Simplifying Algebraic Expressions
An ADE Mathematics Lesson
Days 21-27

Author	ADE Content Specialists
Grade Level	9th grade
Duration	Seven days

Aligns To

Mathematics HS:
Strand 3: Patterns, Algebra, and Functions
Concept 3: Algebraic Representations
PO 1. Create and explain the need for equivalent forms of an equation or expression.
PO 8. Simplify and evaluate polynomials, rational expressions, expressions containing absolute value, and radicals.
PO 9. Multiply and divide monomial expressions with integer exponents.
PO 10. Add, subtract, and multiply polynomial and rational expressions.

Strand 5: Structure and Logic
Concept 2: Logic, Reasoning, Problem Solving, and Proof
PO 2. Solve problems by formulating one or more strategies, applying the strategies, verifying the solution(s), and communicating the reasoning used to obtain the solution(s).
PO 3. Evaluate a solution for reasonableness and interpret the meaning of the solution in the context of the original problem.
PO 4. Generalize a solution strategy for a single problem to a class of related problems; explain the role of generalizations in inductive and deductive reasoning.
PO 7. Find structural similarities within different algebraic expressions and geometric figures.

Connects To

Mathematics HS:
Strand 1: Number and Operations
Concept 2: Numerical Operations
PO 1. Solve word problems involving absolute value, powers, roots, and scientific notation.
PO 2. Summarize the properties of and connections between real number operations; justify manipulations of expressions using the properties of real number operations.

Strand 3: Patterns, Algebra, and Functions
Concept 3: Algebraic Representations
PO 12. Factor quadratic polynomials in the form of ax^2+bx+c where *a, b*, and *c* are integers.

Strand 5: Structure and Logic
Concept 2: Logic, Reasoning, Problem Solving, and Proof
PO 1. Analyze a problem situation, determine the question(s) to be answered, organize given information, determine how to represent the problem, and identify implicit and explicit assumptions that have been made.

Overview
Simplifying algebraic expressions is an important skill. There are several types of problems that will be explored in this lesson. The first steps are to represent algebraic expressions in equivalent forms and simplify rational expressions. Next, you will explore multiplication and division of monomial expressions with whole number exponents. Lastly, you will solve problems with addition, subtraction, and multiplication of polynomial and rational expressions.

Purpose

Simplifying algebraic expressions allows you to work with more complicated algebraic equations and concepts. Adding, subtracting, multiplying, and dividing monomial and polynomial expressions allows you to simplify work before moving on to more complicated mathematical tasks.

Materials

- Algebraic worksheets

Objectives

Students will:

- Simplify algebraic expressions with whole number exponents.
- Simplify rational expressions.
- Multiply and divide monomial expressions with whole number exponents.
- Add, subtract, and multiply polynomial and rational expressions.

Lesson Components

Prerequisite Skills: This lesson builds on grade 7 and 8 skills of simplifying and solving equations as well as simplifying numeric expressions with positive exponents.
You must be able to factor trinomials in order to complete Session 2.

Vocabulary: *monomial, binomial, polynomial, exponent, zero exponent, rational expression, leading coefficient, degree of polynomial*

Session 1: Multiplying Monomials, Dividing Monomials, and Zero Exponents (4 days)

1. Simplify algebraic expressions with whole number expressions.
2. Simplify rational expressions.
3. Multiply and divide monomial expressions with whole number exponents.

Session 2 (3 days)

1. Add, subtract, and multiply polynomial and rational expressions.

Assessment

There are two assessments that will help pinpoint misconceptions before moving on to more complex algebraic expressions and equations.

Simplifying Algebraic Expressions
Background Vocabulary

There are many terms with which you have already become familiar. Before moving on to simplifying algebraic expressions and working with rational expressions, let's review some of these terms. In the space provided define each term in your own words. Afterwards, compare your answer to the definitions provided. Are you saying the same thing? It is beneficial to understand terms in your own words as you gain a deeper understanding of the concept.

