algebra, and Functions
Concept 2: Functions and Relationships
PO 1. Sketch and interpret a graph that models a given context, make connections between the graph and the context, and solve maximum and minimum problems using the graph.
PO 7. Determine domain and range of a function from an equation, graph, table, description, or set of ordered pairs.
Concept 3: Algebraic Representations
PO 3. Write an equation given a table of values, two points on the line, the slope and a point on the line, or the graph of the line.
PO 4. Determine from two linear equations whether the lines are parallel, perpendicular, coincident, or intersecting but not perpendicular.

Strand 4: Geometry and Measurement
Concept 3: Coordinate Geometry
PO 6. Describe how changing the parameters of a linear function affect the shape and position of its graph.

Strand 5: Structure and Logic
Concept 1: Algorithms and Algorithmic Thinking
PO 1. Select an algorithm that explains a particular mathematical process; determine the purpose of a simple mathematical algorithm.

Overview

It is often necessary to solve two equations with two unknowns. Many problems require the use of two variables. In this lesson, you will learn how to solve systems of equations by graphing, substitution, and elimination. You will learn how to set up and solve contextual problems that involve two equations with two unknowns.

Purpose

Solving systems of equations allows you to solve problems that involve more than one unknown. This lesson shows that there are many different ways to solve systems of equations.

Materials

- Systems of equations worksheets
- Ruler
- Graph paper

Objectives

Students will:

- Solve systems of equations by graphing.
- Solve systems of equations by substitution.
- Solve systems of equations by elimination/linear combination.
- Set up and solve contextual problems that involve two variables.

Lesson Components

Prerequisite Skills: To complete this lesson, you need to be proficient at graphing and solving equations with one or two unknowns. This lesson builds on skills of setting up equations from contextual problems. During the lesson you will identify parallel lines, intersecting lines, points of intersection, and lines that are coincident (lie on the same line). Prior experience with finding the slope of a line is needed.

Vocabulary: *equation, system of equations, simultaneous equations, graphing, no solution, one solution, infinite number of solutions, substitution, elimination, system of inequalities*

Session 1: Solving Systems of Equations using Graphing (1 day)

1. Use graphing to solve systems of two linear equations in two variables.

Session 2: Solving Systems of Equations using Substitution (2 ays)

1. Use substitution to solve systems of two linear equations in two variables.

Session 3: Solving Systems of Equations using Elimination (2 days)

1. Use elimination/linear combination to solve systems of two linear equations in two variables.

Session 4: Using Systems of Equations to Solve Problems (2 days)

1. Set up and solve contextual problems that involve systems of two linear equations in two variables.

Session 5: Solving Systems of Equations (1 day)

1. Choose any method to solve systems of two linear equations in two variables.

Assessment

There are multiple assessments on solving systems of two linear equations in two variables. These assessments will help you identify errors before you move on the next lesson.

Solving Systems of Equations
Overview

We can solve a system of equations in many ways. A **system of equations** is a set of two or more equations that must all be true for the same value(s) (note: also referred to as simultaneous equations). For this lesson, we will graph a system containing only two equations. A **solution to a system of equations** is the value(s) that hold true for all equations in the system.

A system of equations may have **no solution**, **one solution**, or an **infinite number of solutions**. The system solution methods can include but are not limited to graphical, elimination/linear combination, and substitution. Systems can be written algebraically or can be represented in context.

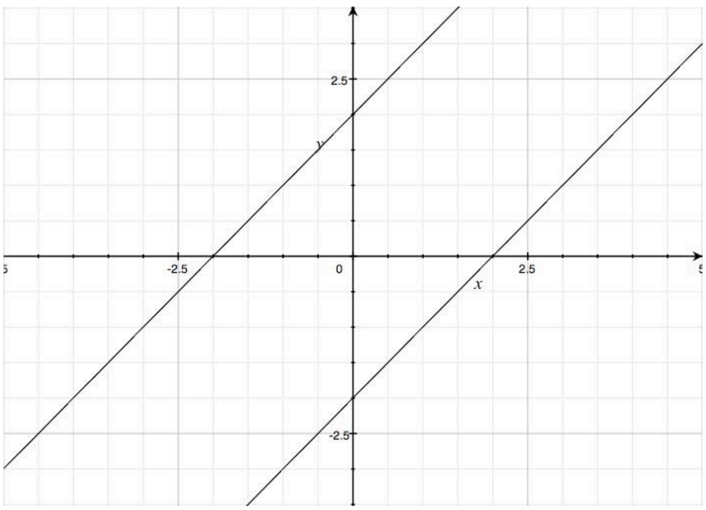
We will learn how to solve a system of two linear equations by graphing in Session 1 and by substitution in Session 2. We will learn how to solve a system of two linear equations by elimination/linear combination in Session 3. We will learn how to solve contextual problems that involve setting up a system of equations in Session 4. In Session 5, you must determine which method to use to solve the system of equations. There is not always a best way to solve a system of equations. Often, it is just personal preference and you must determine which way works best for you to solve the system of equations.

Solving Systems of Equations
Session 1 – Solving Systems of Equations using Graphing

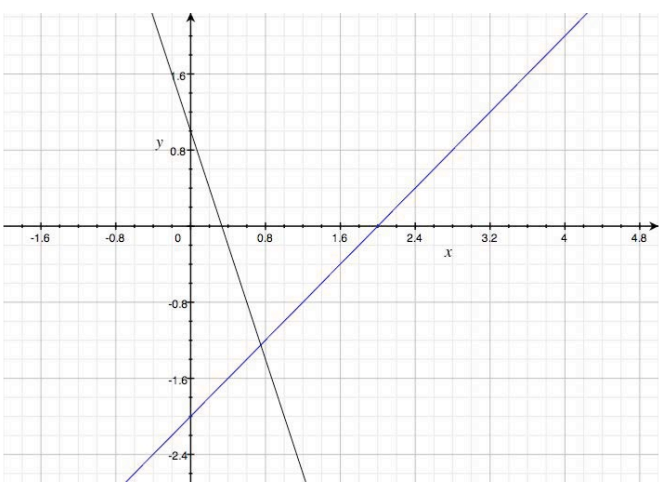
For this session, we solve a system of two linear equations by graphing each line.

When we look at the lines that are graphed, we can determine if they do not meet at all, if they meet in one place, or if they are the same line and therefore meet in all places.

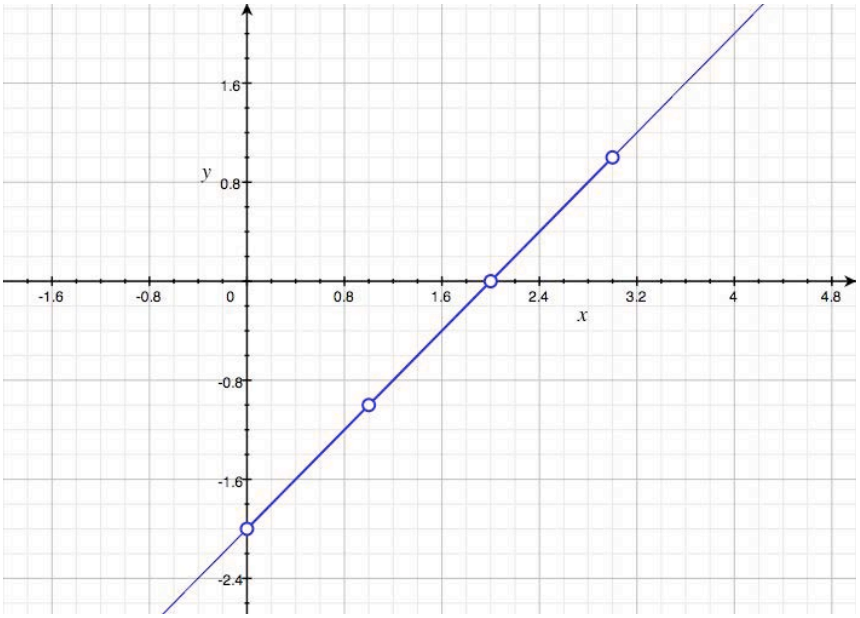
Study the graph of two parallel lines below. How many times do the lines intersect?



Study the graph of two intersecting lines below. How many times do the lines intersect?



Study the graph of two coincident lines (lines that share the same location) below. How many times do the lines intersect?



The definition of a solution for a system of linear equations is, the value(s) that hold true for all equations in the system, the value(s) the lines have in common. Let's investigate what a solution would look like on a graph.

