Graphing Linear Functions – Slope-Intercept
An ADE Mathematics Lesson
Days 16-20

Author	ADE Content Specialists
Grade Level	9^{th} grade
Duration	Five days

Aligns To

Mathematics HS:
Strand 3: Patterns, Algebra, and Functions
Concept 2: Functions and Relationships
PO 1. Sketch and interpret a graph that models a given context, make connections between the graph and the context, and solve maximum and minimum problems using the graph.
PO 4. Use equations, graphs, tables, descriptions, or sets of ordered pairs to express a relationship between two variables.
Concept 3: Algebraic Representations
PO 1. Create and explain the need for equivalent forms of an equation or expression.
PO 3. Write an equation given a table of values, two points on the line, the slope and a point on the line, or the graph of the line.
PO 4. Determine from two linear equations whether the lines are parallel, perpendicular, coincident, or intersecting but not perpendicular.
Concept 4: Analysis of Change
PO 1. Determine the slope and intercepts of the graph of a linear function, interpreting slope as a constant rate of change.

Strand 4: Geometry and Measurement
Concept 3: Coordinate Geometry
PO 6. Describe how changing the parameters of a linear function affect the shape and position of its graph.

Strand 5: Structure and Logic
Concept 2: Logic, Reasoning, Problem Solving, and Proof
PO 1. Analyze a problem situation, determine the question(s) to be answered, organize given information, determine how to represent the problem, and identify implicit and explicit assumptions that have been made.
PO 2. Solve problems by formulating one or more strategies, applying the strategies, verifying the solution(s), and communicating the reasoning used to obtain the solution(s).

Connects To

Mathematics HS:
Strand 2: Data Analysis, Probability, and Discrete Mathematics
Concept 1: Data Analysis (Statistics)
PO 2. Organize collected data into an appropriate graphical representation with or without technology.

Strand 3: Patterns, Algebra, and Functions
Concept 1: Patterns
PO 1. Recognize, describe, and analyze sequences using tables, graphs, words, or symbols; use sequences in modeling.

Strand 4: Geometry and Measurement
Strand 2: Coordinate Geometry
PO 1. Determine how to find the midpoint between two points in the coordinate plane.
PO 3. Determine the distance between two points in the coordinate plane.
PO 4. Verify characteristics of a given geometric figure using coordinate formulas for distance, midpoint, and slope to confirm parallelism, perpendicularity, and congruence.
Concept 4: Measurement
PO 2. Determine the new coordinates of a point when a single transformation is performed on a 2-dimensional figure.
PO 3. Sketch and describe the properties of a 2-dimensional figure that is the result of two or more transformations.

Strand 5: Structure and Logic
Concept 2: Logic, Reasoning, Problem Solving, and Proof
PO 5. Summarize and communicate mathematical ideas using formal and informal reasoning.

Aligns To	Connects To
Mathematics HS: **Strand 5: Structure and Logic** **Concept 2: Logic, Reasoning, Problem Solving, and Proof** **PO 3.** Evaluate a solution for reasonableness and interpret the meaning of the solution in the context of the original problem. **PO 6.** Synthesize mathematical information from multiple sources to draw a conclusion, make inferences based on mathematical information, evaluate the conclusions of others, analyze a mathematical argument, and recognize flaws or gaps in reasoning.	

Overview

Graphing linear functions is very important. Representing linear functions in several different ways builds conceptual understanding of graphing.

Purpose

This lesson emphasizes graphing linear functions by using a table of values, plotting the x- and y-intercept or by using the slope-intercept form of the equation of a line.

Materials

- Graphing worksheets
- Graph paper
- Ruler

Objectives

Students will:

- Use equations, graphs, tables, descriptions, or sets of ordered pairs to express a relationship between two variables.
- Graph a linear equation in two variables.
- Determine the slope and intercepts of the graph of a linear function, interpreting slope as a constant rate of change.
- Describe how changing the parameters of a linear function affect the shape and position of its graph.
- Write an equation given a table of values, two points on the line, the slope and a point on the line, or the graph of the line.
- Sketch and interpret a graph that models a given context, make connections between the graph and the context.

Lesson Components

Prerequisite Skills: The lesson requires the basics of graphing in a coordinate plane. This lesson builds on the grade 6 skill of graphing ordered pairs in any quadrant of the coordinate plane. This lesson builds on the grade 7 skills of using a table of values to graph an equation or proportional relationship and to describe the graph's characteristics. This lesson builds on grade 8 skill of determining if a relationship represented by a graph or table is a function.

Vocabulary: *Cartesian coordinate system, coordinate plane, graph, ordered pairs, origin, x-coordinate, y-coordinate, quadrant, axes, element, domain, range, independent variable set, dependent variable set, rule of correspondence, x-intercept, y-intercept, slope, rate of change, relation, function, linear function, vertical line, horizontal line, slope-intercept form of the equation of a line, rise/run, parallel lines, perpendicular lines, coincident lines, intersecting lines.*

Session 1 (2 days)

1. Graph linear functions by using the slope-intercept form of the equation of a line.
2. Determine the slope and intercepts of the graph of a linear function, interpreting slope as a constant rate of change.

Session 2 (2 days)

1. Determine the slope and intercepts of the graph of a linear function, interpreting slope as a constant rate of change.
2. Discover how changing the parameters of a linear function change the shape of the graph.
3. Sketch and interpret a graph that models a given context, make connections between the graph and the content.

Session 3 (1 day)

1. Determine from two linear equations whether the lines are parallel, perpendicular, coincident, or intersecting but not perpendicular.

Assessment

There are three assessments, one after each session, which will help pinpoint misconceptions before moving on to more complex comparisons.