Term	Definition
algebraic expression	
monomial	
binomial	
polynomial	
degree of a polynomial	
exponent	
leading coefficient	
rational expression	

Term	Definition
algebraic expression	a group of numbers, symbols, and variables that express a single or series of mathematical operations (e.g., $2x+4-16y$)
monomial	an algebraic expression consisting of a single term that does not require any addition or subtraction (e.g., 5y)
binomial	an algebraic expression consisting of two terms (e.g., $x+3$, $4a-6$)
polynomial	an expression containing more than one monomial connected by addition or subtraction(e.g., $3x^2+2x+7$, $4x^5-9x^3+2x+7$)
degree of a polynomial	the degree of the highest term of the polynomial (e.g., The degree of $3x+2x^2+4-7x^5-3x+10x^4$ is 5 because 5 is the greatest exponent)
exponent	a number placed to the right and above (superscript) a non-zero base that indicates the operation of repeated multiplication (e.g., in 5^7 the exponent is 7)
leading coefficient	the coefficient of the term of the highest degree in a polynomial (e.g., in the expression $15x-10x^3-11x^6+7x^2+3.5$ -11 is the leading coefficient)
rational expression	the quotient of two polynomials in the form $\frac{A}{B}$, where A and B are polynomials and where *B* can **never** equal 0.(e.g., $\frac{2x+1}{3x^2-9}$, $3x^2-9\neq 0$)

If you are unfamiliar with any of these terms, review them before proceeding with this lesson. It may be helpful to write more examples for each word as part of your review. It is important that you understand these terms in order to successfully complete this lesson.

Simplifying Algebraic Expressions
Session 1 Part 1– Multiplying Monomials

Example 1: Study the following examples and then answer the questions that follow.

$x^2 = x \bullet x$

$y^2 = y \bullet y$

$x^2 y^2 = x \bullet x \bullet y \bullet y$

$(xy)^2 = (xy)(xy)$
$= x \bullet x \bullet y \bullet y$
$= x^2 \bullet y^2$

$(x^2 y)^2 = (x^2 y)(x^2 y)$
$= (x \bullet x \bullet y)(x \bullet x \bullet y)$
$= x^4 y^2$

$(x^2 y^2)^2 = (x^2 y^2)(x^2 y^2)$
$= (x \bullet x \bullet y \bullet y)(x \bullet x \bullet y \bullet y)$
$= (x \bullet x \bullet x \bullet x)(y \bullet y \bullet y \bullet y)$
$= x^4 y^4$

$(x^2 y^2)^3 = (x^2 y^2)(x^2 y^2)(x^2 y^2)$
$= (x \bullet x \bullet y \bullet y)(x \bullet x \bullet y \bullet y)(x \bullet x \bullet y \bullet y)$
$= (x \bullet x \bullet x \bullet x \bullet x \bullet x)(y \bullet y \bullet y \bullet y \bullet y \bullet y)$
$= x^6 y^6$

Do you see any patterns in the above examples? If so, what are they?

Explain your answer.

Can you make any generalizations from the examples shown that would help you to simplify monomial expressions?

Example 2: Study the following examples and then answer the questions that follow.

$2x^2 = 2 \bullet x \bullet x$

$3x^2 = 3 \bullet x \bullet x$

$(2x^2)(3x^3) = (2 \bullet x \bullet x)(3 \bullet x \bullet x \bullet x)$
$= 2 \bullet 3(x \bullet x \bullet x \bullet x \bullet x)$
$= 6x^5$

$(2x^2 \bullet 3x^3)^2 = (2x^2 \bullet 3x^3)(2x^2 3x^3)$
$= (2 \bullet x \bullet x \bullet 3 \bullet x \bullet x \bullet x)(2 \bullet x \bullet x \bullet 3 \bullet x \bullet x \bullet x)$
$= (2 \bullet 3 \bullet 2 \bullet 3)(x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x)$
$= 36x^{10}$

$(2x^2 \bullet 3x^3)^3 = (2x^2 \bullet 3x^3)(2x^2 \bullet 3x^3)(2x^2 \bullet 3x^3)$
$= (2 \bullet x \bullet x \bullet 3 \bullet x \bullet x \bullet x)(2 \bullet x \bullet x \bullet 3 \bullet x \bullet x \bullet x)(2 \bullet x \bullet x \bullet 3 \bullet x \bullet x \bullet x)$
$= (2 \bullet 3 \bullet 2 \bullet 3 \bullet 2 \bullet 3)(x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x)$
$= 216x^{15}$

Do you see any patterns in the above examples? If so, what are they?

Explain your answer.

Can you make any generalizations from the examples shown that would help you to simplify monomial expressions?