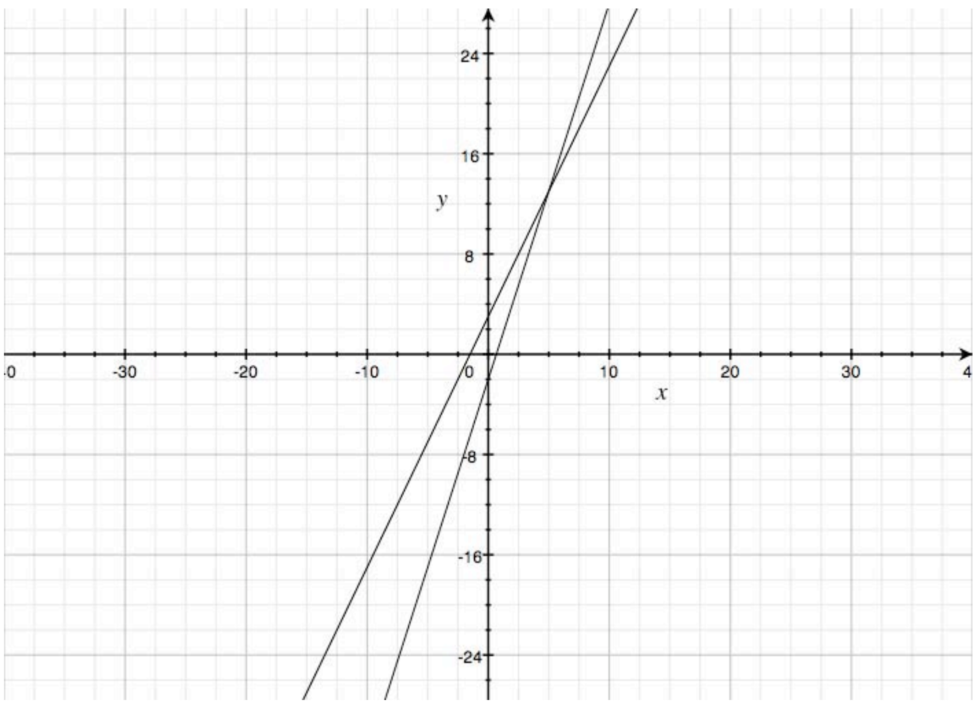
Fill in the tables below.

y = 3x -2	
x	y
0	-2
1	1
2	
3	
4	
5	
6	

y = 2x + 3	
x	y
0	3
1	5
2	
3	
4	
5	
6	

What value do both equations have in common?

Now let's look at the graph of the two lines: y=3x-2 and y=2x+3.



Locate that value the equations had in common. What do you notice?

Now let's think back to the graphs we started with.

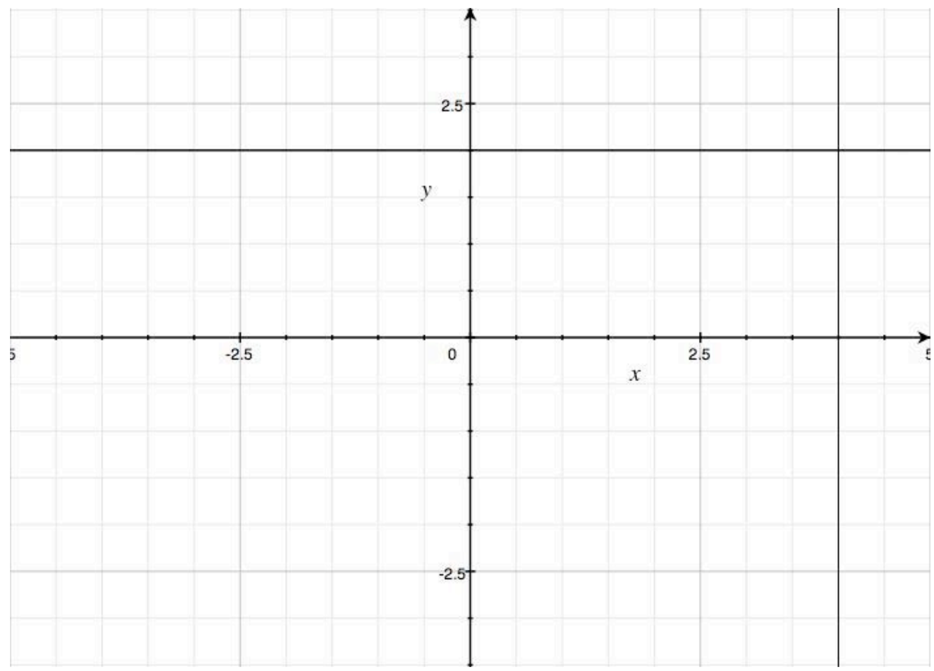
Use what you learned to fill in the table.

Number of solutions	Lines are	Slopes of the lines
	Parallel	The same
	Intersecting	Different
	Coincident	The same

Consider the following examples:

Example 1: Solve the following system of equations by graphing: $\begin{cases} y = 2 \\ x = 4 \end{cases}$

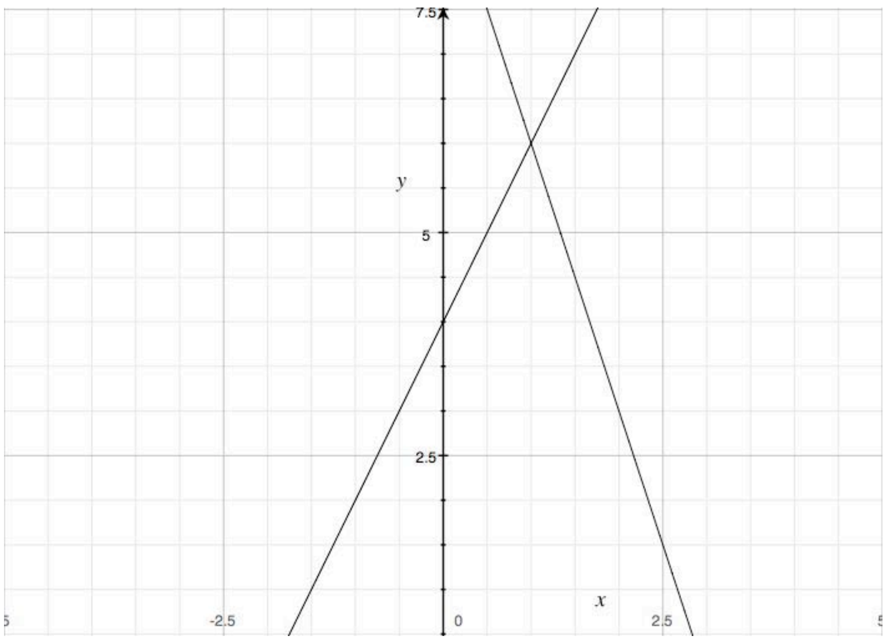
- Graph each equation on the same set of axes.
- After graphing the equations, look to see if the lines intersect.
- If they intersect, note the location where they intersect.
- The place where the lines intersect is the solution to the system of equations.



Study the graphs of $y = 2$ and $x = 4$. The graph of $y = 2$ is a horizontal line with a slope of 0. The graph of $x = 4$ is a vertical line with an undefined slope. The lines intersect at (4, 2). The point (4, 2) is the solution to the system of equations $\begin{cases} y = 2 \\ x = 4 \end{cases}$.

Example 2: Solve the following system of equations by graphing: $\begin{cases} y = 2x + 4 \\ 3x + y = 9 \end{cases}$

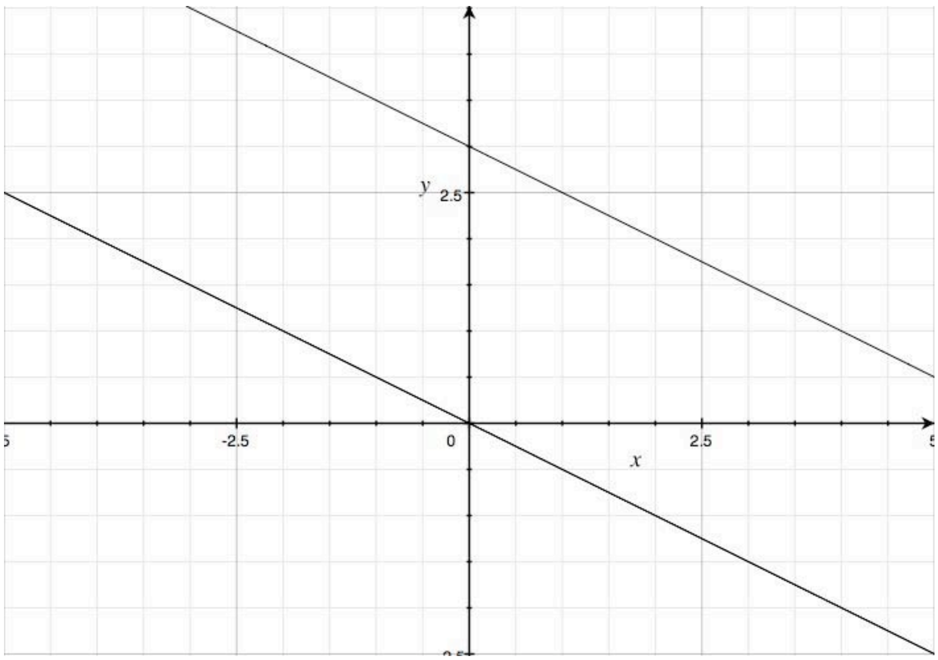
Recall the steps outlined above. Let's graph each equation on the same set of axes. We can graph these equations using a table of values or by placing the equation in slope-intercept form and then graphing.



