

Fundamentals of Graphing Linear Functions

An ADE Mathematics Lesson

Days 11-15

Author	ADE Content Specialists
Grade Level	9 th grade
Duration	Five days

Aligns To

Mathematics HS:
Strand 3: Patterns, Algebra, and Functions
Concept 2: Functions and Relationships
PO 1. Sketch and interpret a graph that models a given context, make connections between the graph and the context, and solve maximum and minimum problems using the graph.
PO 2. Determine if a relationship represented by an equation, graph, table, description, or set of ordered pairs is a function.
PO 4. Use equations, graphs, tables, descriptions, or sets of ordered pairs to express a relationship between two variables.
PO 7. Determine domain and range of a function from an equation, graph, table, description, or set of ordered pairs.

Strand 3: Patterns, Algebra, and Functions
Concept 3: Algebraic Representations
PO 1. Create and explain the need for equivalent forms of an equation or expression.
PO 3. Write an equation given a table of values, two points on the line, the slope and a point on the line, or the graph of the line.
Concept 4: Analysis of Change
PO 1. Determine the slope and intercepts of the graph of a linear function, interpreting slope as a constant rate of change.

Strand 4: Geometry and Measurement
Concept 3: Coordinate Geometry
PO 5. Graph a linear equation or linear inequality in two variables.

Strand 5: Structure and Logic
Concept 2: Logic, Reasoning, Problem Solving, and Proof
PO 1. Analyze a problem situation, determine the question(s) to be answered, organize given information, determine how to represent the problem, and identify implicit and explicit assumptions that have been made.

Connects To

Mathematics HS:
Strand 2: Data Analysis, Probability, and Discrete Mathematics
Concept 1: Data Analysis (Statistics)
PO 2. Organize collected data into an appropriate graphical representation with or without technology.

Strand 3: Patterns, Algebra, and Functions
Concept 1: Patterns
PO 1. Recognize, describe, and analyze sequences using tables, graphs, words, or symbols; use sequences in modeling.

Strand 4: Geometry and Measurement
Concept 2: Transformation of Shapes
PO 2. Determine the new coordinates of a point when a single transformation is performed on a 2-dimensional figure.
PO 3. Sketch and describe the properties of a 2-dimensional figure that is the result of two or more transformations.
Concept 3: Coordinate Geometry
PO 1. Determine how to find the midpoint between two points in the coordinate plane.
PO 3. Determine the distance between two points in the coordinate plane.

Strand 5: Structure and Logic
Concept 2: Logic, Reasoning, Problem Solving, and Proof
PO 5. Summarize and communicate mathematical ideas using formal and informal reasoning.

Aligns To

Mathematics HS:

Strand 5: Structure and Logic

Concept 2: Logic, Reasoning, Problem Solving, and Proof

PO 2. Solve problems by formulating one or more strategies, applying the strategies, verifying the solution(s), and communicating the reasoning used to obtain the solution(s).

PO 3. Evaluate a solution for reasonableness and interpret the meaning of the solution in the context of the original problem.

PO 6. Synthesize mathematical information from multiple sources to draw a conclusion, make inferences based on mathematical information, evaluate the conclusions of others, analyze a mathematical argument, and recognize flaws or gaps in reasoning.

Connects To

Overview

Graphing linear functions is very important. Representing linear functions in several different ways enables you to gain a conceptual understanding of graphing.

Purpose

In this lesson you will graph linear functions by using a table of values or plotting the x- and y-intercept. You will learn how to differentiate functions from relations and how to find the domain and range of a function in different ways.

Materials

- Graphing worksheets
- Ruler
- Graph paper

Objectives

Students will:

- Use equations, graphs, tables, descriptions, or sets of ordered pairs to express a relationship between two variables.
- Graph a linear equation in two variables.
- Determine how to identify functions.
- Determine domain and range of a function from an equation, graph, table, description, or set of ordered pairs.

Lesson Components

Prerequisite Skills: This lesson builds upon prior skills of graphing in a coordinate plane. In grade 6, you graphed ordered pairs in any quadrant of the coordinate plane. In grade 7, you used a table of values to graph an equation or proportional relationship and to describe the graph's characteristics. In grade 8, you determined if a relationship represented by a graph or table is a function.

Vocabulary: *Cartesian coordinate system, coordinate plane, graph, ordered pairs, origin, x-coordinate, y-coordinate, quadrant, axes, element, domain, range, independent variable set, dependent variable set, rule of correspondence, x-intercept, y-intercept, slope, rate of change, relation, function, linear function, vertical line, horizontal line, slope-intercept form of the equation of a line, rise, run*

Session 1 Part 1: Fundamentals of Graphing (1 day)

1. Review graphing points on a coordinate plane.

Session 1 Part 2: Functions (2 days)

1. Determine if a relation is a function.
2. Determine whether a function is a linear function or one of a higher degree.

Session 2 Part 1: Linear Functions and Graphing using a Table of Values or x- and y-intercepts (1 day)

1. Graph linear functions using a table of values or the x-intercept and the y-intercept.

Session 2 Part 2: Domain and Range of a Function (1 day)

1. Determine the domain and range of a linear function.

Assessment

There are multiple assessments scattered throughout the lessons that will help pinpoint misconceptions before moving on to more complex graphs such as the graphs of quadratic functions.

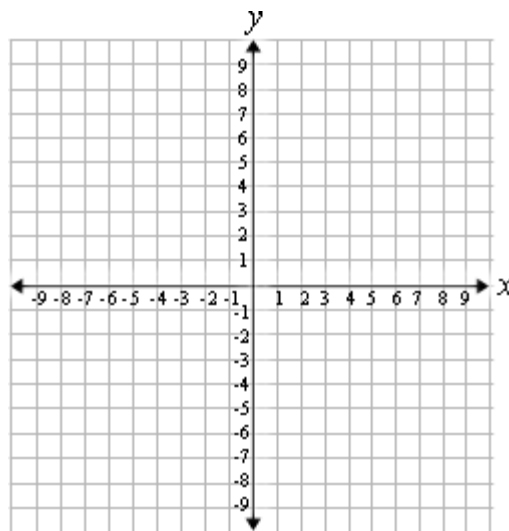
Fundamentals of Graphing Linear Functions

Session 1 Part 1 - Fundamentals of Graphing

Graphing is a technique that is required in many mathematical applications. It is very important to be able to graph straight lines accurately and to understand how to interpret the graph once constructed. To be able to do so requires a good grasp of graphing concepts. This lesson is designed to help you understand graphing concepts from a very elementary level to a more sophisticated level of graphing. It reviews basic graphing techniques and how to determine if a relation is a function.

There are many **fundamental terms** that we need to understand to be able to grasp the concepts. See if you can complete the table with the definition of terms that are associated with graphing. The definitions follow the table so that you can check your work. Note that the definitions should be in your own words.