Session 1 – Graphing Linear Functions – Slope-Intercept

There are many different ways to graph linear functions. We have learned how to graph them by using a table of values and by using the x- and y-intercepts. A third way that we can graph an equation is by placing it in slope-intercept form $f(x) = mx + b$. The *m* represents the slope and the *b* is the y-intercept. We first draw a point where the line is going to intercept the y-axis. From this point, we use $\frac{rise}{run}$ to determine a second point. If the run is positive, we move to the right from the y-intercept. If it is negative, we move to the left. If the rise is positive, we move up and if it's negative, we move down. You may do the rise first or the run first and then do the other from where the new point would be. If an equation is not already in slope-intercept form, solve it so that it is in that form.

To graph an equation using slope-intercept form follow this process:

- Place the equation in slope intercept form.
- Graph the y-intercept.
- Using rise over run, find the 2nd point.
- Draw a line connecting the points using the representation of an infinite line.

Remember all the suggestions for making a well-drawn graph. Fundamentals of Graphing Linear Function reviewed these suggestions and they are repeated here.

When graphing, there are some **expectations** to which we need to adhere. These include:

1. Use graph paper and graph using a pencil.
2. Use a straightedge to make sure that your lines are straight for linear equations.
3. The units on the axes need to be at equal intervals and the units need to be marked so that others reading your graph will know what each tick mark represents on both the x- and y-axes. The units on the x-axis do not need to be the same as the units on the y-axis.
4. The x- and y-axes represent infinite lines so place arrows on the end of each axis.
5. The graph of a linear function is a line not a line segment. Place arrows at the end of the line to indicate the infinite length.
6. Make sure your graph crosses both axes when you extend the line.
7. If you are graphing points, make sure to indicate the points by making them large enough to be visible to the reader. When working with points, it's a good idea to list the coordinates of the point near the graph of the point.
8. Mark the equation of the line on or near the line.
9. If you graph the equation correctly, it should look exactly the same as another person graphing the same equation.
10. Do not graph more than one equation on a set of axes unless the problem asks you to do so. If you are graphing more than one line on the same set of axes, color code or use a legend to indicate which line is the graph of which equation.
11. If you are sketching a graph after having graphed an equation on a graphing calculator, indicate what window you used and the scale.

Example 1:

Graph $f(x) = 2x + 4$ using slope-intercept.

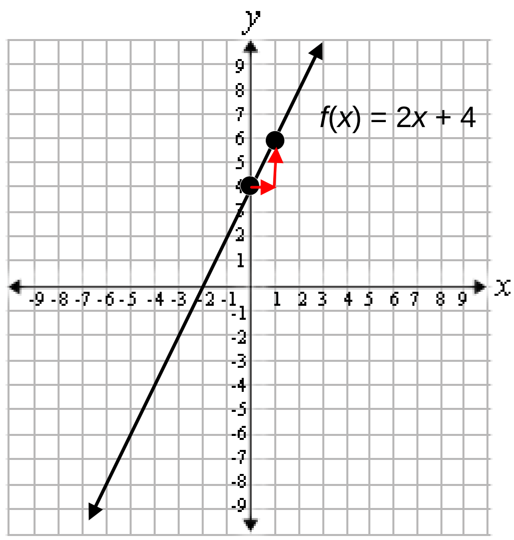
Solution:

Remember that the proper form for slope-intercept equation of a line is $f(x) = mx + b$ where m is the slope and b is the y-intercept.

$$\begin{array}{c} f(x) = 2x + 4 \\ \uparrow \quad \uparrow \\ f(x) = mx + b \end{array}$$

In this case, the slope is 2 which is the same as $\frac{2}{1}$ and the y-intercept is 4.

Begin at 4 on the y-axis. Remember that $\frac{2}{1} = \frac{rise}{run}$. Since the run is positive, move 1 unit to the right from the 4 on the y-axis. Since the rise is positive, move 2 units up from the run. Draw a line connecting the points (0, 4) and (1, 6).



Example 2:

Graph $f(x) = -3x + 2$ using slope-intercept.

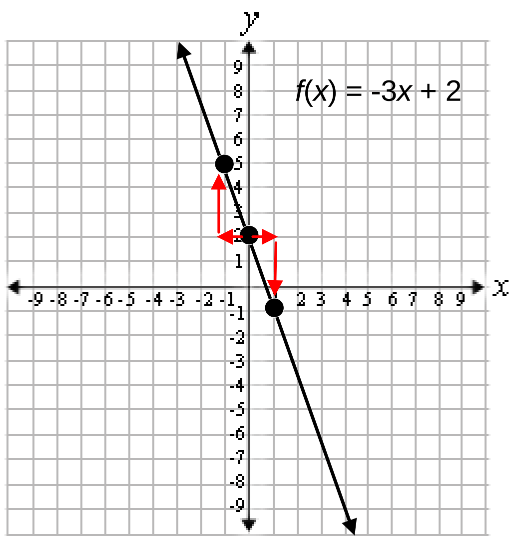
Solution:

Remember that the proper form for slope-intercept equation of a line is $f(x) = mx + b$ where m is the slope and b is the y-intercept.

$$f(x) = -3x + 2$$
$$\uparrow \quad \uparrow$$
$$f(x) = mx + b$$

In this case, the slope is -3, which is the same as $\frac{-3}{1}$ and the y-intercept is 2.

Begin at 2 on the y-axis. Remember that $\frac{-3}{1} = \frac{rise}{run}$. Since the run is positive, move 1 unit to the right from the 2 on the y-axis. Since the rise is negative, move 3 units down from the run. Draw a line connecting the points (0, 2) and (1, -1). Note that since $\frac{3}{-1} = \frac{-3}{1}$, it is also possible to move one unit to the left of 2 and then move 3 units up from that point. That point is (-1, 5) and is on the graph of the linear function as well.