We observe (1, 6) is the Point of Intersection, the solution for the system of equations.

Example 3: Solve the following system of equations by graphing: $\begin{cases} y = -\frac{1}{2}x \\ x + 2y = 6 \end{cases}$

We can graph these equations using a table of values or by placing the equation in slope-intercept form and then graphing.



Study the graphs of these equations. It appears that the lines are parallel. If we place the second equation in slope-intercept form, we can prove what we observe graphically.

$$x + 2y = 6$$

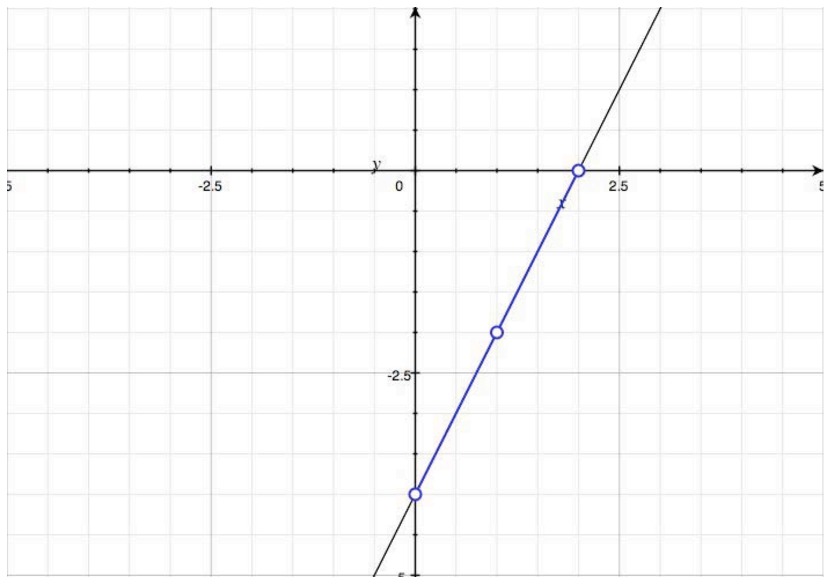
$$2y = -x + 6 \qquad \text{Subtract x from both sides.}$$

$$\frac{2y}{2} = -\frac{x}{2} + \frac{6}{2} \qquad \text{Divide all terms by 2}$$

$$y = -\frac{1}{2}x + 3 \qquad \text{Simplify}$$

Both equations have the same slope proving that <u>the lines are parallel</u>. Therefore there is no point of intersection and <u>there is no solution</u> to this system of equations.

Example 4: Solve the following system of equations by graphing: $\begin{cases} 2x - y = 4 \\ 4x - 8 = 2y \end{cases}$



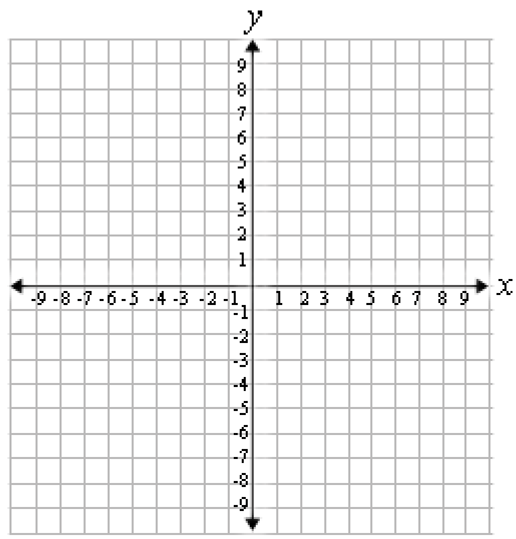
When we graph these equations, we observe that we have the same line. We can manipulate the second equation to look like the first equation in order to prove they are coincident.

$4x - 8 = 2y$	
$\frac{4x-8}{2} = \frac{2y}{2}$	Divide all terms by 2
$2x - 4 = y$	Simplify
$2x = y + 4$	Add four to both sides
$2x - y = 4$	Subtract y from both sides

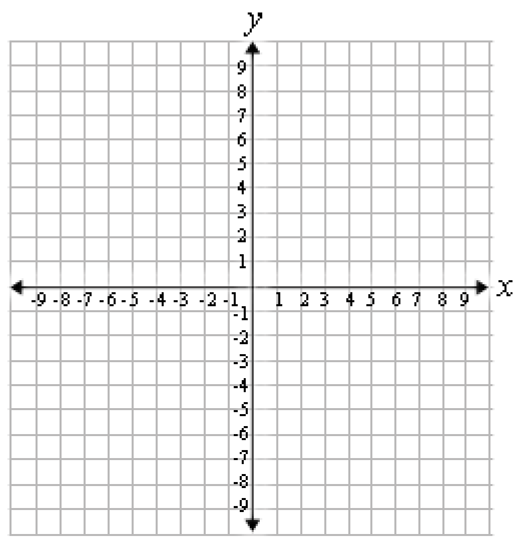
Therefore, any ordered pair that will work in the first equation, $2x - y = 4$, will also work in the second equation, $4x - 8 = 2y$. We have an infinite number of solutions.

Solve these systems of equations by <u>graphing</u>. Indicate the solution to the systems of equations and justify your answer.

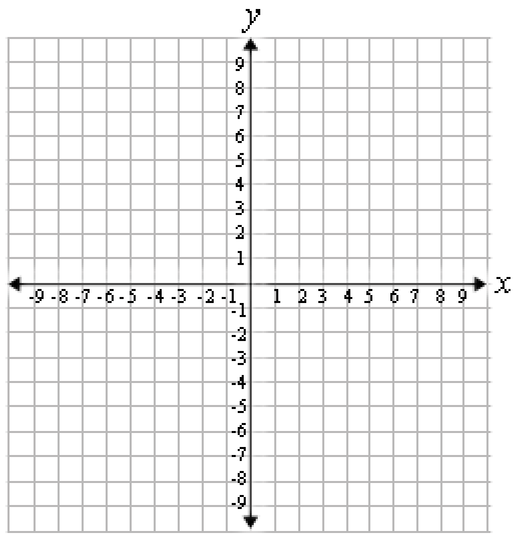
1. Solve the following system of equations: $\begin{cases} x = -3 \\ y = 1 \end{cases}$



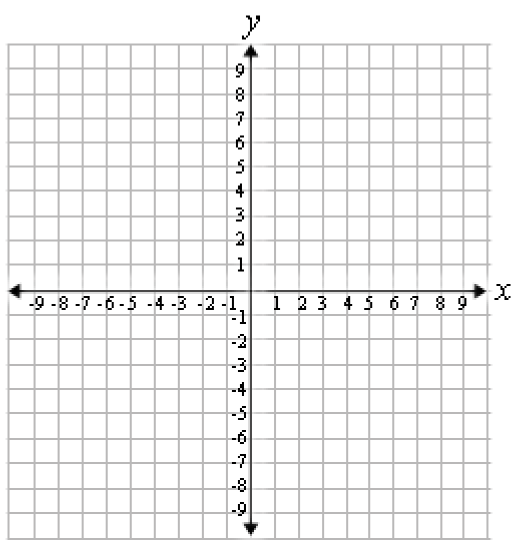
2. Solve the following system of equations: $\begin{cases} x + y = 5 \\ 2y - 4 = x \end{cases}$



3. Solve the following system of equations: $\begin{cases} x + 2y = 4 \\ 2y = 6 - x \end{cases}$



4. Solve the following system of equations: $\begin{cases} 3x - y = 4 \\ 6x + 2y = -8 \end{cases}$



Solving Systems of Equations
Session 2 – Solving Systems of Equations using Substitution

We can solve a system of equations using **substitution**. To solve a system of two linear equations by substitution, follow these steps:

- Solve one equation for one unknown.
- Substitute the value for the unknown in the other equation.
- Simplify the equation and solve for the unknown variable.
- Substitute this value into one of the equations and solve for the other variable.
- Write your solution as an ordered pair.
- Check your answer in both original equations. It is important that you check your answer in both original equations as you may have made a mistake when simplifying your work. If you substitute in at this point, and do the algebra correctly, you may think you have come up with a correct answer when you have not. It is also important that you check your answer in both original equations as your answer may work in one equation but not the other.