The Cartesian Coordinate Plane is divided into 4 quadrants. Place the name of each quadrant in its proper location indicating the signs for ordered pairs in that quadrant (+, +) etc. Indicate the origin on the graph.



Does your answer indicate that ordered pairs in Quadrant 1 are (+, +), in Quadrant 2 are (-, +), in Quadrant 3 are (-, -), and in Quadrant 4 are (+, -)? The origin is at (0, 0).

Define the following graphing terms in your own words. Check your answer with the definitions that follow the table.

Term	Definition
ordered pair	
x-intercept	
x-coordinate	
y-intercept	
y-coordinate	
origin	
graph	
quadrant	
axis	
slope of a line	
vertical line	
horizontal line	
Cartesian Coordinate System	

1. **Ordered Pair** - a pair of numbers used to locate and describe points in the coordinate plane in the form (x, y)
2. **x-intercept** - the coordinate at which the graph of a line intersects the x-axis
3. **x-coordinate** - 1st value in an ordered pair
4. **y-intercept** - the coordinate at which the graph of a line intersects the y-axis
5. **y-coordinate** - 2nd value in an ordered pair
6. **Origin** - the intersection of the axes in a coordinate grid, often defined as $(0, 0)$ in two-dimensions
7. **Graph** - a representation of an algebraic equation applied to a coordinate grid
8. **Quadrant** - one of the four sections into which the coordinate plane is divided by the x- and y-axes
9. **Axis (axes: plural)** (in two-dimensions) - one of two perpendicular number lines used to form a coordinate system
10. **Slope of a line** - the measure of steepness of a line calculated as the change in y divided by the change in x (the rise over the run)
11. **Vertical Line** - a line perpendicular to the x-axis with an undefined slope ($x = a$)
12. **Horizontal line** - a line parallel to the x-axis with a slope of 0 ($y = b$)
13. **Cartesian Coordinate System** - a plane containing points identified by their distance from the origin in ordered pairs along two perpendicular lines referred to as axes (note: also referred to as coordinate plane and rectangular coordinate plane)

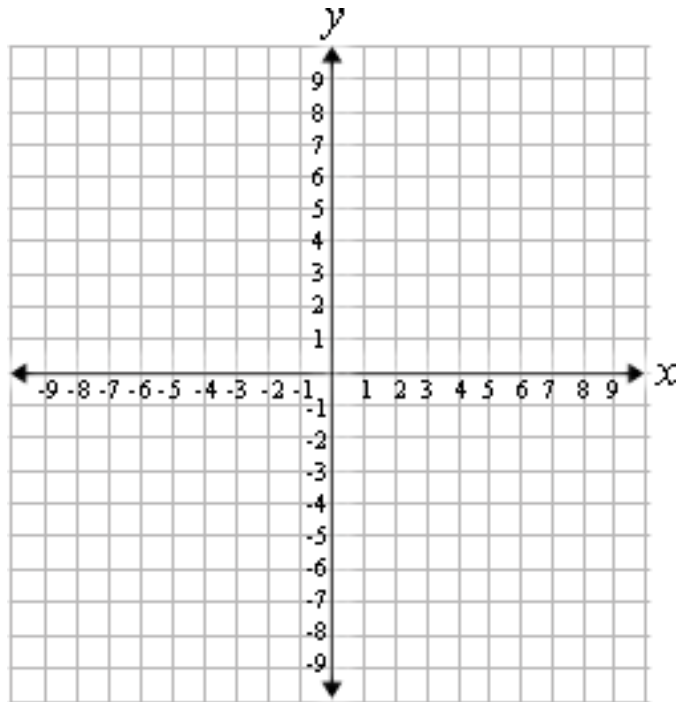
Please note that there are **many terms that describe exactly the same thing** but may vary from textbook to textbook. For example, the following terms are identical in meaning:

- ordered pair, point
- 1st value, x-value, abscissa, input variable
- 2nd value, y-value, ordinate, output variable

To graph a point, the x-value would be indicated on the horizontal axis and the y-value is indicated on the vertical axis. If the x-value is positive, move to the right and if it is negative, move it to the left. If the y-value is positive, move up and if it is negative, move down.

Graph the following points on the graph provided. Place the letter of each point on the graph in the appropriate place.

A (2, 3) B (-1, 8) C (3, -4) D (-7, -7) E (0, 0) F (5, 0) G (-8, -2)



Fundamentals of Graphing Linear Functions Assessment 1

Define each of the following terms in the space provided.

Term	Definition
relation	
function	
linear function	
quadratic function	
independent variable	
dependent variable	
rule of correspondence	
domain	
range	
slope of a line	
x-intercept	
y-intercept	

Fundamentals of Graphing Linear Functions

Session 1 Part 2 - Functions

A **relation** is a correspondence between a first set, the domain, and a second set, the range, such that each member of the domain corresponds to at least one member of the range. We can express the relationship between two sets in many different ways. Consider the relations:

$$\{4, 2, 0, -2, -4\} \Rightarrow \{2, 1, 0, -1, -2\}$$

There are 5 **elements** in each set. The first set contains the elements 4, 2, 0, -2, and 4 and is called the **domain** of the relation. The second set contains the elements 2, 1, 0, -1, -2 and is called the **range** of the relation. In this case, each element in the range is one-half of the same element in the domain. This is called a **correspondence**.

Study the following sets and diagrams. Determine why some relations are labeled “yes” and some are labeled “no”.

$$\{4, 2, 0, -2, -4\} \Rightarrow \{2, 1, 0, -1, -2\} \text{ yes}$$

$$\{1, 2, 3, 4, 5\} \Rightarrow \{1, 4, 9, 16, 25\} \text{ yes}$$

$$\{25, 16, 9, 4, 1\} \Rightarrow \{5, -5, 4, -4, 3, -3, 2, -2, 1, -1\} \text{ no}$$

$$\{\text{Arizona, Phoenix}\} \Rightarrow \{\text{Diamondbacks, Cardinals, Coyotes, Suns}\} \text{ no}$$

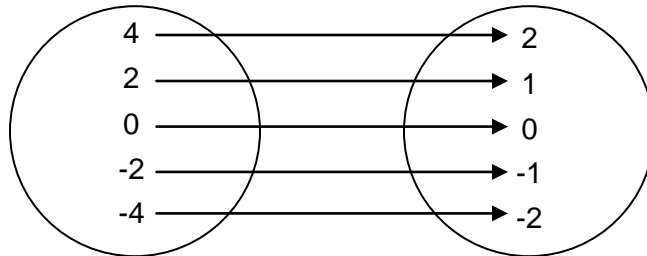
$$\{\text{Arizona, New York}\} \Rightarrow \{\text{Phoenix, Tucson, Buffalo, Albany}\} \text{ no}$$

$$\{1, 1, 2, 2, 3, 3\} \Rightarrow \{2, 4, 6\} \text{ yes}$$

Let's look at the correspondence for each relation.