Example 3: Study the following examples and then answer the questions that follow.

$$a^3 = a \bullet a \bullet a$$

$$a^2 = a \bullet a$$

$$a^3 a^2 = a \bullet a \bullet a \bullet a \bullet a = a^5$$

$$(a^4)^3 = a^4 \bullet a^4 \bullet a^4$$
$$= a \bullet a \bullet a \bullet a \bullet a \bullet a \bullet a \bullet a \bullet a \bullet a \bullet a \bullet a = a^{12}$$

$$(a^3b^2)^2 = (a^3b^2)(a^3b^2)$$
$$= (a \bullet a \bullet a \bullet b \bullet b)(a \bullet a \bullet a \bullet b \bullet b)$$
$$= (a \bullet a \bullet a \bullet a \bullet a \bullet a)(b \bullet b \bullet b \bullet b) = a^6b^4$$

$$(a^3b^2)^3$$
$$= (a^3b^2)(a^3b^2)(a^3b^2)$$
$$= (a \bullet a \bullet a \bullet b \bullet b)(a \bullet a \bullet a \bullet b \bullet b)(a \bullet a \bullet a \bullet b \bullet b)$$
$$= (a \bullet a \bullet a \bullet a \bullet a \bullet a \bullet a \bullet a \bullet a)(b \bullet b \bullet b \bullet b \bullet b \bullet b) = a^9b^6$$

Do you see any patterns in the above examples? If so, what are they?

Explain your answer.

Can you make any generalizations from the examples shown that would help you to simplify monomial expressions?

Example 4: Study the following examples and then answer the questions that follow.

$m^2 = m \bullet m$

$m^4 = m \bullet m \bullet m \bullet m$

$(m^2)(m^4) = m \bullet m \bullet m \bullet m \bullet m \bullet m$
$= m^6$

$(m^2n^4)^2$
$= (m^2n^4)(m^2n^4)$
$= (m \bullet m \bullet n \bullet n \bullet n \bullet n)(m \bullet m \bullet n \bullet n \bullet n \bullet n)$
$= (m \bullet m \bullet m \bullet m)(n \bullet n \bullet n \bullet n \bullet n \bullet n \bullet n \bullet n)$
$= m^4n^8$

$(m^2n^4)^3$
$= (m^2n^4)(m^2n^4)(m^2n^4)$
$= (m \bullet m \bullet n \bullet n \bullet n \bullet n)(m \bullet m \bullet n \bullet n \bullet n \bullet n)(m \bullet m \bullet n \bullet n \bullet n \bullet n)$
$= (m \bullet m \bullet m \bullet m \bullet m \bullet m)(n \bullet n \bullet n \bullet n \bullet n \bullet n \bullet n \bullet n \bullet n \bullet n \bullet n \bullet n)$
$= m^6n^{12}$

Do you see any patterns in the above examples? If so, what are they?

Explain your answer.

Can you make any generalizations from the examples shown that would help you to simplify monomial expressions?

Was there a common theme to your answers in each of the four examples? As you can tell, the process of simplifying a monomial raised to a power is not difficult but it can be very tedious. There are rules of exponents that show what happens when monomials are raised to a power.

Rule	Example
$a^m \bullet a^n = a^{m+n}$	$a^6 \bullet a^3 = a^{6+3} = a^9$ $5x^3 \bullet 2x^2 = (5 \bullet 2)(x^{3+2}) = 10x^5$
$(a^m)^n = a^{mn}$	$\left(x^2\right)^3 = x^{2 \bullet 3} = x^6$ $(5^3)^8 = 5^{3 \bullet 8} = 5^{24}$
$(a \bullet b)^n = a^n b^n$	$(2n^3)^2 = (2^2)(n^{3 \bullet 2}) = 4n^6$

Look through the examples shown previously. Explain in the space provided how applying the rules of exponents would affect simplifying each example.

Simplify the following algebraic expressions showing your work in the space provided.

1. $(x^2 \bullet y^4 \bullet z)^2$

2. $(-4x \bullet 5x^4)$

3. $(2ab^2)^3$

4. $(-3ab^2c^3)^2(abc)^2$

5. $(-2x^2yz^3)^3$

6. $(-mn^2)(m^2n)(m^3n^3)$

7. $(2y^2z)^2(xz)^3(-4x^2y)^2$

Simplifying Algebraic Expressions
Session 1 Part 2 – Dividing Monomials

We can also divide monomial expressions with whole number exponents. Study the examples and then answer the questions that follow.

Example 1: Study the examples and then answer the questions that follow.

$$\frac{x^4}{x^3}$$

$$= \frac{x \bullet x \bullet x \bullet x}{x \bullet x \bullet x}$$

$$= \frac{\cancel{x} \bullet \cancel{x} \bullet \cancel{x} \bullet x}{\cancel{x} \bullet \cancel{x} \bullet \cancel{x}}$$

$$= x$$

$$\frac{12x^4}{3x^3}$$

$$= \frac{2 \bullet 2 \bullet 3 \bullet x \bullet x \bullet x \bullet x}{3 \bullet x \bullet x \bullet x}$$

$$= \frac{2 \bullet 2 \bullet \cancel{3} \bullet \cancel{x} \bullet \cancel{x} \bullet \cancel{x} \bullet x}{\cancel{3} \bullet \cancel{x} \bullet \cancel{x} \bullet \cancel{x}}$$