Example 1: Solve the following system of equations using **substitution:** $\begin{cases} y = x + 2 \\ 4x + y = 7 \end{cases}$

Step 1: Solve one equation for one unknown. In this system, we have one equation already solve for one variable, $y = x + 2$.

Step 2: Substitute the value for y in the second equation.

$$4x + (x + 2) = 7$$

Step 3: Simplify the resulting equation.

$4x + (x + 2) = 7$	
$(4x + x) + 2 = 7$	Associative Property
$5x + 2 = 7$	Simplify
$5x = 5$	Subtract 2 from both sides
$x = 1$	Divide both sides by 5

Step 4: Now substitute this in one equation and solve for *y* (you can choose either equation. We chose y = x + 2).

$y = x + 2$	
$y = 1 + 2$	Substitute x = 1
$y = 3$	Simplify

Step 5: Write your answer as an ordered pair.

The answer to the system of equations is (1, 3) because x = 1 and y = 3 in both equations.

Step 6: Check your answer in both original equations.

$$\begin{aligned} y &= x + 2 \\ 3 &= 1 + 2 \\ 3 &= 3 \end{aligned}$$

$$\begin{aligned} 4x + y &= 7 \\ 4(1) + 3 &= 7 \\ 4 + 3 &= 7 \\ 7 &= 7 \end{aligned}$$

Example 2: Solve the following system of equations using **substitution:** $\begin{cases} 2x - y = 6 \\ 3x + y = 4 \end{cases}$

Step 1: Solve one equation for one unknown. You can choose either equation.
We chose 3x + y = 4.

$$3x + y = 4$$
$$y = 4 - 3x \qquad \text{Subtract 3 x from both sides}$$

Step 2: Substitute the value for y in the second equation.

$$2x - y = 6$$
$$2x - (4 - 3x) = 6$$

Step 3: Simplify the resulting equation.

$$2x - (4 - 3x) = 6$$
$$2x - 4 + 3x = 6 \qquad \text{Distribute the negative through the parentheses}$$
$$5x - 4 = 6 \qquad \text{Combine like terms}$$
$$5x = 10 \qquad \text{Add 4 to both sides}$$
$$x = 2 \qquad \text{Divide both sides by 5}$$

Step 4: Now substitute this value into one equation and solve for *y.* You can choose either equation. We chose 2x – y = 6.

$$2x - y = 6$$
$$2(2) - y = 6$$
$$4 - y = 6$$
$$-y = 2$$
$$y = -2$$

Step 5: Write your answer as an ordered pair. The answer to the system of equations is $(2,-2)$.

Step 6: Check your answer in both original equations.

$$2x - y = 6$$
$$2(2) - (-2) = 6$$
$$4 + 2 = 6$$
$$6 = 6$$

$$3x + y = 4$$
$$3(2) + (-2) = 4$$
$$6 - 2 = 4$$
$$4 = 4$$

Example 3: Solve the following system of equations using **substitution:** $\begin{cases} y = x + 3 \\ y - x = 5 \end{cases}$

Step 1: Solve one equation for one unknown. Since the first equation, $y = x + 3$,is already solved for y , let's use this equation to substitute into the second equation, $y - x = 5$.

Step 2: Substitute the value for y in the second equation.

$$y - x = 5$$
$$(x + 3) - x = 5$$

Step 3: Simplify the resulting equation.

$$(x + 3) - x = 5$$
$$x - x + 3 = 5$$
$$3 = 5$$

Since we know that 3 does not equal 5, we know that there is no solution to this equation.

Step 4: The lines must be parallel in this system of equations since there is no solution. If we put the second equation in slope-intercept form, we can determine if the lines are parallel.

$$y - x = 5$$
$$y = x + 5 \qquad \text{the second equation in slope - intercept form}$$

$$\text{y} = \text{x} + 3 \qquad \text{first original equation}$$
$$\text{y} = \text{x} + 5 \qquad \text{second original equation in slope - intercept form}$$

Step 5: The coefficient of x in both equations is 1. The slope is 1 for both equations. Lines with the same slope and different y-intercepts are parallel. Therefore, there is no solution because the lines will never intersect.

The answer to the system of linear equations is, "**no solution**".

Step 6: You can check the solution by graphing each line and observing that they are parallel or by finding the slope as we did in step 4.

Example 4: Solve the following system of equations using **substitution:** $\begin{cases} 3x-5y=15 \\ x-5=y \end{cases}$

Step 1: Solve one equation for one unknown. Let's solve the second equation for x and use it to substitute into the first equation.

$$x-5=y$$
$$x=y+5 \qquad \text{add 5 to both sides}$$

Step 2: Substitute the value for x into the first equation.

$$3x-5y=15$$
$$3(y+5)-5y=15$$

Step 3: Simplify the resulting equation.

$$3(y+5)-5y=15$$
$$3y+15-5y=15$$
$$-2y=0$$
$$y=0$$

Step 4: Now substitute this in one equation and solve for y. (you can choose either equation to substitute the value into. We chose the second equation.)

$$x-5=y$$
$$x-5=0$$
$$x=5$$

Step 5: The answer to the system of equations is (5, 0).

Step 6: Check your answer in both original equations.

$$3x-5y=15$$
$$3(5)-5(0)=15$$
$$15-0=15$$
$$15=15$$

$$x-5=y$$
$$5-5=0$$
$$0=0$$

Example 5:

Solve the following system of linear equations using **substitution:** $\begin{cases} 2x+5=2y-1 \\ 3x-2y=7 \end{cases}$

Step 1: Solve one equation for one unknown. Let's solve the first equation for x and use it to substitute into the second equation.

$$\begin{aligned} &2x+5=2y-1 \\ &2x=2y-6 \quad \text{subtract 5 from both sides} \\ &x=y-3 \quad \text{divide all terms by 2} \end{aligned}$$

Step 2: Substitute the value for x into the second equation.

$$3(y-3)-2y=7$$

Step 3: Simplify the resulting equation.

$$\begin{aligned} &3(y-3)-2y=7 \\ &3y-9-2y=7 \\ &3y-2y-9=7 \\ &y-9=7 \\ &y=16 \end{aligned}$$

Step 4: Now substitute the value for y into one equation and solve for x. (You can choose either equation to substitute the value into. We chose the second equation.)

$$\begin{aligned} &3x-27=7 \\ &3x-2(16)=7 \\ &3x-32=7 \\ &3x=39 \\ &x=13 \end{aligned}$$

Step 5: The answer to the system of equations is (13, 16).

Step 6: Check your answer in both original equations.

$$\begin{aligned} &2x+5=2y-1 \\ &2(13)+5=2(16)-1 \\ &26+5=32-1 \\ &31=31 \end{aligned}$$

$$\begin{aligned} &3x-2y=7 \\ &3(13)-2(16)=7 \\ &39-32=7 \\ &7=7 \end{aligned}$$

Solve these systems of two linear equations using <u>substitution</u>. Indicate the solution to the systems of equations and check your answer if possible.

1. Solve the following system of linear equations : $\begin{cases} x = -1 \\ y = 4 \end{cases}$

2. Solve the following system of linear equations : $\begin{cases} x - 2y = 5 \\ 2x + y = 15 \end{cases}$

3. Solve the following system of linear equations : $\begin{cases} y - x = -1 \\ x + y = -5 \end{cases}$

4. Solve the following system of linear equations : $\begin{cases} x + 2y = 4 \\ 2x + 4y = -1 \end{cases}$

5. Solve the following system of linear equations : $\begin{cases} 3x - 3y = -6 \\ x - y = -2 \end{cases}$

Solving Systems of Equations
Session 3 – Solving Systems of Equations using Elimination

We can solve a system of equations using **elimination.** This method is also called **linear combination**. To solve a system of two linear equations using elimination we have to eliminate a variable. We can do that by adding the equations when the coefficient of one variable in one equation is the opposite of the coefficient of the same variable in the second equation.

- If the coefficient of one variable in one equation is the opposite of the same variable in the second equation, add the equations.
- The result will be one equation with one variable.
- Solve the resulting equation for the variable.
- Substitute this value in either original equation and solve for the other variable.
- Check your answer by substituting the solution into both original equations.

Example 1: Solve the following system of equations using elimination. $\begin{cases} x+y=4 \\ x-y=2 \end{cases}$

Step 1: Make sure that the coefficient of one variable in one equation is the opposite of the coefficient of the same variable in the other equation.

The coefficients of y are opposites.