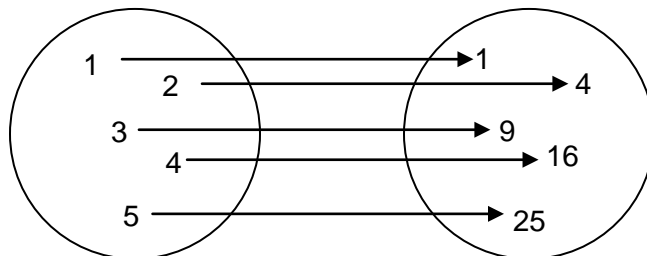
In the first relation, each element in the second set is one-half the corresponding element in the first set.

$$4 \Rightarrow 2, 2 \Rightarrow 1, 0 \Rightarrow 0, -2 \Rightarrow -1, -4 \Rightarrow -2$$



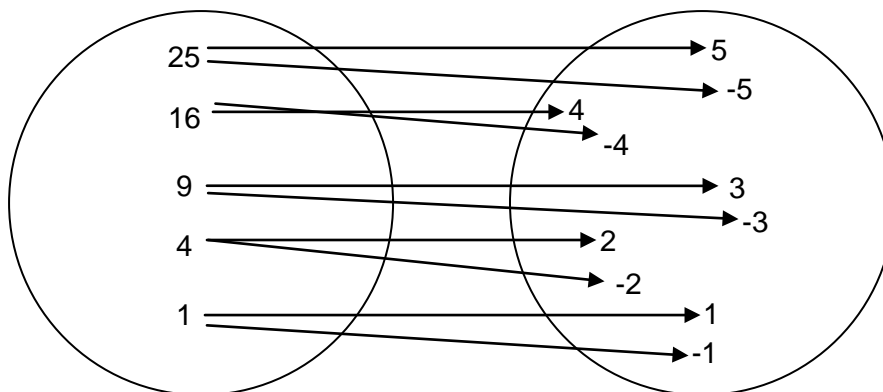
In the second relation, each element in the second set is the square of the corresponding element in the first set.

$$1 \Rightarrow 1, 2 \Rightarrow 4, 3 \Rightarrow 9, 4 \Rightarrow 16, 5 \Rightarrow 25$$



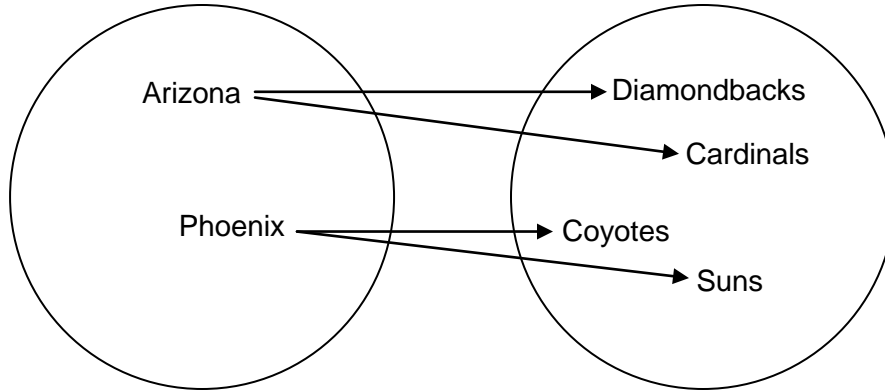
In the third relation, each element in the second set is the square root of the corresponding element in the first set.

$$25 \Rightarrow 5 \text{ and } -5, 16 \Rightarrow 4 \text{ and } -4, 9 \Rightarrow 3 \text{ and } -3, 4 \Rightarrow 2 \text{ and } -2, 1 \Rightarrow -1 \text{ and } 1$$



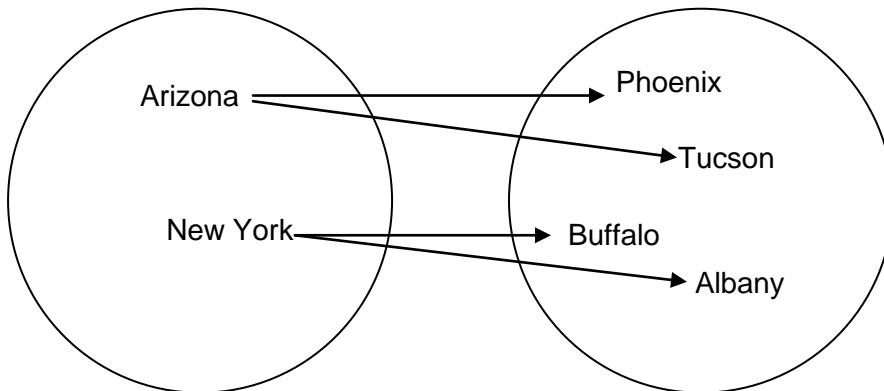
In the fourth relation, each element in the second set is a sports team named by the place in the first set.

Arizona \Rightarrow Diamondbacks and Cardinals, Phoenix \Rightarrow Coyotes and Suns



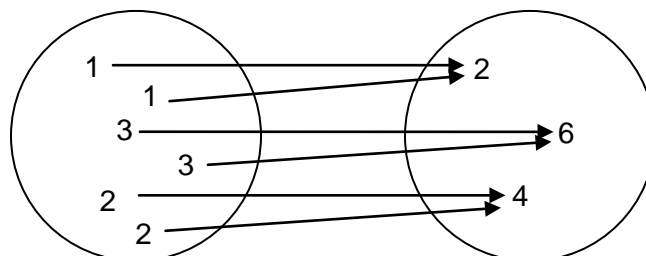
In the fifth relation, each element in the second set is a city located in the state listed in the first set.

Arizona \Rightarrow Phoenix and Tucson, New York \Rightarrow Buffalo and Albany



In the sixth relation, each element in the second set is twice that of an element listed in the first set.

1 \Rightarrow 2, 2 \Rightarrow 4, 3 \Rightarrow 6



In each instance the domain is the first set listed and the range is the second set listed. The rule stated in each relation is the correspondence between the first set and the second set. Have you determined why some relations are marked “yes” and some relations are marked “no”? If not, look the example again to help you.

A **function** is a rule that defines a relationship between two sets of numbers in that for each value of the **independent variable set** there is only one value in the **dependent variable set**. Another way to define function is that it is a **correspondence** between a first set, called the **domain**, and a second set, called the **range**, such that each member of the domain corresponds to **exactly one** member of the range.

The domain is the same as the independent variable set. We can choose any number for the domain. However, because there is some rule or correspondence from the domain to the range, the value of the range is dependent on the values chosen for the domain. Therefore the range is the dependent variable set.