Example 3:

Graph $f(x) = \frac{2}{5}x - 4$ using slope-intercept.

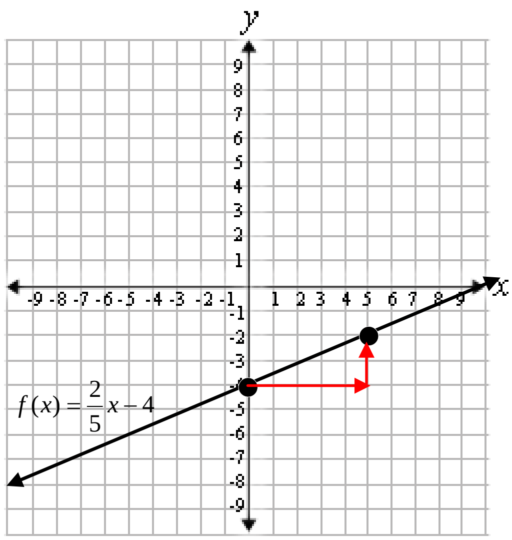
Solution:

Remember that the proper form for slope-intercept equation of a line is $f(x) = mx + b$ where m is the slope and b is the y-intercept.

$$f(x) = \frac{2}{5}x - 4$$
$$\uparrow \quad \uparrow$$
$$f(x) = mx + b$$

In this case, the slope is $\frac{2}{5}$ and the y-intercept is -4.

Begin at -4 on the y-axis. Remember that $\frac{2}{5} = \frac{rise}{run}$. Since the run is positive, move 5 units to the right from the -4 on the y-axis. Since the rise is positive, move 2 units up from the run. Draw a line connecting the points (0, -4) and (5, -2).



Example 4:

Graph $f(x) = -6$ using slope-intercept.

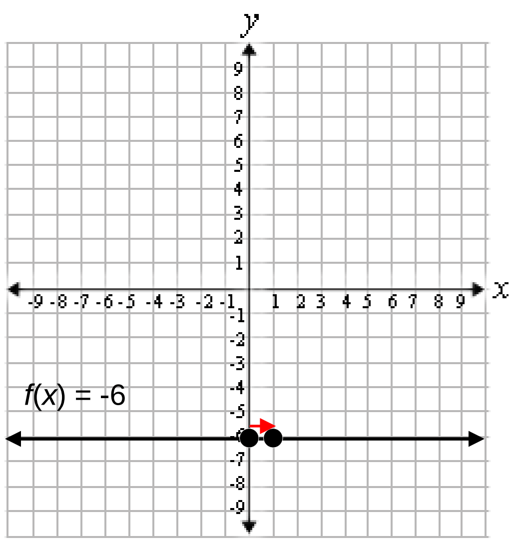
Solution:

Remember that the proper form for slope-intercept equation of a line is $f(x) = mx + b$ where m is the slope and b is the y-intercept.

$$f(x) = 0x - 6$$
$$\uparrow \qquad \uparrow$$
$$f(x) = mx + b$$

In this case, the slope is 0 and the y-intercept is --6.

Begin at --6 on the y-axis. Remember that $\frac{0}{1} = \frac{rise}{run}$. Since the run is positive, move 1 unit to the right from the -6 on the y-axis. Since the rise is 0, move no units up from the run. Draw a line connecting the points (0, -6) and (1, -6).



Example 5:

Graph $f(x)+2=3-x$ using slope-intercept.

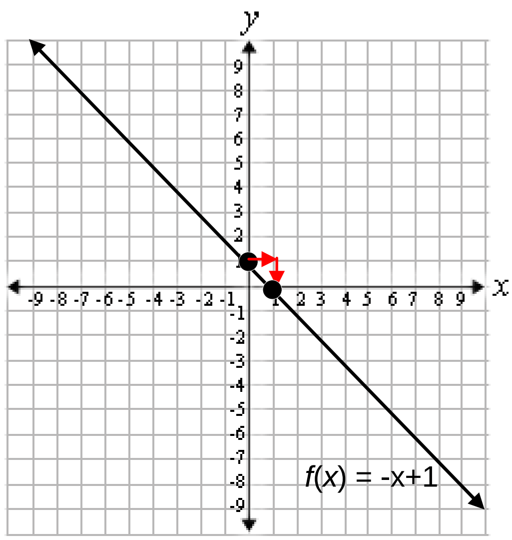
Solution:

First place the equation in slope-intercept form.

$$f(x)+2=3-x \Rightarrow f(x)+2=-x+3 \Rightarrow f(x)=-x+3-2 \Rightarrow f(x)=-x+1$$

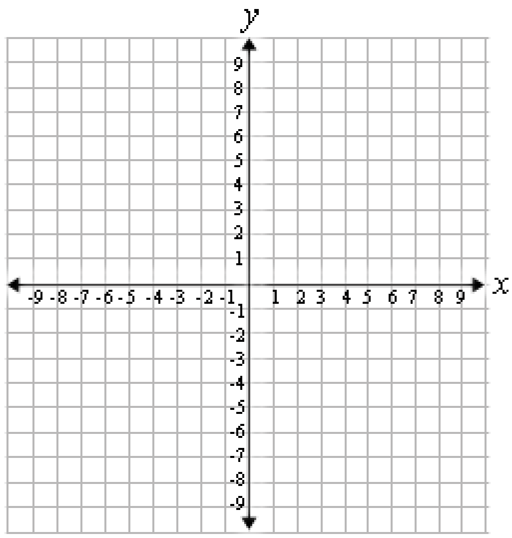
Remember that the proper form for slope-intercept equation of a line is $f(x)=mx+b$ where m is the slope and b is the y-intercept. In this case, the slope is $-1=\frac{-1}{1}$ and the y-intercept is 1.