Step 2: Eliminate one variable by adding the equations. The coefficient of the y in the first equation is 1. The coefficient of the y in the second equation is -1. These are opposites so we can add the equations together.

$$\begin{array}{r} x+y=4 \\ \underline{x-y=2} \end{array}$$

$$2x=6$$

Step 3: Solve the resulting equation for the variable.

$$\frac{2x}{2}=\frac{6}{2}$$
$$x=3$$

Step 4: Substitute the value into either original equation and solve for the other variable.

$$x+y=4$$
$$3+y=4$$
$$y=1$$

Step 5: Write the solution as an ordered pair.

The solution is (3, 1)

Step 6: Check your answer by substituting the values into both original equations.

$x+y=4$	$x-y=2$
$3+1=4$	$3-1=2$
$4=4$	$2=2$

Example 2: Solve the following system of equations using elimination. $\begin{cases} 3x+36=5y \\ -9-3x=4y \end{cases}$

Step 1: Make sure that the coefficient of one variable in one equation is the opposite of the coefficient of the same variable in the other equation. If we rewrite the second equation using the commutative property we can better see if we have opposites.

$$3x+36=5y \qquad \textit{and} \quad -9-3x=4y$$
$$-3x-9=4y$$

Now let's write them on top of each other. We have opposite coefficients for x.

$$3x+36=5y$$
$$-3x-9=4y$$

Step 2: Eliminate one variable by adding the equations. The coefficient of the y in the first equation is 1. The coefficient of the y in the second equation is -1. These are opposites so we can add the equations together.

$$\begin{array}{r} 3x+36=5y \\ \underline{-3x-9=4y} \\ 27=9y \end{array}$$

Step 3: Solve the resulting equation for the variable.

$$27=9y$$
$$3=y$$

Step 4: Substitute the value into either original equation and solve for the other variable.

$$3x+36=5y$$
$$3x+36=5(3)$$
$$3x+36=15$$
$$3x=15-36$$
$$3x=-21$$
$$x=-7$$

Step 5: Write the solution as an ordered pair. The solution is (-7, 3)

Step 6: Check your answer by substituting the values into both original equations.

$$3x+36=5y$$
$$3(-7)+36=5(3)$$
$$-21+36=15$$
$$15=15$$

$$-9-3x=4y$$
$$-9-3(-7)=4(3)$$
$$-9+21-12$$
$$12=12$$

Sometime we must use a **linear combination** to make sure that the coefficients of one variable are opposites. To solve a system of two linear equations when the coefficient of a variable in the first equation is not the opposite of the coefficient of the same variable in the second equation, follow these steps which describe a linear combination.

- Choose which variable you would like to eliminate.
- Find the Least Common Multiple (LCM) of that variable by looking at the coefficients of the variable in each equation. For example, if the coefficient of x in one equation is 5 and the coefficient of the x in the second equation is 3, the LCM would be 15.
- Multiply the first equation by the factor that will result in the LCM for the chosen variable.
- Multiply the second equation by the factor that will result in the opposite of the LCM (in this case it would be -15) for the same variable.
- Add the equations which will result in one equation with one variable.
- Solve the equation for the variable.
- Substitute this value into one of the original equations and solve for the other variable.
- Check your solution by substituting the solution into both original equations.

Example 3: Solve the following system of equations using **linear combination**: $\begin{cases} 3x - 5y = 1 \\ x + 2y = 4 \end{cases}$

Step 1: Since the LCM of the coefficients of the variable x is 3, and the LCM of the coefficients of the variable y is 10, we can choose either x or y to eliminate. Let's eliminate x. To make the coefficients of the x term opposites, we need to multiply the second equation by -3. The coefficient of the x in the second equation will become -3. These are opposites so we can add the equations together.

$$\begin{aligned} &x + 2y = 4 \\ &-3(x + 2y) = -3(4) \quad \text{Multiply all terms by - 3} \\ &-3x - 6y = -12 \qquad \text{Simplify} \end{aligned}$$

The rewritten system: $\begin{cases} 3x - 5y = 1 \\ -3x - 6y = -12 \end{cases}$

Step 2: Add the equations and solve for the variable.

$$\begin{cases} 3x - 5y = 1 \\ -3x - 6y = -12 \end{cases}$$
$$\overline{\quad -11y = -11 \quad}$$

Step 3: Solve the resulting equation for the variable.

$$\begin{aligned} -11y &= -11 \\ \frac{-11y}{-11} &= \frac{-11}{-11} \\ y &= 1 \end{aligned}$$

Step 4: Substitute the value for the variable in one of the original equations and solve for the second variable.

$$\begin{aligned} &x + 2y = 4 \\ &x + 2(1) = 4 \\ &x + 2 = 4 \\ &x = 2 \end{aligned}$$

Step 5: The solution to the system of equations is (2, 1).

Step 6: Check the solution in both original equations.

$$\begin{aligned} &3x - 5y = 1 \\ &3(2) - 5(1) = 1 \\ &6 - 5 = 1 \\ &1 = 1 \end{aligned}$$

$$\begin{aligned} &x + 2y = 4 \\ &2 + 2(1) = 4 \\ &4 = 4 \end{aligned}$$

Let's solve the same system of equations $\begin{cases} 3x-5y=1 \\ x+2y=4 \end{cases}$ but eliminate the variable *y* first.

Step 1: Find a linear combination to eliminate y. The LCM of 5 and 2 is 10. We can multiply the first equation by 2 and the second equation by 5 and our *y* coefficients should be -10 and 10.

$$\begin{cases} 3x-5y=1 \\ x+2y=4 \end{cases}$$

$$2(3x)-2(5y)=2(1)$$
$$5(x)+5(2y)=5(4)$$

$$\begin{cases} 6x-10y=2 \\ 5x+10y=20 \end{cases}$$

Step 2: Add the equations and solve for the variable.

$$\begin{cases} 6x-10y=2 \\ 5x+10y=20 \end{cases}$$

$$11x=22$$

Step 3: Solve for x.

$$11x=22$$
$$\frac{11x}{11}=\frac{22}{11}$$
$$x=2$$

Step 4: Substitute the value for the variable in one of the original equations and solve for the second variable.

$$3x-5y=1$$
$$3(2)-5y=1$$
$$6-5y=1$$
$$-5y=-5$$
$$\frac{-5y}{-5}=\frac{-5}{-5}$$
$$y=1$$

Step 5: The solution to the system of equations is (2, 1).

Step 6: Check the solution in both original equations.

$$3x-5y=1$$
$$3(2)-5(1)=1$$
$$6-5=1$$
$$1=1$$

$$x+2y=4$$
$$2+2(1)=4$$
$$4=4$$

Note – We arrived at the very same solution whether we eliminated x first or y first.

Example 4: Solve the following system of equations by **elimination**: $\begin{cases} 6x - 3y = 0 \\ x + y = 1 \end{cases}$

Step 1: Find a linear combination to eliminate y. The LCM of 3 and 1 is 3.

$$\begin{cases} 6x - 3y = 0 \\ x + y = 1 \end{cases}$$

$$\begin{aligned} &6x - 3y = 0 \\ &3(x) + 3(y) = 3(1) \end{aligned}$$

$$\begin{cases} 6x - 3y = 0 \\ 3x + 3y = 3 \end{cases}$$

Step 2: Add the equations and solve for the variable.

$$\frac{\begin{cases} 6x - 3y = 0 \\ 3x + 3y = 3 \end{cases}}{9x = 3}$$

Step 3: Solve for x.

$$\begin{aligned} &9x = 3 \\ &\frac{9}{9}x = \frac{3}{9} \\ &x = \frac{1}{3} \end{aligned}$$

Step 4: Substitute the value for the variable in one of the original equations and solve for the second variable.

$$\begin{aligned} &6x - 3y = 0 \\ &6\left(\frac{1}{3}\right) - 3y = 0 \\ &2 - 3y = 0 \\ &-3y = -2 \\ &\frac{-3y}{-3} = \frac{-2}{-3} \\ &y = \frac{2}{3} \end{aligned}$$

Step 5: The solution to the system of equations is $\left(\frac{1}{3}, \frac{2}{3}\right)$.

Step 6: Check the solution in both original equations.

$$6x - 3y = 0$$
$$6\left(\frac{1}{3}\right) - 3\left(\frac{2}{3}\right) = 0$$
$$2 - 2 = 0$$
$$0 = 0$$

$$x + y = 1$$
$$\frac{1}{3} + \frac{2}{3} = 1$$
$$1 = 1$$

Solve this same system of equations $\begin{cases} 6x - 3y = 0 \\ x + y = 1 \end{cases}$ by eliminating "x" first. Remember, you should arrive at the same solution, $\left(\frac{1}{3}, \frac{2}{3}\right)$.

Example 5: Solve the following system of equations by **elimination**: $\begin{cases} 5m + 2n = -8 \\ 4m + 3n = 2 \end{cases}$

1. Explain how to eliminate "m" first in the system of equations above.

2. Explain how to eliminate "n" first in the system of equations above.

3. In the space below, solve the system of equations by eliminating either variable first. Remember to check your solution in both original equations.