Each of the relations previously listed that had “yes” next to them were functions. Those that had “no” next to them were not functions. For every element in the domain of each relation listed, all those that had only one element in the range were labeled with “yes”. Note that all functions are relations, but not all relations are functions.

Determine which of the following relations are functions and justify your answer. State the correspondence for each relation.

1. $\{1, 2, 3, 4, 5\} \Rightarrow \{3, 6, 9, 12, 15\}$

2. $\{\text{California}\} \Rightarrow \{\text{Los Angeles, San Francisco, San Diego}\}$

3. $\{100\} \Rightarrow \{1, 2, 5, 10, 20, 25, 50, 100\}$

4. $\{1, 2, 3, 4, 6, 8, 12, 24\} \Rightarrow \{24\}$

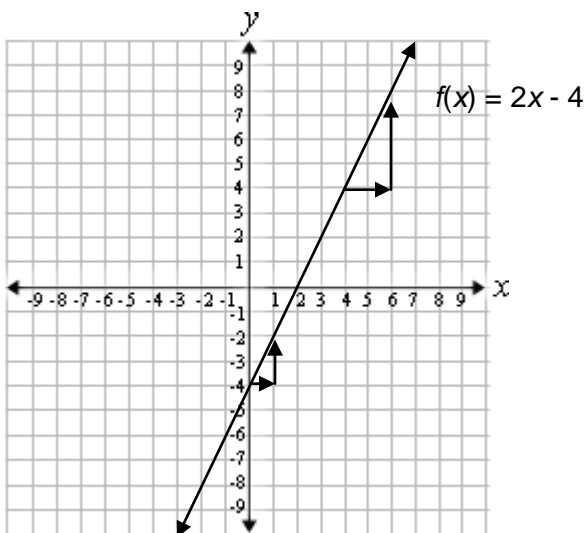
5. $\{\text{Beagle, Poodle, Chow, German Shepherd}\} \Rightarrow \{\text{Dog}\}$

6. $\{100, 80, 60, 40, 20\} \Rightarrow \{5, 4, 3, 2, 1\}$

Fundamentals of Graphing Linear Functions Session 2 Part 1 – Linear Functions

Linear functions can be written in the form, $f(x) = mx + b$. A **linear function** is a function that has a constant rate of change and can be modeled by a straight line. In the equation, $f(x) = mx + b$, m is called the slope and b is the y-intercept. An example of a linear function is $f(x) = 2x - 4$.

In this function, the slope is 2 and the y-intercept is -4. Study the graph of $f(x) = 2x - 4$.



The graph of the linear function intercepts the y-axis at (0, -4). The slope is 2 which means for every 1 unit we move horizontally, we move 2 units vertically. Note the slopes indicated on the graph. (4, 4) becomes (6, 8). The change in x is 2 units. The change in y is 4 units. $\frac{4}{2}$ equals 2 which is the slope of the function, $f(x) = 2x - 4$. The point (0, -4) becomes (1, -2). The change in x is 1 unit and the change in y is 2 units. Again, this is the slope of the function, $f(x) = 2x - 4$.

A linear function has no exponent for a variable greater than 1. The function $f(x) = x^2 + 2x + 1$ will be a parabola when graphed as this is a quadratic function.

When the highest exponent of a variable is 2 in a function, it is a quadratic function.

Determine the slope and y-intercept of each linear function.

Function	Slope	y-intercept
$f(x) = 4x + 2$		
$f(x) = -3x - 1$		
$f(x) = -x + 6$		
$f(x) = 7x$		
$f(x) = x - 5$		
$f(x) = 2x + 9$		

Determine if each function is linear or quadratic. Justify your answer.

Function	Linear or Quadratic
$f(x) = 4x - 3$	
$f(x) = x^2$	
$f(x) = x$	
$f(x) = x^2 + 3x - 4$	

Fundamentals of Graphing Linear Functions

Session 2 Part 1

Graphing Using a Table of Values or x- and y-intercepts

When graphing, there are some **expectations** to which we need to adhere. These include:

1. Use graph paper and graph using a pencil.
2. Use a straightedge to make sure that your lines are straight for linear equations.
3. The units on the axes need to be at equal intervals and the units need to be marked so that others reading your graph will know what each tick mark represents on both the x- and y-axes. The units on the x-axis do not need to be the same as the units on the y-axis.
4. The x- and y-axes represent infinite lines so place arrows on the end of each axis.
5. The graph of a linear function is a line not a line segment. Place arrows at the end of the line to indicate the infinite length.
6. Make sure your graph crosses both axes when you extend the line.
7. If you are graphing points, make sure to indicate the points by making them large enough to be visible to the reader. When working with points, it's a good idea to list the coordinates of the point near the graph of the point.
8. Mark the equation of the line on or near the line.
9. If you graph the equation correctly, it should look exactly the same as another person graphing the same equation.
10. Do not graph more than one equation on a set of axes unless the problem asks you to do so. If you are graphing more than one line on the same set of axes, color code or use a legend to indicate which line is the graph of which equation.

There are many different approaches to graphing an equation. Let's review some of them.

1. **We can graph an equation by using a table of values.** Two points determine a line but it is good to have three points and use one as a check. You may choose any x value (or y value) and solve for the other value. Then make up a table and graph these points. Now, draw a line connecting these three points indicating the infinite length with arrows.
2. **We can graph an equation by using the x- and y-intercepts.** When using this method, we find the x-value when y is 0 (this is the x-intercept). Then, find the y-value when x is 0 (this is the y-intercept). Graph these two points and draw a line connecting them. Extend the line and draw arrows on each end to indicate the infinite length.
3. **We can graph an equation by placing it in slope-intercept form: $y = mx + b$ or $f(x) = mx + b$.** The m represents the slope and the b is the y-intercept. We first draw a point where the line is going to intercept the y-axis. From this point, we use $\frac{\text{rise}}{\text{run}}$ to determine a second point. If the run is positive, we move to the right from the y-intercept. If it is negative, we move to the left. If the rise is positive, we move up and if it's negative, we move down. You may do the rise first or the run first and then do the other from where the new point would be.

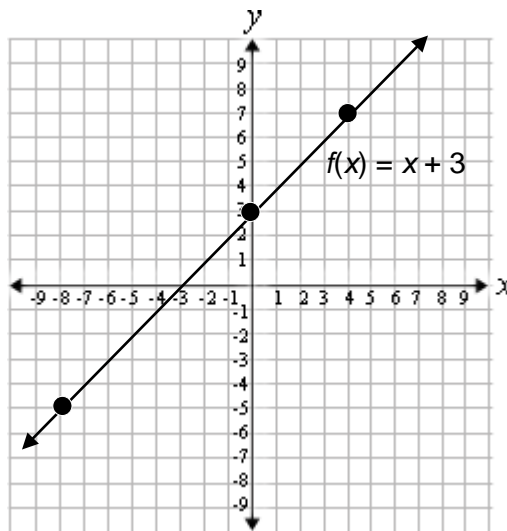
Example 1: Graph $f(x) = x + 3$

Make sure to follow all the expectations enumerated above.