$$= 2 \bullet 2 \bullet x = 4x$$

$$\frac{12x^4y^3}{3x^3y^2}$$

$$= \frac{2 \bullet 2 \bullet 3 \bullet x \bullet x \bullet x \bullet x \bullet y \bullet y \bullet y}{3 \bullet x \bullet x \bullet x \bullet y \bullet y}$$

$$\frac{2 \bullet 2 \bullet \cancel{3} \bullet \cancel{x} \bullet \cancel{x} \bullet \cancel{x} \bullet x \bullet \cancel{y} \bullet \cancel{y} \bullet y}{\cancel{3} \bullet \cancel{x} \bullet \cancel{x} \bullet \cancel{x} \bullet \cancel{y} \bullet \cancel{y}}$$

$$= 2 \bullet 2 \bullet x \bullet y = 4xy$$

$$\frac{8x^3y^4}{2x^5y^3}$$

$$= \frac{2 \bullet 2 \bullet 2 \bullet x \bullet x \bullet x \bullet y \bullet y \bullet y \bullet y}{2 \bullet x \bullet x \bullet x \bullet x \bullet x \bullet y \bullet y \bullet y}$$

$$\frac{2 \bullet 2 \bullet \cancel{2} \bullet \cancel{x} \bullet \cancel{x} \bullet \cancel{x} \bullet y \bullet \cancel{y} \bullet \cancel{y} \bullet \cancel{y}}{\cancel{2} \bullet \cancel{x} \bullet \cancel{x} \bullet \cancel{x} \bullet x \bullet x \bullet \cancel{y} \bullet \cancel{y} \bullet \cancel{y}}$$

$$= \frac{4y}{x^2}$$

Do you see any patterns in the above examples? If so, what are they?

Explain your answer.

Can you make any generalizations from the examples shown that would help you to simplify monomial expressions?

Example 2: Study the examples and then answer the questions that follow.

$$\frac{xy^3z^2}{xyz}$$

$$= \frac{x \bullet y \bullet y \bullet y \bullet z \bullet z}{x \bullet y \bullet z}$$

$$= \frac{\cancel{x} \bullet \cancel{y} \bullet y \bullet y \bullet \cancel{z} \bullet z}{\cancel{x} \bullet \cancel{y} \bullet \cancel{z}}$$

$$= y^2z$$

$$\frac{3xy^3z^2}{15xyz}$$

$$= \frac{3 \bullet x \bullet y \bullet y \bullet y \bullet z \bullet z}{3 \bullet 5 \bullet x \bullet y \bullet z}$$

$$= \frac{\cancel{3} \bullet \cancel{x} \bullet y \bullet y \bullet \cancel{y} \bullet \cancel{z} \bullet z}{\cancel{3} \bullet 5 \bullet \cancel{x} \bullet \cancel{y} \bullet \cancel{z}}$$

$$= \frac{y^2z}{5}$$

$$\left(\frac{xy^3z^2}{xyz}\right)^2$$

$$=\left(\frac{xy^3z^2}{xyz}\right)\bullet\left(\frac{xy^3z^2}{xyz}\right)$$

$$=\left(\frac{x\bullet y\bullet y\bullet y\bullet z\bullet z}{x\bullet y\bullet z}\right)\left(\frac{x\bullet y\bullet y\bullet y\bullet z\bullet z}{x\bullet y\bullet z}\right)$$

$$=\frac{x\bullet x\bullet y\bullet y\bullet y\bullet y\bullet y\bullet y\bullet z\bullet z\bullet z\bullet z}{x\bullet x\bullet y\bullet y\bullet z\bullet z}$$

$$=\frac{x\bullet x\bullet y\bullet y\bullet y\bullet y\bullet y\bullet y\bullet z\bullet z\bullet z\bullet z}{x\bullet x\bullet y\bullet y\bullet z\bullet z}$$

$$=y\bullet y\bullet y\bullet y\bullet z\bullet z$$

$$=y^4z^2$$

If the same fraction is simplified first, <u>the result is the same</u>.

$$\left(\frac{xy^3z^2}{xyz}\right)^2$$

$$=\left(\frac{x\bullet y\bullet y\bullet y\bullet z\bullet z}{x\bullet y\bullet z}\right)^2$$

$$=\left(y^2z\right)^2$$

$$=(y\bullet y\bullet z)(y\bullet y\bullet z)$$

$$=(y\bullet y\bullet y\bullet y\bullet z\bullet z)$$

$$=y^4z^2$$

Do you see any patterns in the examples? If so, what are they?