Check: The correct solution to the system of equations $\begin{cases} 5m + 2n = -8 \\ 4m + 3n = 2 \end{cases}$ is (-4, 6).

Example 6: Solve the following system of equations using **elimination**: $\begin{cases} 2x - y = 3 \\ 2y - 3x = -2 \end{cases}$

Sometimes it is necessary to rearrange the terms in an equation to be able to add the equations together. The LCM of y is 2.

Step 1:

$$\begin{cases} 2x - y = 3 \\ 2y - 3x = -2 \end{cases}$$

$\begin{cases} 2x - y = 3 \\ -3x + 2y = -2 \end{cases}$ Rewrite the second equation using the commutative property.

$\begin{matrix} 2(2x) - 2(y) = 2(3) \\ -3x + 2y = -2 \end{matrix}$ Multiply all terms in the first equation by 2 so the coefficients of y are opposites.

$\begin{cases} 4x - 2y = 6 \\ -3x + 2y = -2 \end{cases}$ Our rewritten system is shown.

Step 2:

$$\begin{array}{c} \begin{cases} 4x - 2y = 6 \\ -3x + 2y = -2 \end{cases} \\ \hline x = 4 \end{array}$$

Step 3: The equation is already solved for x.

$$x = 4$$

Step 4:

$$\begin{aligned} &2x - y = 3 \\ &2(4) - y = 3 \\ &8 - y = 3 \\ &y = 5 \end{aligned}$$

Step 5: The solution to the system of equations is (4, 5).

Step 6:

$$\begin{aligned} &2x - y = 3 && 2y - 3x = -2 \\ &2(4) - 5 = 3 && 2(5) - 3(4) = -2 \\ &8 - 5 = 3 && 10 - 12 = -2 \\ &3 = 3 && -2 = -2 \end{aligned}$$

Example 7: Solve the following system of equations using **elimination**: $\begin{cases} 3x+6y=12 \\ 6x+4y=-8 \end{cases}$

Sometimes it is necessary to multiply by a negative number in order to get *opposite* coefficients.

Step 1:

$$\begin{cases} 3x+6y=12 \\ 6x+4y=-8 \end{cases}$$

6 is the LCM for the x coefficients but **BOTH** are positive. I will need to multiply the first equation by -2 to make sure they are opposites. Make sure to multiply all terms in the first equation by 2 so the coefficients of y are opposites.

$$\begin{aligned} &-2(3x+6y)=-2(12) \\ &-6x-12y=-24 \end{aligned}$$

$\begin{cases} -6x-12y=-24 \\ 6x+4y=-8 \end{cases}$ Our rewritten system is shown.

Step 2:

$$\begin{cases} -6x-12y=-24 \\ \underline{6x+4y=-8} \end{cases}$$

$$-8y=-32$$

Step 3: Solve for y.

$$\begin{aligned} &-8y=-32 \\ &\frac{-8y}{-8}=\frac{-32}{-8} \\ &y=4 \end{aligned}$$

Step 4: Substitute it back into an original equation.

$$\begin{aligned} &6x+4y=-8 \\ &6x+4(4)=-8 \\ &6x+16=-8 \\ &6x=-16-8 \\ &6x=-24 \\ &x=-4 \end{aligned}$$

Step 5: The solution to the system of equations is (-4, 4).

Step 6: Check the solution by substituting it into both original equations.

$$\begin{aligned} &3x+6y=12 \\ &3(-4)+6(4)=12 \\ &-12+24=12 \\ &12=12 \end{aligned}$$

$$\begin{aligned} &6x+4y=-8 \\ &6(-4)+4(4)=-8 \\ &-24+16=-8 \\ &-8=-8 \end{aligned}$$

Solve these systems of equations using <u>elimination</u>. Indicate the solution to the systems of equations and check your answer if possible.

1. Solve the following system of equations : $\begin{cases} 6x + 4y = 24 \\ 2x - 4y = 8 \end{cases}$

2. Solve the following system of equations : $\begin{cases} x - 2y = 5 \\ 2x + y = 15 \end{cases}$

3. Solve the following system of equations : $\begin{cases} c + d = -1 \\ -2c - 3d = 0 \end{cases}$

4. Solve the following system of equations : $\begin{cases} x + 2y = 4 \\ 2x + 4y = -1 \end{cases}$

5. Solve the following system of equations : $\begin{cases} 3m - 3n = -6 \\ m - 2n = -2 \end{cases}$

Solving Systems of Equations
Session 4 – Using Systems of Equations to Solve Problems

It is often necessary to set up two equations with two variables to solve a problem. Each of the problems below can be solved using a system of equations. Follow the steps below to solve each problem.

- Read the problem twice to determine the context of the problem and to identify the key components of the problem. There is very often extraneous (unnecessary) information in the problem so be careful to thoroughly understand the problem.
- Identify the unknown values and assign different variables to each.
- Set up a system of equations to identify each given fact.
- Solve the system of equations using an appropriate method.
- Answer all the questions in the original problem.
- Check your answers in the original problem. You may have set up the equation incorrectly. If you check your answers using the equation you made then it might indicate you have a correct answer when you do not.

Example 1: Find two numbers whose sum is 64 and whose difference is 42.

Solution:

The problem is asking us to find two numbers that when added equal 64 and when subtracted equal 42.

- Let x = one number.   Let y = the second number.
- Set up a system of equations based on the information given. $\begin{cases} x+y=64 \\ x-y=42 \end{cases}$
- Solve the system of equations using any appropriate method. In this case, since the coefficients of “y” are opposites, we can use elimination very easily.

$$\begin{cases} x+y=64 \\ x-y=42 \end{cases}$$

$$2x=106$$

$$\frac{2x}{2}=\frac{106}{2}$$

$$x=53$$

$$x+y=64$$

$$53+y=64$$

$$y=11$$

The numbers are 53 and 11. To check the solution, go back to the words of the problem.

Find two numbers whose sum is 64 53+11=64

and whose difference is 42 53-11=42

We know that we have the correct answer.

Example 2:

The sum of two numbers is 16. Three times the greater number equals the sum of four times the lesser number increased by 6. Find the numbers.

Solution: The problem is asking us to find two numbers that when added equal 16. We know that multiplying 3 times the larger number is 4 times the smaller number plus 6.

- Let x = larger number. Let y = the smaller number.
- Set up a system of equations based on the information given. The first equation will be $x + y = 16$. The second equation is $3x = 4y + 6$. Our system of equations is

  $$\begin{cases} x + y = 16 \\ 3x = 4y + 6 \end{cases}$$

- Let's use substitution to solve this problem.

$$x + y = 16$$

$$x = 16 - y$$

$$3x = 4y + 6$$

$$3(16 - y) = 4y + 6$$

$$48 - 3y = 4y + 6$$

$$42 - 3y = 4y$$

$$42 = 7y$$

$$y = 6$$

$$x + y = 16$$

$$x + 6 = 16$$

$$x = 10$$

The numbers are 10 and 6.

Check the answers in the original word problem. Two numbers total 16 $10 + 6 = 16$

Three times the larger number is 4 times the smaller number increased by 6. 3 times 10 is 30. 4 times 6 is 24. 24 increased by 6 is 30.

30=30

Since the arithmetic works for both facts in the original problem, we know that we have the correct answer.

Example 3: Larissa has $1.95 in quarters and dimes. She has a total of 9 coins. How many coins are quarters and how many are dimes?

Solution: The problem is telling us that Larissa has $1.95 but that she only has quarters and dimes. We are to find the number of each coin that Larissa has.

- Let q represent the number of quarters that Larissa has and d represents the number of dimes that Larissa has.
- Set up the equations. $q + d = 9$ represents the fact that Larissa has 9 coins. Since a quarter is worth $.25 and a dime is worth $.10 we can set up the second equation using this information. $.25q + .10d = 1.95$. Then our system of equations becomes

$$\begin{cases} q + d = 9 \\ .25q + .10d = 1.95 \end{cases}$$

- Now we can solve this system of equations using any method we wish. Let's use elimination. If we multiply the second equation by -10, we have opposite numbers for the coefficients of d.

$$\begin{cases} q + d = 9 \\ .25q + .10d = 1.95 \end{cases}$$

$$\begin{cases} q + d = 9 \\ -10(.25)q - 10(.10d) = (-10)1.95 \end{cases}$$

$$\begin{cases} q + d = 9 \\ -2.5q - d = -19.5 \end{cases}$$

$$\begin{cases} q + d = 9 \\ -2.5q - d = -19.5 \end{cases}$$

$$-1.5q = -10.5$$

$$\frac{-1.5}{-1.5}q = \frac{-10.5}{-1.5}$$

$$q = 7$$

$$q + d = 9$$

$$7 + d = 9$$

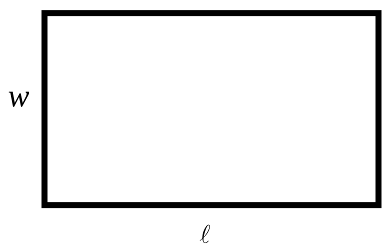
$$d = 2$$

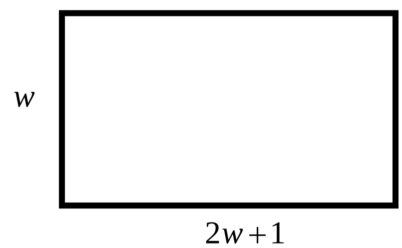
The solution to the problem is that Larissa has 7 quarters and 2 dimes. Check the solution in the original equation. $7 + 2 = 9$. $7(\$.25) = \$1.75$ and $2(\$.10) = \$.20$. $\$1.75 + .20 + \$1.95$. Since the arithmetic fits for the original problem, we know we have the correct solution.