Begin at 1 on the y-axis. Remember that $\frac{-1}{1}=\frac{rise}{run}$. Since the run is positive, move 1 unit to the right from 1 on the y-axis. Since the rise is negative, move 1 unit down from the run. Draw a line connecting the points (0, 1) and (1, 0).

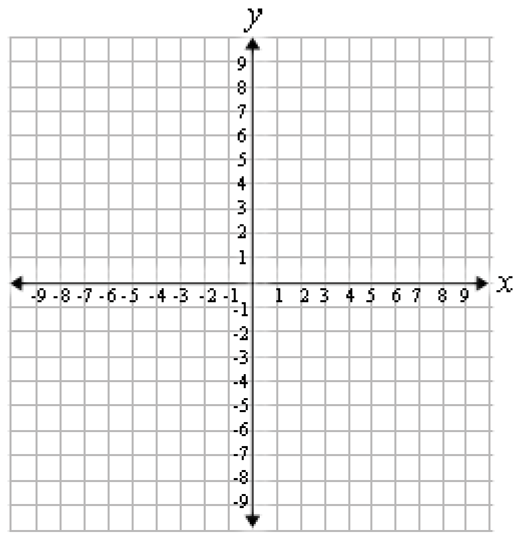


For each of the following problems, graph the linear function using slope-intercept. In the work space that follows each graph, explain how you created the graph. Be very specific.