Explain your answer.

Can you make any generalizations from the examples shown that would help you to simplify monomial expressions?

Example 3: Study the examples and then answer the questions that follow.

$$\frac{100a^2b^3c^4}{30a^2b^4c^5}$$

$$= \frac{2 \bullet 2 \bullet 5 \bullet 5 \bullet a \bullet a \bullet b \bullet b \bullet b \bullet c \bullet c \bullet c \bullet c}{2 \bullet 3 \bullet 5 \bullet a \bullet a \bullet b \bullet b \bullet b \bullet b \bullet c \bullet c \bullet c \bullet c \bullet c}$$

$$= \frac{\cancel{2} \bullet 2 \bullet \cancel{5} \bullet 5 \bullet \cancel{a} \bullet \cancel{a} \bullet \cancel{b} \bullet \cancel{b} \bullet \cancel{b} \bullet \cancel{c} \bullet \cancel{c} \bullet \cancel{c} \bullet \cancel{c}}{\cancel{2} \bullet 3 \bullet \cancel{5} \bullet \cancel{a} \bullet \cancel{a} \bullet b \bullet \cancel{b} \bullet \cancel{b} \bullet \cancel{b} \bullet c \bullet \cancel{c} \bullet \cancel{c} \bullet \cancel{c} \bullet \cancel{c}}$$

$$= \frac{2 \bullet 5}{3 \bullet b \bullet c}$$

$$= \frac{10}{3bc}$$

$$\left(\frac{3a^2b^3}{a^3b}\right)^2$$

$$=\left(\frac{3a^2b^3}{a^3b}\right)\left(\frac{3a^2b^3}{a^3b}\right)$$

$$=\left(\frac{3\bullet a\bullet a\bullet b\bullet b\bullet b}{a\bullet a\bullet a\bullet b}\right)\left(\frac{3\bullet a\bullet a\bullet b\bullet b\bullet b}{a\bullet a\bullet a\bullet b}\right)$$

$$=\frac{3\bullet 3\bullet a\bullet a\bullet a\bullet a\bullet b\bullet b\bullet b\bullet b\bullet b\bullet b}{a\bullet a\bullet a\bullet a\bullet a\bullet a\bullet b\bullet b}$$

$$=\frac{3\bullet 3\bullet \cancel{a}\bullet \cancel{a}\bullet \cancel{a}\bullet \cancel{a}\bullet b\bullet b\bullet \cancel{b}\bullet \cancel{b}\bullet b\bullet b}{\cancel{a}\bullet \cancel{a}\bullet \cancel{a}\bullet \cancel{a}\bullet a\bullet a\bullet \cancel{b}\bullet \cancel{b}}$$

$$=\frac{9b^4}{a^2}$$

Simplify the same problem first and obtain the same answer.

$$\left(\frac{3a^2b^3}{a^3b}\right)^2$$

$$=\left(\frac{3\bullet \cancel{a}\bullet \cancel{a}\bullet b\bullet \cancel{b}\bullet b}{\cancel{a}\bullet \cancel{a}\bullet a\bullet \cancel{b}}\right)^2$$

$$=\left(\frac{3\bullet b\bullet b}{a}\right)^2$$

$$=\left(\frac{3\bullet b\bullet b}{a}\right)\left(\frac{3\bullet b\bullet b}{a}\right)$$

$$=\frac{3\bullet 3\bullet b\bullet b\bullet b\bullet b}{a\bullet a}$$

$$=\frac{9b^4}{a^2}$$

Do you see any patterns in the above examples? If so, what are they?

Explain your answer.

Can you make any generalizations from the examples shown that would help you to simplify monomial expressions?

Was there a common theme to your answers in each of the examples above? As you can tell, the process of simplifying a rational exponent is not difficult but it can be very tedious. Let's add some new rules to the previous rules of exponents that we discovered.

Rule	Example
$a^m \bullet a^n = a^{m+n}$	$a^6 \bullet a^3 = a^{6+3} = a^9$
$(a^m)^n = a^{mn}$	$\left(x^2\right)^3 = x^{2\bullet 3} = x^6$
$(a \bullet b)^n = a^n b^n$	$(2n^3)^2 = 2^2 n^{3\bullet 2}) = 4n^6$
$\dfrac{a^m}{a^n} = a^{m-n}$	$\dfrac{a^7}{a^3} = a^{7-3} = a^4$
$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$	$\left(\dfrac{x}{3}\right)^3 = \dfrac{x^3}{3^3} = \dfrac{x^3}{27}$

Does it matter when simplifying rational expressions whether the expression is simplified before being raised to a power or after being raised to a power? Justify your answer.