Example 4: The perimeter of a rectangle is 20 inches. The length of the rectangle is one more than twice its width. What is the area of the rectangle?

Solution: This problem is telling us that we have a rectangle and gives us information about the relationship between the width of the rectangle and its length. Note that the problem is asking us to find the area of the rectangle so we must first find the dimensions of it and then find its area. It helps to draw a diagram when solving this type of problem.

- Let w equal the width of the rectangle and l equal the length of the rectangle.
- Now draw a diagram.





- Set up the equations. Perimeter is 2 times the width plus 2 times the length.

$$P = 2w + 2\ell$$

$$20 = 2w + 2\ell$$

$$\ell = 2w + 1$$

- Solve the system of equations. $\begin{cases} 20 = 2w + 2\ell \\ \ell = 2w + 1 \end{cases}$
- Simplify the first equation:

$$20 = 2w + 2\ell$$

$$20 = 2(w + \ell)$$

$$\frac{20}{2} = \frac{2(w + \ell)}{2}$$

$$10 = w + \ell$$

- Now use substitution to solve the system of equations:

$$10 = w + \ell$$

$$10 = w + 2w + 1$$

$$10 = 3w + 1$$

$$9 = 3w$$

$$w = 3$$

- Substitute in this value to find the length.

  $2w + 1 = \ell \qquad 2(3) + 1 = 7 \qquad 6 + 1 = \ell \qquad 7 = \ell$

  The dimensions of the rectangle are 3 by 7. Check this solution first in the original problem before finding the area of the rectangle. Perimeter equals

  $2w + 2\ell = 20 \qquad 2(3) + 2(7) = 20 \qquad 2w + 1 = \ell \qquad 2(3) + 1 = 7$

- Now find the area of the rectangle. $A = \ell \bullet w$ $3 \bullet 7 = 21$ square units.

Solve the following problems using a system of two linear equations. Show your work. Make certain to answer all the questions posed in the problems.

1. The tens digit of a two-digit number is two more than twice the units digit. The sum of the digits is 11. Use a system of equations and any method to find the two-digit number.

2. Billy bought a pair of shorts and a shirt. His bill without tax was $35.00. The shirt cost $10 less than twice the cost of the shorts. What was the cost of each item of clothing?

3. At the supermarket this week, butter cost four times more than margarine. Together, the margarine and butter cost $7.25. What did the butter and the margarine cost individually?

Solving Systems of Equations
Session 5 – Solving Systems of Equations

Solve the following system of two linear equations showing your work. You may use any method you wish. Check your solution for at least two of these systems graphically.

1. $\begin{cases} x+5y=4 \\ 3x+15y=-1 \end{cases}$

2. $\begin{cases} 4a+5b=6 \\ 6a-9b=-2 \end{cases}$

3. $\begin{cases} s=3t-3 \\ 2s-6t-=6 \end{cases}$

4. $\begin{cases} x+y=3 \\ 2y=1-2x \end{cases}$

5. $\begin{cases} \frac{1}{2}x+2y=12 \\ x-2y=6 \end{cases}$

6. $\begin{cases} 2x-y=-4 \\ x+y=4 \end{cases}$

7. $\begin{cases} 2x+5y=7 \\ y=-\frac{2}{5}x+\frac{7}{5} \end{cases}$

8. $\begin{cases} \frac{1}{2}x-2y=9 \\ y=3-x \end{cases}$

9. $\begin{cases} 8m - 7n = -10 \\ 4m + 4n = 3 \end{cases}$

10. $\begin{cases} x = 2y \\ 4x + 4y = 3 \end{cases}$

11. $\begin{cases} x + y = \frac{1}{2} \\ x = -2y \end{cases}$

Solving Systems of Equations
Assessment 1

1. Describe how to solve a system of two linear equations by graphing.

2. Describe how to solve a system of two linear equations by substitution.

3. Describe how to solve a system of two linear equations by elimination/linear combination.

4. Explain what the graph shows when there is no solution to a system of two linear equations. Give an example of a system of two linear equations that has no solution (sketch a graph or write equations).

5. Explain what the graph shows when there are an infinite number of solutions to a system of two equations. Give an example of a system of two equations that has an infinite number of solutions (sketch a graph or write equations).

6. Explain what the graph shows when there is one solution to a system of two equations. Give an example of a system of two equations that has exactly one solution (sketch a graph or write equations).

Solving Systems of Equations
Assessment 2

Consider the following system of linear equations: $\begin{cases} 4x + y = 0 \\ x + 2y = -7 \end{cases}$

1. Solve the above system using elimination.

2. Check your answer to #1 using both original equations.

3. Solve the system using substitution.

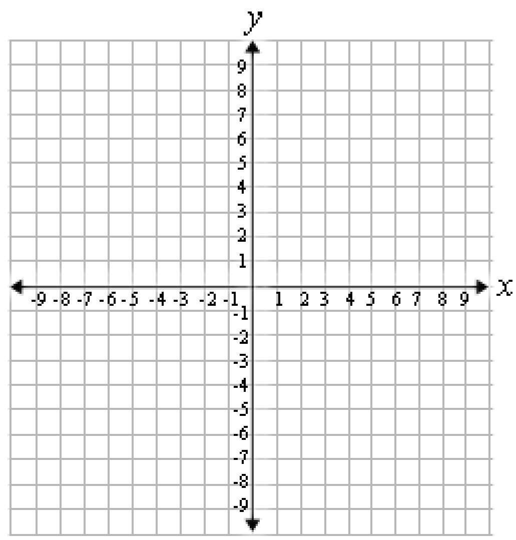
4. Solve the system of equations graphically. Make certain to mark the units on the axes and to write your answer as an ordered pair. Use graph paper to show your work and solution.

Solving Systems of Equations
Assessment 3

Answer the following questions completely **showing your work.**

1. Solve the following system of equations **graphically**.

$$\begin{cases} 3x - y = 4 \\ 6x + 2y = -8 \end{cases}$$



2. Solve the following system of equations using **substitution**.

$$\begin{cases} x - 2y = 5 \\ 3x - 5y = 8 \end{cases}$$

3. Solve the following system of equations using **elimination**.

$$\begin{cases} 5m + 2n = -8 \\ 4m + 3n = 2 \end{cases}$$

4. Find two numbers whose sum is 50 and whose difference is 28. Solve the problem using a system of linear equations.

Extensions

1. Solve systems of two linear inequalities in two variables.

Solving Systems of Equations
Extension

We can also have a system of linear inequalities. We usually solve these systems by graphing each inequality and determining the regions of overlap.

A **solution to a system of linear inequalities** is the set of all values that make each inequality true. When we graph inequalities, remember that a solid line indicates that the line is included in the inequality. A dotted line indicates that the line is not included in the inequality.

To solve a system of linear inequalities, follow these steps

- Graph each line that is part of the system on the same coordinate grid. It is very helpful to graph each inequality using a separate color.
- Graph a solid line if the inequality is ≤ or ≥ since this indicates that the solution includes the line.
- Graph a dotted line if the inequality is < or > since this indicates that the solution does not include the line.
- Choose one point on the graph. If that point makes the inequality a correct statement, color all points on the side of the line that includes that point.
- If that point makes the inequality an incorrect statement, color all points on the side of the line that does not include that point.
- The region (set of points) that has both shades (where the color overlaps) is the solution set for the system of inequalities.