1. Graph $f(x) = x + 3$ using a table of values:
 - a. Find at least three points that satisfy the equation. Two points determine a line but the third point is a check for accuracy. Make a table. You may choose any points you like but remember that you need an appropriate scale for the axes.

x	$f(x)$
0	3
4	7
-8	-5

- b. Graph these points and connect them using the representation of an infinite line.



- c. Label the line appropriately.

2. Graph $f(x) = x + 3$ using the intercepts:

- a. Substitute 0 in for x in the equation and solve for $f(x)$. This is the y-intercept (the place where the function crosses the y-axis).

$$f(x) = x + 3 \Rightarrow f(x) = 0 + 3 \Rightarrow f(x) = 3$$

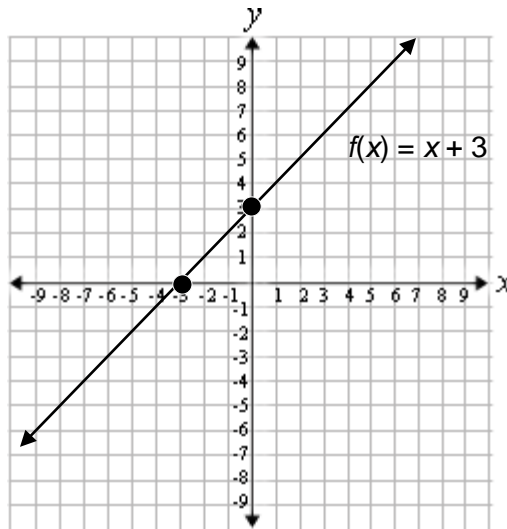
- b. Substitute 0 in for $f(x)$ in the equation and solve for x . This is the x-intercept (the place where the function crosses the x-axis).

$$f(x) = x + 3 \Rightarrow 0 = x + 3 \Rightarrow x = -3$$

- c. Plot these two points.

x	$f(x)$	intercept
0	3	y-intercept
-3	0	x-intercept

- d. Draw a line connecting the intercepts using the representation of an infinite line.



- e. Label the line appropriately.

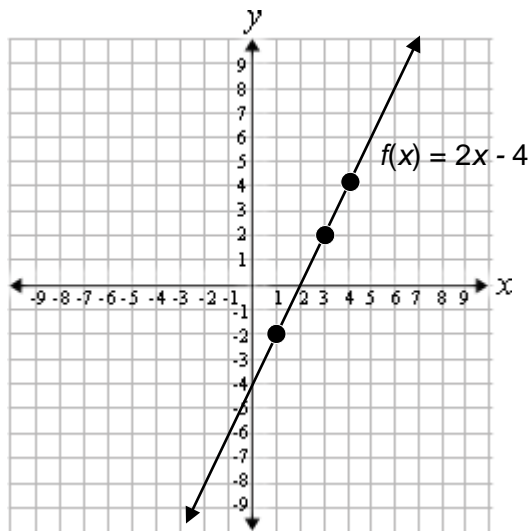
Example 2: Graph $f(x) = 2x - 4$

Make sure to follow all the expectations enumerated above.

1. Graph $f(x) = 2x - 4$ using a table of values:
 - a. Find at least three points that satisfy the equation. Two points determine a line but the third point is a check for accuracy. Make a table. You may choose any points you like but remember that you need an appropriate scale for the axes.

x	$f(x)$
1	-2
4	4
3	2

- b. Graph these points and connect them using the representation of an infinite line.



- c. Label the line appropriately.

2. Graph $f(x) = 2x - 4$ using the intercepts:

- a. Substitute 0 in for x in the equation and solve for $f(x)$. This is the y-intercept (the place where the function crosses the y-axis).

$$f(x) = 2x - 4 \Rightarrow f(x) = 2(0) - 4 \Rightarrow f(x) = -4$$

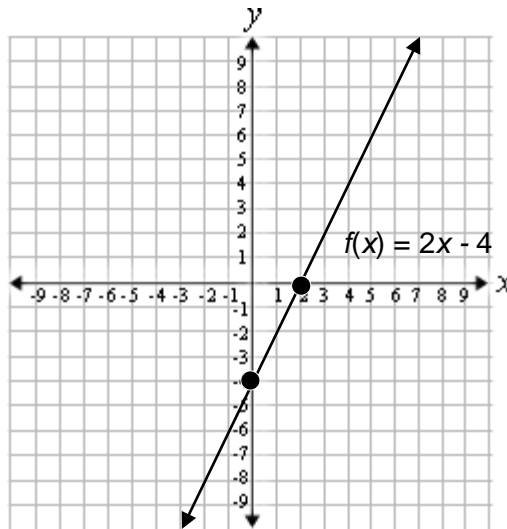
- b. Substitute 0 in for $f(x)$ in the equation and solve for x . This is the x-intercept (the place where the function crosses the x-axis).

$$f(x) = 2x - 4 \Rightarrow 0 = 2x - 4 \Rightarrow 2x = 4 \Rightarrow x = 2$$

- c. Plot these two points.

x	$f(x)$	intercept
0	-4	y-intercept
2	0	x-intercept

- d. Draw a line connecting the intercepts using the representation of an infinite line.

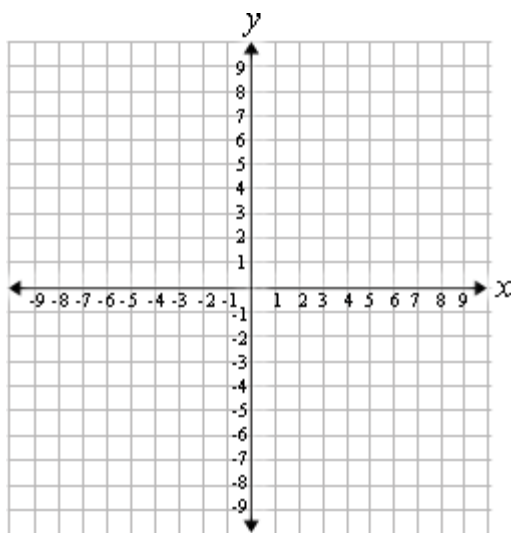


- e. Label the line appropriately.

Now try to graph other functions using both a table of values and the x- and y-intercepts. Remember that in each case, the graph should be identical. All points that are on the first graph made with the table of values should also be on the graph that used the x and y intercepts. Show all your work.