Look through the examples shown previously. Explain in the space provided how applying the rules of exponents would affect simplifying each example.

There may be many ways to simplify an algebraic expression. It is important to understand each step when simplifying an algebraic expression. Which way you choose to simplify an algebraic expression is not as important as understanding the steps as you simplify.

Several previous examples were simplified in more than one way yet we always ended up with the same answer.

Simplify the following algebraic expressions showing your work.

1. $\frac{a^5}{a^3}$

2. $\frac{4x^8y^5}{x^3y}$

3. $\left(\frac{c^2}{4}\right)^2$

4. $\frac{6m^3n^2}{16m^5n^6}$

5. $\left(\frac{2a^2b^3c}{abc}\right)^2$

6. $\left(\frac{10x^2y^2z^2}{25x^3yz}\right)^2$

Simplifying Algebraic Expressions
Session 1 Part 3– Zero Exponents

Example 1: Study the examples and then answer the questions that follow.

$$\frac{x^4}{x^4} = \frac{x \bullet x \bullet x \bullet x}{x \bullet x \bullet x \bullet x} = 1$$

If we apply the rules of exponents that we learned, then we know that $\frac{x^4}{x^4} = x^{4-4} = x^0$.

Since $\frac{x^4}{x^4} = 1$ and $\frac{x^4}{x^4} = x^0$, we learn that $x^0 = 1$.

Similarly,

$\frac{c^3}{c^3} = \frac{c \bullet c \bullet c}{c \bullet c \bullet c} = 1$ and $\frac{c^3}{c^3} = c^{3-3} = c^0$. Therefore $c^0 = 1$.

Similarly,

$\frac{5^2}{5^2} = \frac{5 \bullet 5}{5 \bullet 5} = \frac{1}{1}$ and $\frac{5^2}{5^2} = 5^{2-2} = 5^0$. Therefore $5^0 = 1$

Similarly,

$\frac{a}{a} = 1$ and $\frac{a}{a} = \frac{a^1}{a^1} = a^{1-1} = a^0$. Therefore, $a^0 = 1$

Do you see any patterns in the above examples? If so, what are they?

Explain your answer.

Can you make any generalizations from the examples shown that would help you to simplify monomial expressions when the expression contains a zero exponent?

Was there a common theme to your answers in the example? As you can tell, the process of simplifying a rational exponent is not difficult but it can be very tedious. Let's add some new rules to the previous rules of exponents that we discovered.

Rule	Example
$a^m \bullet a^n = a^{m+n}$	$a^6 \bullet a^3 = a^{6+3} = a^9$
$(a^m)^n = a^{mn}$	$\left(x^2\right)^3 = x^{2\bullet 3} = x^6$
$(a \bullet b)^n = a^n b^n$	$(2n^3)^2 = 2^2 n^{3\bullet 2}) = 4n^6$
$\dfrac{a^m}{a^n} = a^{m-n} \quad (a \neq 0)$	$\dfrac{a^7}{a^3} = a^{7-3} = a^4$
$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n} \quad (b \neq 0)$	$\left(\dfrac{x}{3}\right)^3 = \dfrac{x^3}{3^3} = \dfrac{x^3}{27}$
$a^0 = 1 \quad (a \neq 0)$	$\left(3xy\right)^0 = 1$

Simplify the following algebraic expressions showing your work.

1. $(mn)^0$

2. $\left(\frac{4xy}{5}\right)^0$

3. $\frac{(3abc)^0}{7}$

4. $-4x^0y^0z$

Simplify the following expressions showing your work in the space provided.

1. $(x^2)(y^3)$

2. $(2x^3y)(6x^2y)$

3. $(a^3b^4)(-3a^2b)$

4. $(n^2)^6$

5. $(3x^3)^3$

6. $-5(x^2y^4z^3)^2$

7. $\frac{n^7}{n^3}$

8. $\left(\frac{2xy^2z}{5xy}\right)^2$

9. $\frac{12x^4y^2z^3}{3xyz}$

10. $(xy)^0$

11. $\frac{(a^3b^2c)^0}{2}$

12. $\left(\frac{2cd^4}{12c^2d}\right)^0$

Simplifying Algebraic Expressions
Assessment 1

Match each term to its definition.