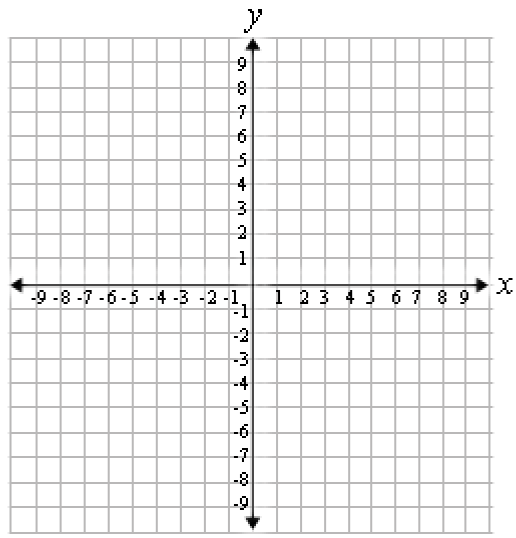
1. $f(x) = 3x - 6$



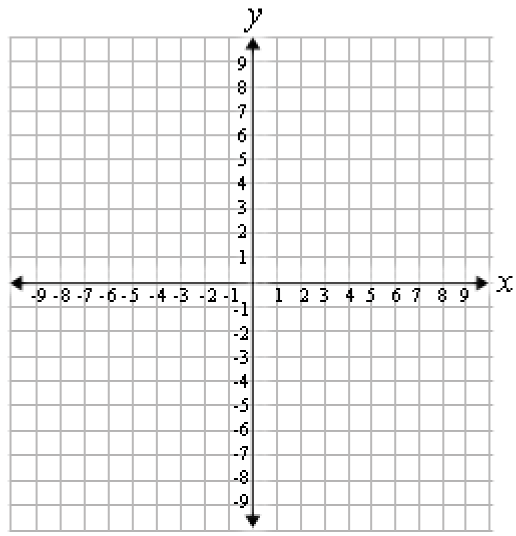
2. $f(x) = -2x - 3$



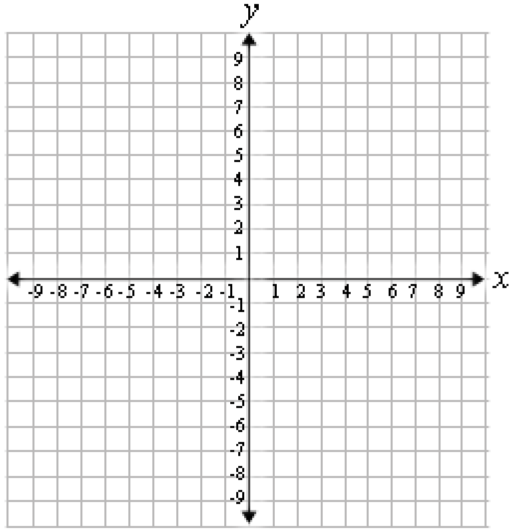
3. $f(x) = \frac{3}{4}x + 2$



4. $f(x) = 3$



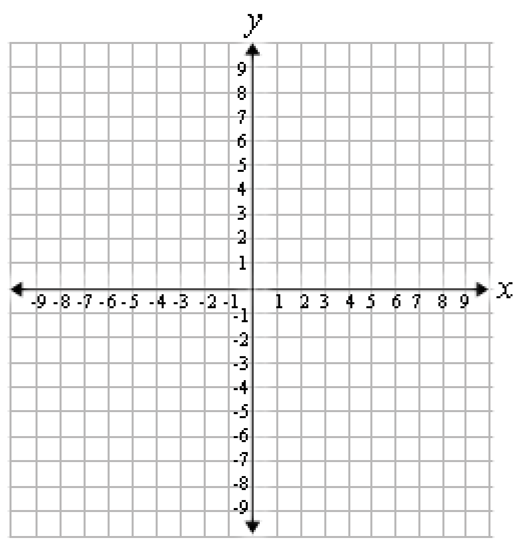
5. $f(x) - 3 = 1 - 2x$



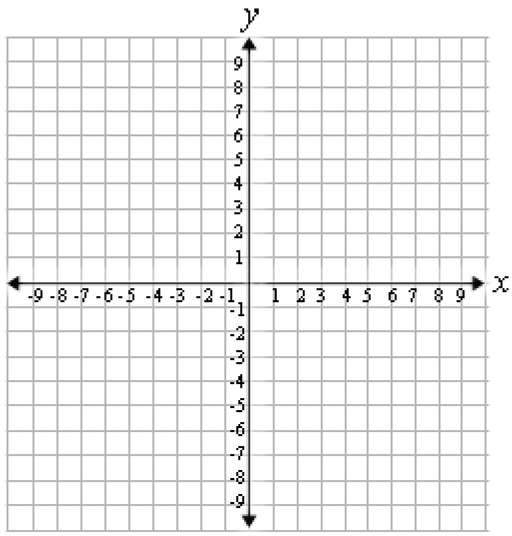
Graphing Linear Functions – Slope-Intercept
Assessment 1

For each of the following problems, graph the linear function using slope-intercept. In the work space that follows each graph, explain how you created the graph. Be very specific.

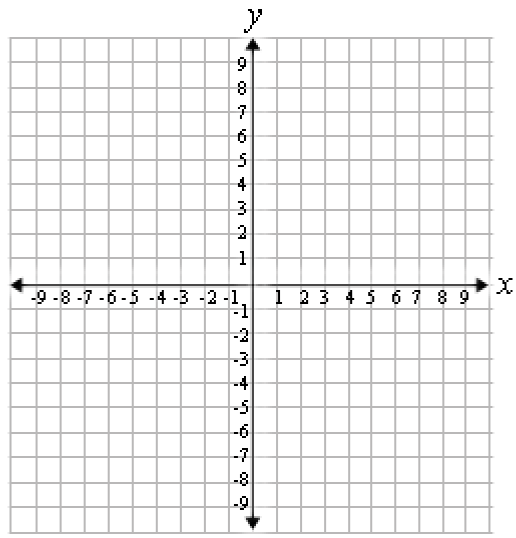
1. $f(x) = 2x + 4$



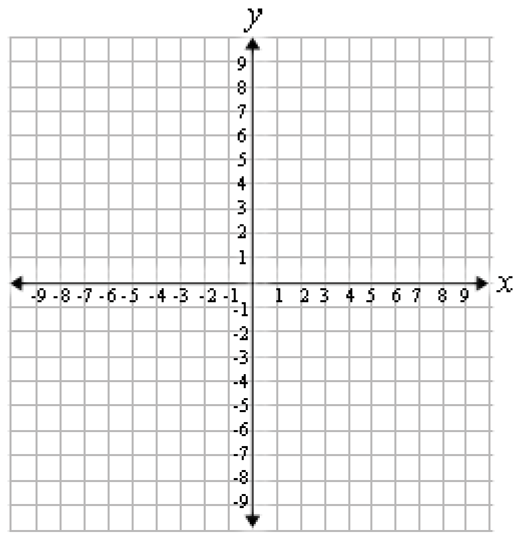
2. $f(x) = -2x + 3$



3. $f(x) = \frac{2}{3}x + 3$



4. $f(x) = -3$



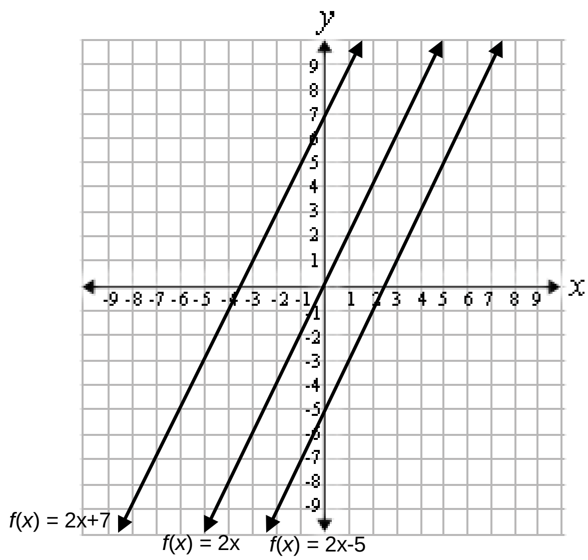
5. In your own words, explain how to graph a function using the slope and y-intercept of that function.

Session 2 – Graphing Linear Functions – Slope-Intercept

Consider what happens to the graph of $f(x) = 2x + 4$ when we change only one parameter. If we change the y-intercept only, the slope will be exactly the same. The difference in the shape of the graph will be where the line intersects the y-axis.

Study the graphs of the functions when the y-intercept is changed:

$f(x) = 2x + 7$
$f(x) = 2x$
$f(x) = 2x - 5$



What happened to the line when the y-intercept changed?

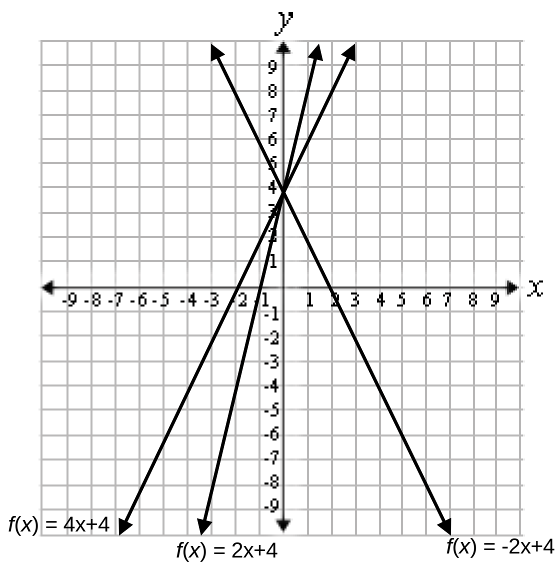
What is the relationship of the graphs of the lines to each other?