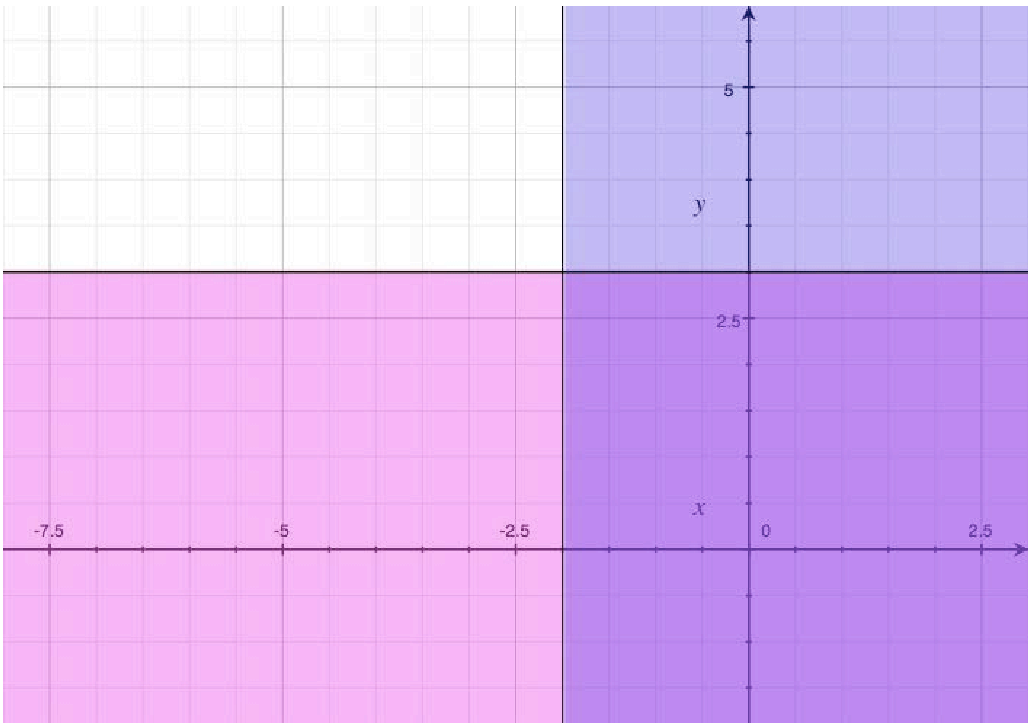
Example 1: Solve the following system of inequalities: $\begin{cases} y \le 3 \\ x \ge -2 \end{cases}$

Solution:

Since the two inequalities contain ≤ and ≥, we will graph the lines $y = 3$ and $x = -2$ using a solid line. Graph each line on the same coordinate grid indicating which line is which (try using different colors). Choose the point (0, 0) to test each inequality.

Choosing (0, 0) with the inequality $y \le 3$ means that $0 \le 3$. This is a true statement so color the side of the line that contains the point (0, 0).

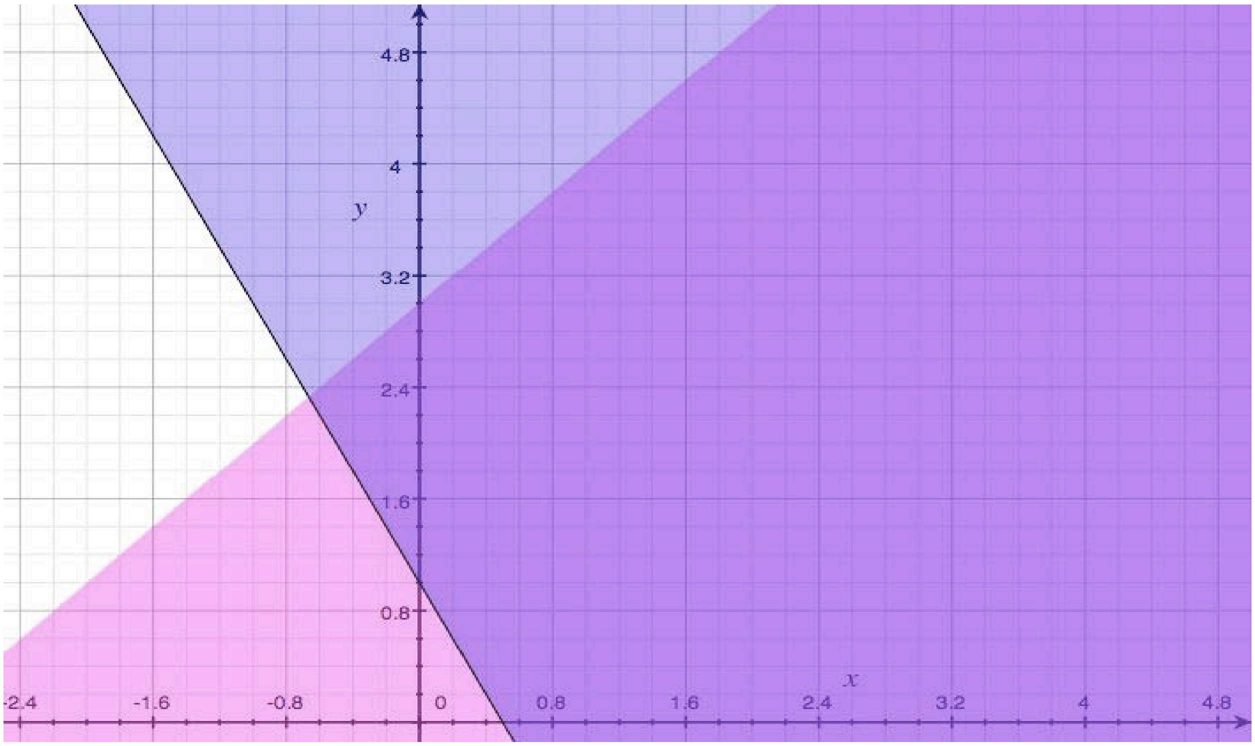
Now try $x \ge -2$ with the same point $0 \ge -2$ This is also true statement so color the side of the line that contains that point.



The solution to the set of inequalities is the set of all points that are both colors.

Example 2: Solve the following system of inequalities: $\begin{cases} y < x+3 \\ y \geq -2x+1 \end{cases}$

Solution: Since the first equation contain <, we will graph the line $y = x+3$ using a dotted line. Since the second equation contains ≥, we will graph the line $y = -2x+1$ using a solid line.



Choose the point (0, 0) as a test point for each inequality.

$y < x+3$

$0 < 0+3$

$0 < 3$

This is true statement so color all the points on the side of the line $y = x+3$ that contains (0, 0).

$y \geq -2x+1$

$0 \geq -2 \bullet 0+1$

$0 \geq 1$

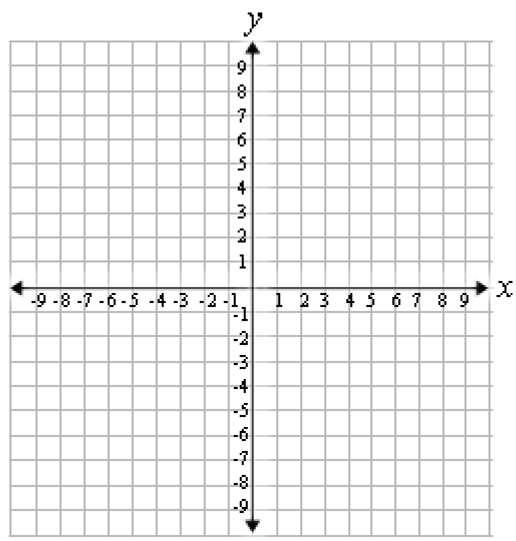
This is not a true statement so color all the points on the side of the line that does not contain the point (0, 0).

The solution to the set of inequalities is the set of all points that are both colors.

Answer the following questions completely <u>showing your work</u>.

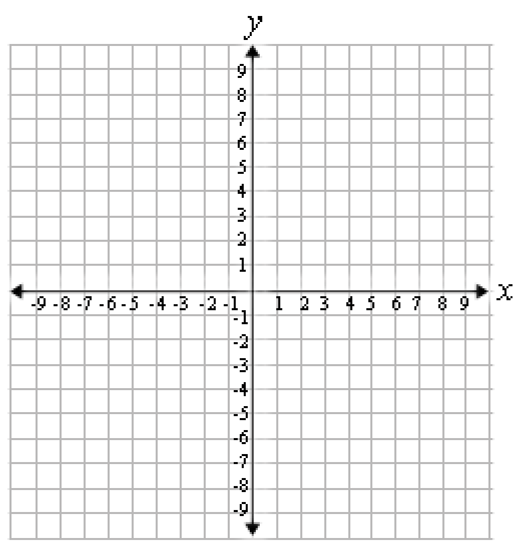
1. Solve the following system of inequalities: $\begin{cases} y \geq -x + 2 \\ y > 2x - 1 \end{cases}$

   Show your work. Use a different color to represent each inequality.



2. Solve the following system of inequalities: $\begin{cases} y < x - 4 \\ y \leq -2x + 3 \end{cases}$

   Show your work. Use a different color to represent each inequality.



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