_____**1. binomial**

_____**2. leading coefficient**

_____**3. Monomial**

_____**4. Exponent**

_____**5. degree of a polynomial**

_____**6. polynomial**

_____**7. algebraic expression**

_____**8. rational expression**

A. an algebraic expression consisting of a single term that does not require any addition or subtraction (e.g., 5y)

B. the degree of the highest term of the polynomial (e.g., The degree of $3x+2x^2+4-7x^5-3x+10x^4$ is 5 because 5 is the greatest exponent)

C. an algebraic expression consisting of two terms (e.g., $x+3$, $4a-6$)

D. a group of numbers, symbols, and variables that express a single or series of mathematical operations (e.g., $2x+4-16y$)

E. a number placed to the right and above (superscript) a non-zero base that indicates the operation of repeated multiplication (e.g., in 5^7 the exponent is 7)

F. the quotient of two polynomials in the form $\frac{A}{B}$, where A and B are polynomials and where B can **never** equal 0.(e.g., $\frac{2x+1}{3x^2-9}$, $3x^2-9\neq 0$)

G. an expression containing more than one monomial connected by addition or subtraction(e.g., $3x^2+2x+7$, $4x^5-9x^3+2x+7$)

H. the coefficient of the term of the highest degree in a polynomial (e.g., in the expression $15x-10x^3-11x^6+7x^2+3.5$ -11 is the leading coefficient)

Simplify the following algebraic expressions showing your work.

9. $\dfrac{a^7}{a^4}$

10. $\dfrac{6x^3y3}{2x^2y}$

11. $\left(\dfrac{b^3}{3}\right)^2$

12. $\dfrac{4m^2n^3}{18m^4n^5}$

13. $\left(\dfrac{-3a^3b^3c^2}{a^2bc}\right)^2$

14. $\left(\dfrac{5x^2y^2z^3}{20x^4yz^2}\right)^2$

15. $(ab)^0$

16. $\frac{(x^3y^2z)^0}{-5}$

17. $\left(\frac{7mn^4}{21m^2n}\right)^0$

18. $\left(\frac{3}{xy^4}\right)^0$

Simplifying Algebraic Expressions
Session 2 – Operations with Rational Expressions
***Note: You must be able to factor trinomials in order to complete this session.**

We can add, subtract, multiply, and divide rational expressions.

Example 1: Study the following problems which add fractions with like denominators.

$$\frac{2}{9}+\frac{5}{9}$$
$$=\frac{2+5}{9}$$
$$=\frac{7}{9}$$

$$\frac{1}{4}+\frac{1}{4}$$
$$=\frac{1+1}{4}$$
$$=\frac{2}{4}$$

This can be simplified to $\frac{1}{2}$

$$\frac{5}{12}+\frac{3}{12}$$
$$=\frac{5+3}{12}$$
$$=\frac{8}{12}$$

This can be simplified to $\frac{2}{3}$

$$-\frac{3}{5}+\frac{4}{5}$$
$$=\frac{-3+4}{5}$$
$$=\frac{1}{5}$$

In the work space provided, write the process used to add fractions with like denominators in your own words.

Example 2: Study the following problems which subtract fractions with like denominators.

$$\frac{3}{4} - \frac{2}{4} = \frac{3-2}{4}$$
$$= \frac{1}{4}$$

$$\frac{8}{9} - \frac{5}{9} = \frac{8-5}{9}$$
$$= \frac{3}{9}$$
$$= \frac{1}{3}$$

$$\frac{5}{12} - \frac{2}{12} = \frac{5-2}{12}$$
$$= \frac{3}{12}$$
$$= \frac{1}{4}$$

$$\frac{1}{3} - \left(-\frac{1}{3}\right) = \frac{1}{3} - \frac{-1}{3}$$

*Remember that a negative fraction means *either* the numerator is negative OR the denominator is negative, but BOTH are not negative.

$$= \frac{1-(-1)}{3}$$
$$= \frac{1+1}{3}$$
$$= \frac{2}{3}$$

In the work space provided, write the process used to subtract fractions with like denominators in your own words.

Example 3: Study the following problems and then answer the question that follows.

$$\frac{2}{x}+\frac{3}{x}=\frac{2+3}{x}$$
$$=\frac{5}{x}$$

$$\frac{2}{x+3}+\frac{3}{x+3}=\frac{2+3}{x+3}$$
$$=\frac{5}{x+3}$$

$$\frac{a}{5}+\frac{7}{5}=\frac{a+7}{5}$$

$$\frac{x^2}{x+1}+\frac{1}{x+1}=\frac{x^2+1}{x+1}$$

$$\frac{2x}{x+3}+\frac{6}{x+3}=\frac{2x+6}{x+3}$$
$$=\frac{2(x+3)}{x+3}$$
$$=\frac{2\cancel{(x+3)}}{\cancel{(x+3)}}$$
$$=2$$

In the work space provided, write the process used to subtract rational expressions with like denominators in your own words.