Summarize what changing the y-intercept does for a linear function.

Consider what happens to the graph of $f(x) = 2x + 4$ when we change only one parameter. If we change the slope only, the y-intercept stays the same. The difference in the shape of the graph will be in its steepness.

Study the graphs of the function when the slope is changed:

$f(x) = 2x + 4$
$f(x) = 4x + 4$
$f(x) = -2x + 4$

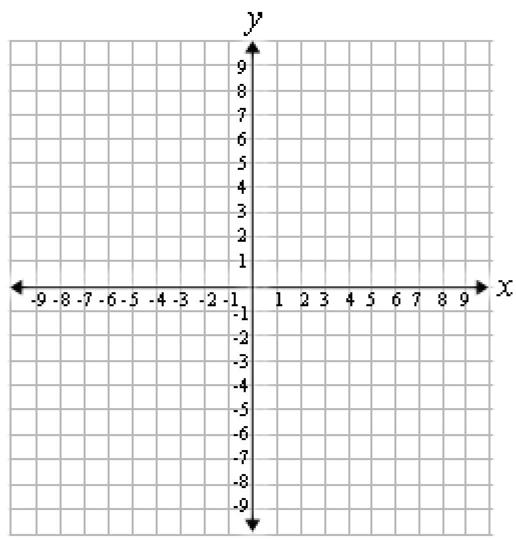


What happened to the line when the slope changed?

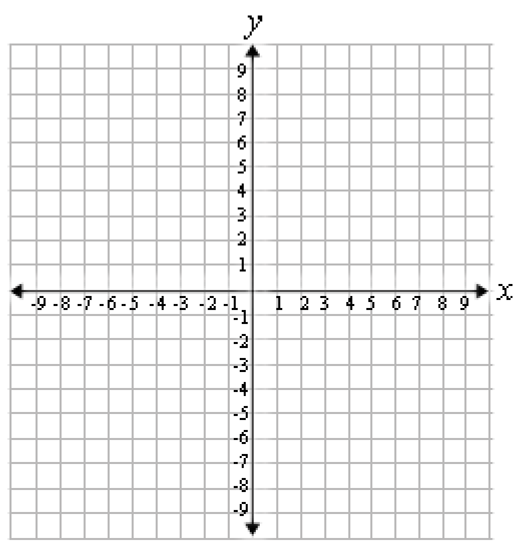
What is the relationship of the graphs of the lines to each other?

Summarize what changing the slope does for a linear function.

In the space below, make up a linear function and graph it. Then change its y-intercept two times and graph each of these new functions. Describe the changes in the graphs of these lines when the y-intercept is changed.



In the space below, make up a linear function and graph it. Then change its slope two times and graph each of these new functions. Describe the changes in the graphs of these lines when the slope is changed.



Many times a graph is used to represent a contextual problem. Study the following problems and the graphs that follow them.

Example 1:

A phone company has many different payment plans for a cell phone. One of them charges a flat rate of $10 per month for the first 200 minutes of use and then 5 cents for each minute after the original 200 minutes. Represent the cost for this cell phone plan graphically.

Solution:

First, define a function that represents the information in the problem.

Let x = the number of minutes used over 200 minutes.

Let f(x) = the monthly cost of the plan.

Then, $f(x) = .05x + 10$

Let's make a table of values.

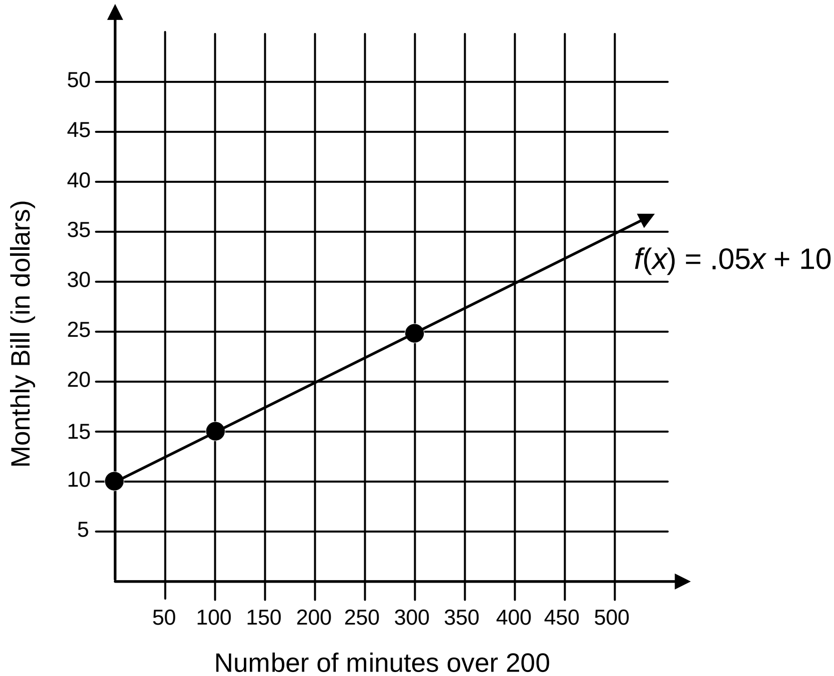
If the customer uses only 200 minutes for the month, the cost is $10.

If the customer uses 300 minutes for the month, the cost increases to $10 plus (300-200) minutes • $.05. This equals $10 + 100 • $.05 which is $15.

If the customer uses 500 minutes for the month, the cost increases to $10 plus (500-200) minutes • $.05. This equals $10 + 300 • $.05 which is $25.

x	**f(x)**
0	$10
100	$15
300	$25

Now let's graph these values to find the line of our function



Answer the following questions about the graph.