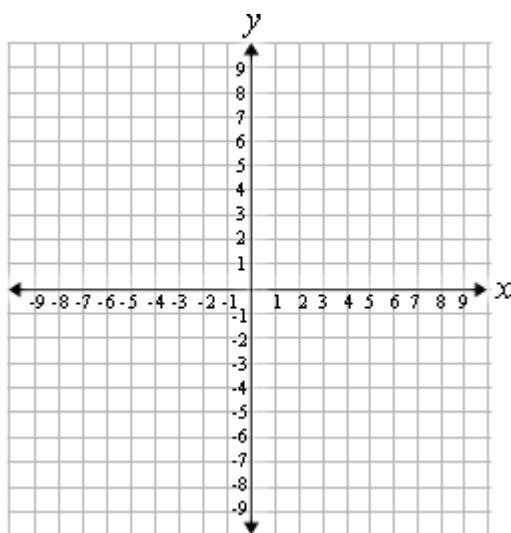
Problem 1a: Graph $f(x) = -x + 2$. Use a table of values.

x	$f(x)$



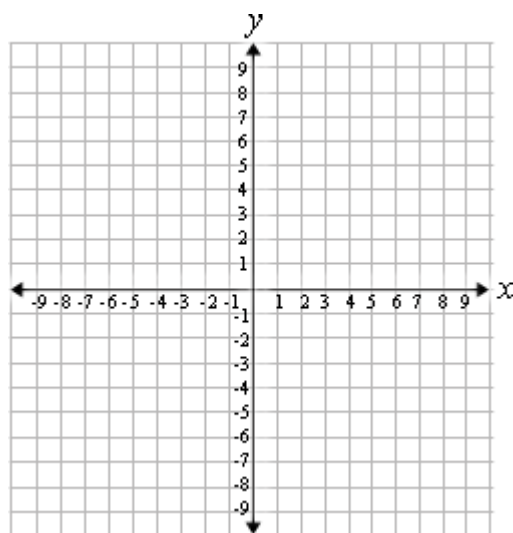
Problem 1b: Graph $f(x) = -x + 2$. Use the x-intercept and y-intercept.

x	$f(x)$	intercept
		y-intercept
		x-intercept



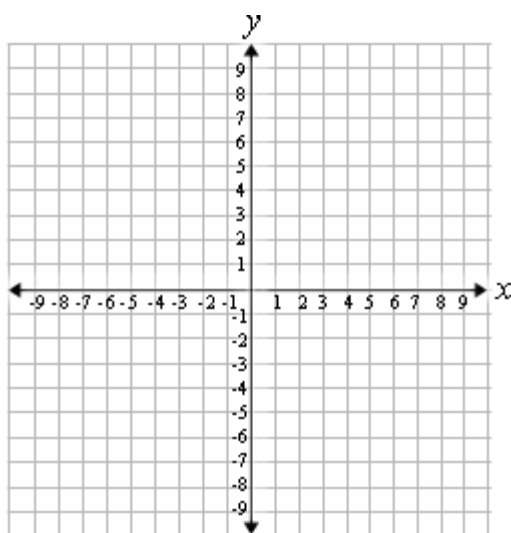
Problem 2a: Graph $f(x) = 3x + 6$. Use a table of values.

x	$f(x)$
-1	
-2	
-3	



Problem 2b: Graph $f(x) = 3x + 6$. Use the x-intercept and y-intercept.

x	$f(x)$	intercept
		y-intercept
		x-intercept



Fundamentals of Graphing Linear Functions Assessment 2

Answer the following short answer questions about the fundamentals of graphing linear functions. State whether the following statements are true or false. If false, state why.

1. All relations are functions.
2. The domain of a function is the set of dependent variables.
3. The range of a function is the set of output variables for a function.
4. The x-intercept is the point $(0, y)$.
5. A function is a relation that has exactly one y-value for every x-value.

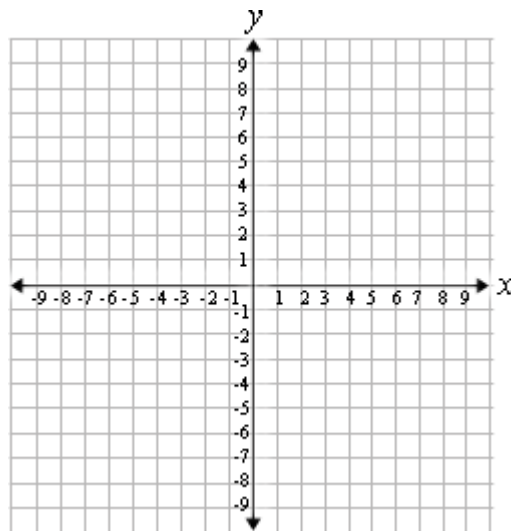
Complete the following sentences with the correct word(s).

6. The _____ variable is the same as the x-value of an ordered pair.
7. The _____ of the absolute value function, $f(x) = |x|$ is the set of all real numbers that is equal to or greater than 0.
8. Graphs in the Cartesian Coordinate System are divided into four areas. Each area is called a _____.
9. To graph a function using a table of values,
_____.
10. To graph a function using the x-intercept and the y-intercept,
_____.

Fundamentals of Graphing Linear Functions Assessment 3

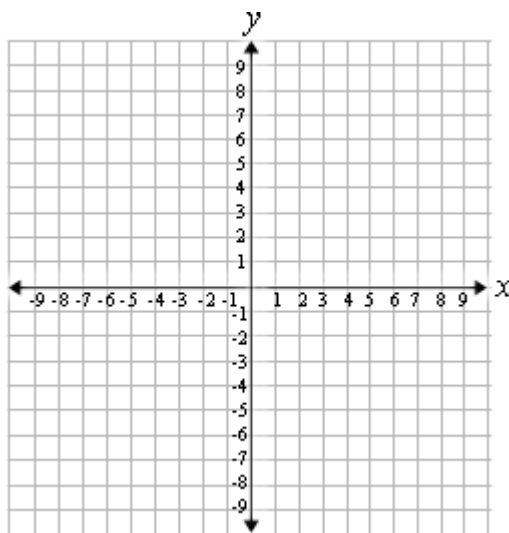
1. Graph $f(x) = 3x - 3$ using a table of values.

x	$f(x)$



2. Graph $f(x) = -2x + 8$ using the x and y-intercepts. Show your work.

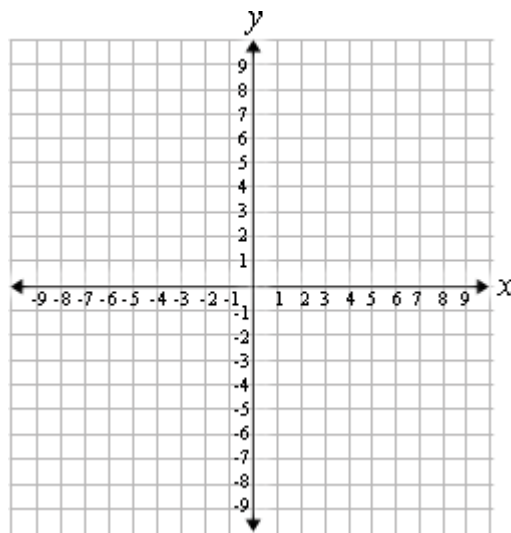
x	$f(x)$	intercept
		y-intercept
		x-intercept



3. Graph $f(x) = \frac{1}{2}x + 5$ using a table of values and using the x and y-intercepts.

x	$f(x)$

x	$f(x)$	intercept
		y-intercept
		x-intercept



Fundamentals of Graphing Linear Functions

Session 2 Part 2 – Domain and Range of a Function

The **domain of a function** is the set of values for the independent variable (input value) of a function.