The process used to add and subtract rational expressions with the same denominator follows the same process as adding or subtracting numerical fractions.

Example 4: Study the following problems and then answer the question that follows.

$$\frac{2}{x}-\frac{3}{x}=\frac{2-3}{x}$$
$$=\frac{-1}{x}$$

$$\frac{5}{x+3}-\frac{x}{x+3}=\frac{5-x}{x+3}$$

$$\frac{a}{5}-\frac{7}{5}=\frac{a-7}{5}$$

$$\frac{x^2}{x+1}-\frac{1}{x+1}=\frac{x^2-1}{x+1}$$

$$=\frac{(x+1)(x-1)}{x+1}$$

*Note : We factored x^2-1 into $(x+1)(x-1)$ because it is a "difference of squares".

$$=\frac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}}$$

* When we factor everything from either the numerator or the denominator it does not become 0, but it is 1.

That is because factoring is division. If you divide 5 by 5 the answer is 1.

Well, if we divide the $(x+1)$ by $(x+1)$ we get 1 as our new denominator.

$$=x-1$$

$$\frac{2x}{x-3}-\frac{6}{x-3}=\frac{2x-6}{x-3}$$
$$=\frac{2(x-3)}{x-3}$$
$$=\frac{2\cancel{(x-3)}}{\cancel{(x-3)}}$$
$$=2$$

$$\frac{n^2}{n-3}-\frac{6n-9}{n-3}=\frac{n^2-(6n-9)}{n-3}$$

$=\dfrac{n^2-6n+9}{n-3}$ * We factored the numerator into (x - 3)(x - 3)

$$=\frac{(n-3)(n-3)}{n-3}$$

$$=\frac{(n\blacklozenge 3)(n-3)}{(n\blacklozenge 3)}$$

$$=n-3$$

In the work space provided, write the process used to add rational expressions with like denominators in your own words.

To add or subtract rational expressions with the same denominator:

- Add or subtract the algebraic expressions in the numerators. Be careful when simplifying an algebraic expression that follows a subtraction sign.
- Keep the same denominator and place the simplified numerator over it.
- Simplify the rational expression to lowest terms by factoring out common factors.

Add the following rational expressions showing your work.

1. $\frac{1}{x}+\frac{3}{x}$

2. $\frac{5}{c+4}+\frac{3}{c+4}$

3. $\frac{-x}{x+1}+\frac{x}{x+1}$

4. $\frac{2}{a}+\frac{b}{a}$

5. $\frac{x^2}{x+2}+\frac{4x+4}{x+2}$

6. $\frac{b}{b^2-9}+\frac{-3}{b^2-9}$

Subtract the following rational expressions showing your work.

1. $\frac{5}{x} - \frac{6}{x}$

2. $\frac{5}{c+4} - \frac{3}{c+4}$

3. $\frac{x}{x+1} - \frac{1}{x+1}$

4. $\frac{x}{x^2-16}-\frac{4}{x^2-16}$

5. $\frac{x^2}{x-2}-\frac{2x+8}{x-2}$

6. $\frac{b}{b^2-36}-\frac{6}{b^2-36}$

Simplifying Algebraic Expressions
Assessment 2

Add the following rational expressions showing your work.

1. $\frac{2}{a}+\frac{4}{a}$

2. $\frac{6}{n+4}+\frac{2}{n+4}$

3. $\frac{-2x}{x+1}+\frac{3x}{x+1}$

4. $\dfrac{x^2}{x+3}+\dfrac{6x+9}{x+3}$

5. $\dfrac{b}{b^2-25}+\dfrac{-5}{b^2-25}$

Subtract the following rational expressions showing your work.

6. $\dfrac{4}{x}-\dfrac{10}{x}$

7. $\frac{17}{r+5} - \frac{12}{r+5}$

8. $\frac{x}{x+1} - \frac{-1}{x+1}$

9. $\frac{x}{x^2-36} - \frac{6}{x^2-36}$

10. $\frac{x^2}{x-3} - \frac{-2x+15}{x-3}$

In the space below, write the process for simplifying rational expressions with like denominators, in your own words.

Extensions

These websites will provide additional explanation and interactive examples with multiplying and dividing polynomials.

- Multiplying Algebraic Expressions
  http://www.wisc-online.com/objects/index_tj.asp?objID=GEM1904
- Dividing Algebraic Expressions
  http://www.wisc-online.com/objects/index_tj.asp?objID=GEM2104

Sources

2008 AZ Mathematics Standards
2000 NCTM Principles and Standards
2008 The Final Report of the National Mathematics Advisory Panel
1999 Bringing the NCTM Standards to Life, Exemplary Practices from High Schools