1. **Why is it important to indicate the function before graphing the information in the problem?**

2. **List some advantages for graphing the function as described by the information in the problem.**

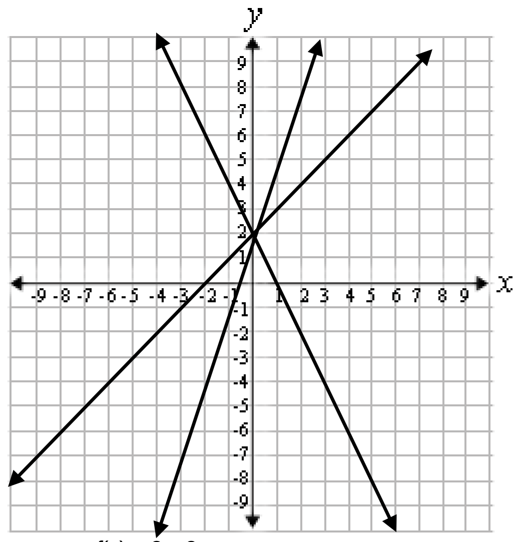
Graphing Linear Functions – Slope-Intercept
Assessment 2

1. Study the graphs of the function when the slope is changed:

$f(x) = 3x + 2$

$f(x) = x + 2$

$f(x) = -2x + 2$



What happened to the line when the slope changed?

What is the relationship of the graphs of the lines to each other?

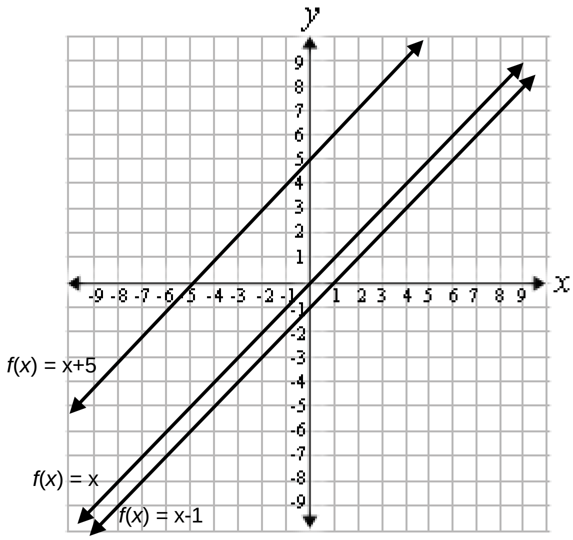
Summarize what changing the slope does for a linear function.

2. Study the graphs of the functions when the y-intercept is changed:

$f(x) = x - 1$

$f(x) = x$

$f(x) = x + 5$



What happened to the line when the y-intercept changed?

What is the relationship of the graphs of the lines to each other?

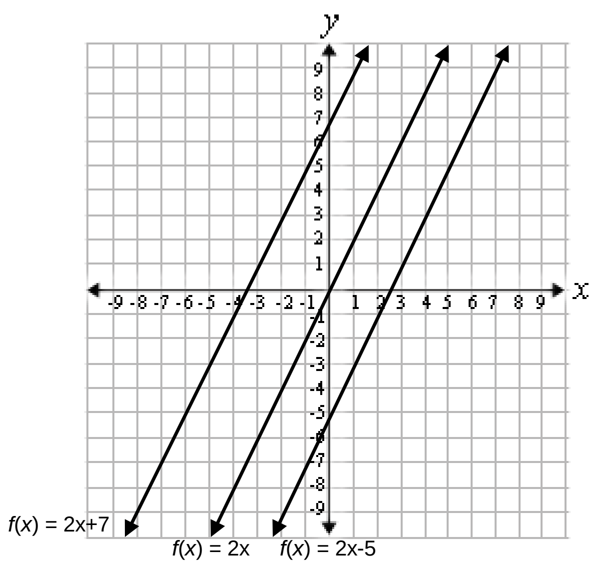
Summarize what changing the y-intercept does for a linear function.

Session 3 – Graphing Linear Functions – Slope-Intercept

It is possible to determine whether two lines are parallel, perpendicular, intersecting, or coincident by looking at the slopes of the lines. Recall the graphs and what they looked like when we changed only the y-intercept. The lines were parallel.

Parallel lines have the same slope.

$f(x) = 2x + 7$
$f(x) = 2x$
$f(x) = 2x - 5$



In each function, the slope of the line is 2. All three lines are parallel.

Example 2:

Consider the set of functions:

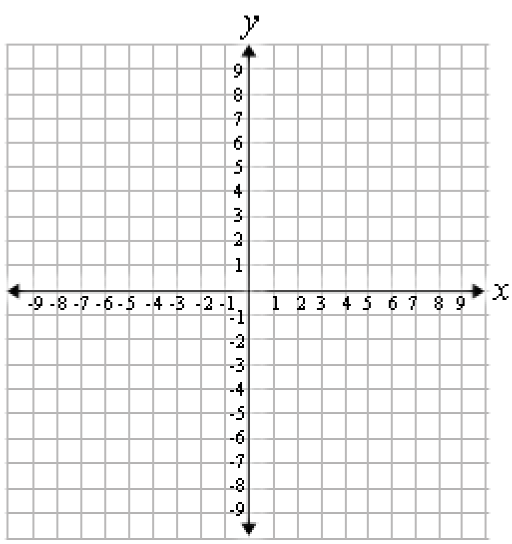
$f(x) = x + 5$

$f(x) = x$

$f(x) = x - 5$

What do you know about this set of functions? What will the graphs of the functions be? How are the three functions related?