The **range of a function** is the set of all possible output values for a function.

For the purposes of this lesson we will only talk about linear functions.

Let's study some examples of domain and range of a function containing ordered pairs. Each example given below is a function as every x-value goes to exactly one y-value.

Example 1:

Find the domain and range of the function: $\{(1, 3), (2, 6), (3, 9), (4, 12), (5, 15)\}$.

Solution:

The set of values for the independent variable is the set of all first values of the ordered pairs. When we choose a first value (input value), we have many choices. Since functions have a rule of correspondence, the second value (output value), is dependent on our first value. Therefore, we call the first or input values the independent variable of the function and the set of second or output values the dependent variable of the function. The set of all input values is the **domain of a function**. The set of all output values is the **range of a function**.

The set of x-values or input values of the set of ordered pairs is $\{1, 2, 3, 4, 5\}$. This is the **domain of our function**.

The set of y-values or output values of the set of ordered pairs is $\{3, 6, 9, 12, 15\}$. This is the **range of our function**.

Example 2:

Find the domain and range of the function: $\{(1, 1), (2, 1), (3, 1), (4, 2), (5, 2), (6, 2)\}$.

Solution:

The set of x-values or input values of the set of ordered pairs is $\{1, 2, 3, 4, 5, 6\}$. This is the **domain of our function**.

The set of y-values or output values of the set of ordered pairs is $\{1, 2\}$. This is the **range of our function**. We do not write 1 and 2 more than one time even though they appear as y-values more than once.

Example 3:

Find the domain, range, and rule of correspondence for the function listed below.

$$\left\{ \left(\frac{1}{2}, 2 \right), \left(\frac{1}{3}, 3 \right), \left(\frac{1}{4}, 4 \right), \left(\frac{1}{5}, 5 \right) \right\}$$

Solution:

The set of x-values or input values of the set of ordered pairs is $\left\{ \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \right\}$. This is the

domain of the function.

The set of y-values or output values of the set of ordered pairs is $\{2, 3, 4, 5\}$. This is the **range of the function**.

What is the relationship between the x-value and the y-value in this function? They are reciprocals of each other. Remember that this is called the **rule of correspondence**.

To find the domain and range of a function:

- List all the x-values (1st values) or independent variable. This is the **domain** of the function.
- List all the y-values (2nd values) or dependent variable. This is the **range** of the function.

Each set of ordered pairs below is a function. Study them and in the work space provided find the domain and range of the function, describe the rule of correspondence, and justify your answers.

1. $\{(1, 2), (2, 4), (3, 6), (4, 8), (5, 10)\}$

2. $\{(1, 1), (2, 4), (3, 9), (4, 16), (5, 25), (6, 36)\}$

3. $\{(4, 9), (8, 17), (12, 25), (16, 33)\}$

4. $\{(3, 3), (-3, 3), (8, 8), (-8, 8), (11, 11), (-11, 11), (20, 20), (-20, 20)\}$

The domain and range of a function can also be discovered through a table of values, its equation, or its graph.

For a function represented by a table, follow the same procedure that we used to determine the domain and range of a set of ordered pairs.

To find the domain and range of a function:

- List all the x-values (1st values) or independent variable. This is the domain of the function.
- List all the y-values (2nd values) or dependent variable. This is the range of the function.

Example 1:

Determine the domain and range of the following function. Explain your reasoning.

x	y
2	-2
4	-4
-6	6
-8	8

Solution:

The set of all x-values is the **domain**: {2, 4, -6, -8}.

The set of all y-values {-2, -4, 6, 8} is the **range**.

Example 2:

What is the domain and the range of $f(x) = x + 3$? Explain your answer.

Solution:

The domain of $f(x) = x + 3$ is the set of all real numbers as we can choose any real number for x .

The range is also the set of all real numbers as all real numbers will result when we choose any number for x .

Example 3:

What is the domain and range of $f(x) = |x|$? Explain your answer.

Solution:

The domain of $f(x) = |x|$ is all real numbers as we can choose any number for x . However, $f(x)$ will always be either 0 or a positive real number so the range of $f(x) = |x|$ will be 0 and the set of positive real numbers. We can represent this by $f(x) \geq 0$.

Example 4:

Find the domain and range of $f(x) = 4$. Justify your answer.

Solution:

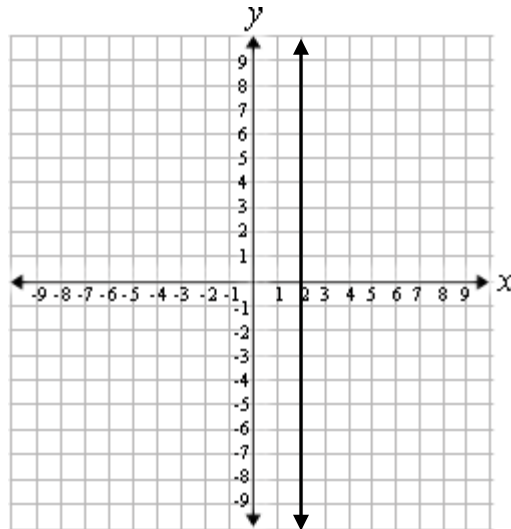
The domain of $f(x) = 4$ is all real numbers as we can choose any number for x .

The range of $f(x) = 4$ will always be 4 as for any value we choose for x , $f(x)$ will always equal 4.

Example 5:

The graph of $x = 2$ is shown below. Determine the domain and range of $x = 2$ using the graph. Justify your answer.

Please note that $x = 2$ is not a function. For every x -value shown (which is 2), $f(x)$ can be anything. So there is not only one y -value for every x -value.