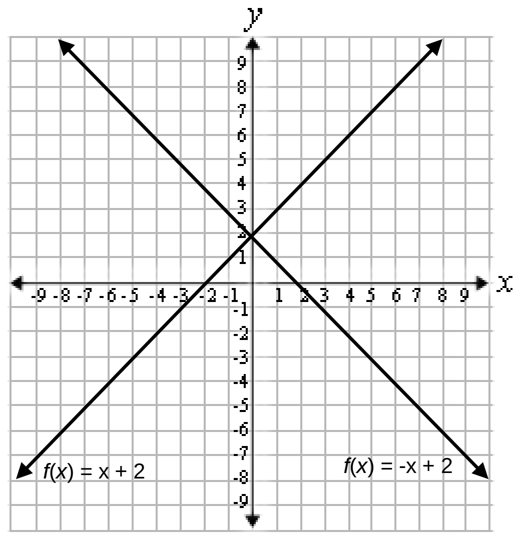
Graph the three functions on the same set of axes to check your answers. Use any method you wish to graph the functions (table of values, intercepts, or slope-intercept) but show your work.



Now, let's look at the graphs of some other functions.

Example 3:

Study the following graph of two functions.



Both graphs have a y-intercept of 2. If we use $\frac{rise}{run}$, the graph that is decreasing has a slope of $\frac{-1}{1}$ while the graph that is increasing has a slope of $\frac{1}{1}$.

The equations of the functions then become

$$f(x) = x + 2$$
$$f(x) = -x + 2$$

Label each graph with its proper equation. When we examine the lines we see that they are perpendicular to each other. The slopes are -1 and 1 which are negative reciprocals of each other.

Parallel lines have the same slope.

The slopes of perpendicular lines are negative reciprocals of each other.

Example 4:

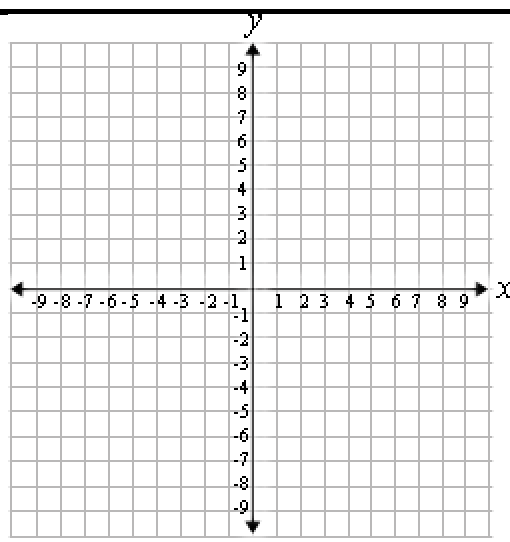
Graph the functions below and determine their relationship to each other.

$$f(x) = \frac{2}{3}x + 3$$

$$f(x) = -\frac{3}{2} - 4$$

Let's look at each function separately. The slope of $f(x) = \frac{2}{3}x + 3$ is $\frac{2}{3}$ and its y-intercept is 3.

The slope of $f(x) = -\frac{3}{2}x - 4$ is $-\frac{3}{2}$ and its y-intercept is -4. Graph each line showing all your work and determine the relationship of each function to the other. Justify your answer.



Lines can also be coincident which means that they are exactly the same line and every point on one line is exactly the same as every point on the other line. If lines are not coincident or parallel, they are intersecting. Intersecting lines do not have to have to be perpendicular. They have one point in common.

Consider the following sets of functions. Determine if they are parallel, perpendicular, coincident, or intersecting and not perpendicular. Justify your answer.

1. $$f(x) = 2x$$
$$f(x) = -2x$$

2. $$f(x) = 3x + 2$$
$$f(x) = 3x - 2$$

3. $$f(x) = \frac{x}{2} + 5$$
$$f(x) = -2x + 3$$

4. $$f(x) - 2 = x + 3$$
$$f(x) - 3 = x + 2$$

5. $f(x) = 6$
$x = 2$

6. $f(x) = 3x + 2$
$f(x) = -2x + 2$

7. Summarize the relationships two lines can have in relationship to each other.

Graphing Linear Functions – Slope-Intercept
Assessment 3

Consider the following sets of functions. Determine if they are parallel, perpendicular, coincident, or intersecting and not perpendicular. Justify your answer.

1. $f(x) = 3x$
   $f(x) = -3x$

2. $f(x) = 4x + 1$
   $f(x) = 4x - 1$

3. $f(x) = \frac{x}{3} + 1$
   $f(x) = -3x + 4$

4. $f(x) - 3 = 2x + 3$
$f(x) - 2 = 2x + 4$

5. $f(x) = 3$
$x = -3$

Extensions

1. Graph functions using a graphing calculator.

Graphing Linear Functions – Slope-Intercept Extension

We can graph an equation using a graphing calculator. We need to put the equation in "$y =$" after you have placed the equation in slope-intercept form. Check to make sure your window is correct for the equation you are graphing. Usually, we use a standard window in which the minimum value for x is –10, the maximum value for x is 10, the minimum value for y is –10 and the maximum value for y is 10. We usually use a scale of one which means that each unit between –10 and 10 is 1. If the scale were 2, then each unit between –10 and 10 would represent 2. After putting in the equation and checking to make sure that the window is correct, hit the "graph" key and the equation will show up on the graphing screen. Make sure to be accurate when sketching the graph on your paper.

Try putting some of the equations you graphed in this lesson into your graphing calculator to see if you obtained the correct shapes of the graphs.

Sources

2008 AZ Mathematics Standards
2000 NCTM Principles and Standards
2008 The Final Report of the National Mathematics Advisory Panel
1999 Bringing the NCTM Standards to Life, Exemplary Practices from High Schools