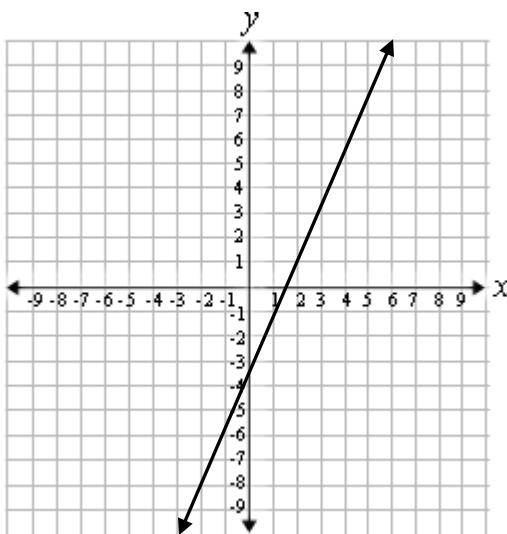
Solution:

The domain of $x = 2$ is $\{2\}$.

The range of $x = 2$ is all real numbers as for every x value, y can be any number. For example, $\{(2, 1), (2, 7), (2, 0), (2, -9)\dots\}$ all are ordered pairs on the graph.

Example 6:

Find the domain and range of the function $f(x) = 2x - 4$ represented on the graph below. Justify your answer.



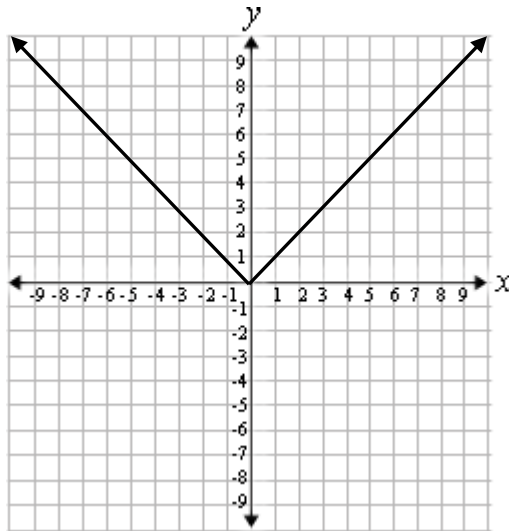
Solution:

The domain for the function $f(x) = 2x - 4$ is all real numbers as any real number may be substituted for x .

The range for the function is all real numbers as once we substitute in a value for x , $f(x)$ will be all real numbers.

Example 7:

Find the domain and range of the function $f(x) = |x|$ represented on the graph below.



Solution:

The domain of the function $f(x) = |x|$ is all real numbers as all real numbers can be used for x-values in the function.

Looking at the graph, we note that the set of ordered pairs satisfying the function will always be in the 1st or 2nd quadrant where the y-value is always positive. Therefore, the range of the function $f(x) = |x|$ is all real numbers greater than or equal to 0 which can be represented by $f(x) \geq 0$.

Determine the domain and range of each of the following in the work space provided. Explain your reasoning.

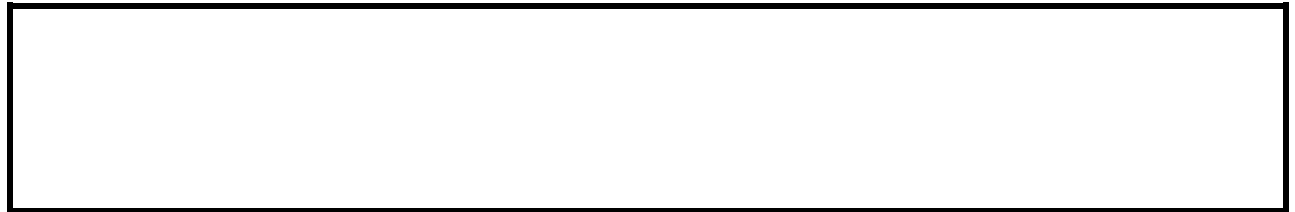
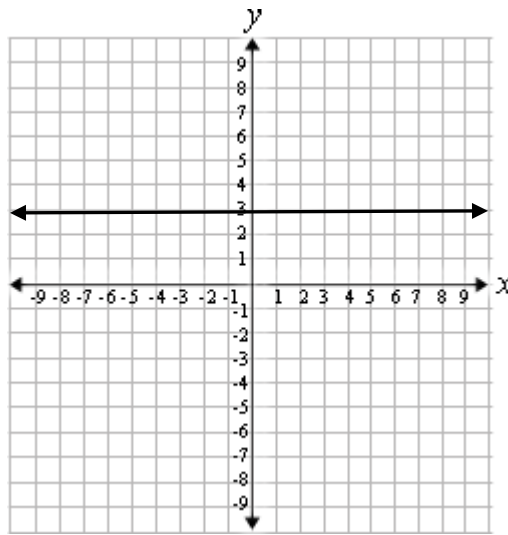
1.

x	y
2	2
12	3
22	4
32	5

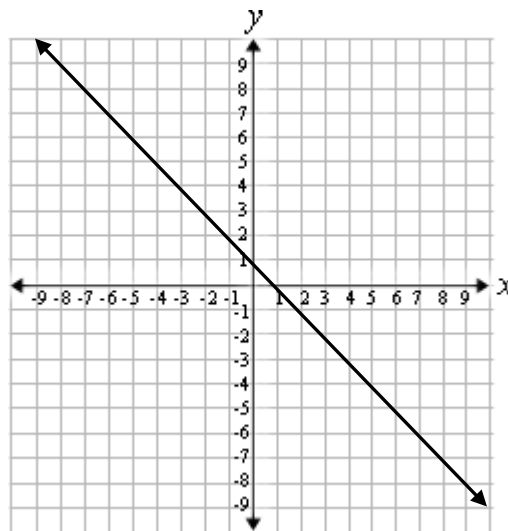
2. $f(x) = -x + 5$

3. $f(x) = 2|x|$

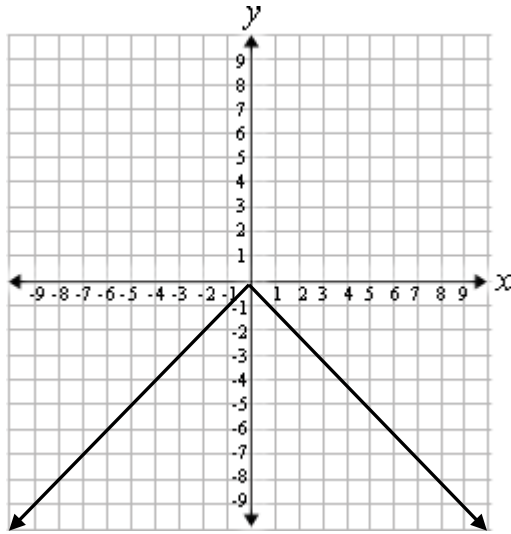
4. $f(x) = 3$



5. $f(x) = -x + 1$



6. $f(x) = -|x|$



Fundamentals of Graphing Linear Functions Assessment 4

Determine the domain and range of each of the following in the work space provided. Explain your reasoning.

1.

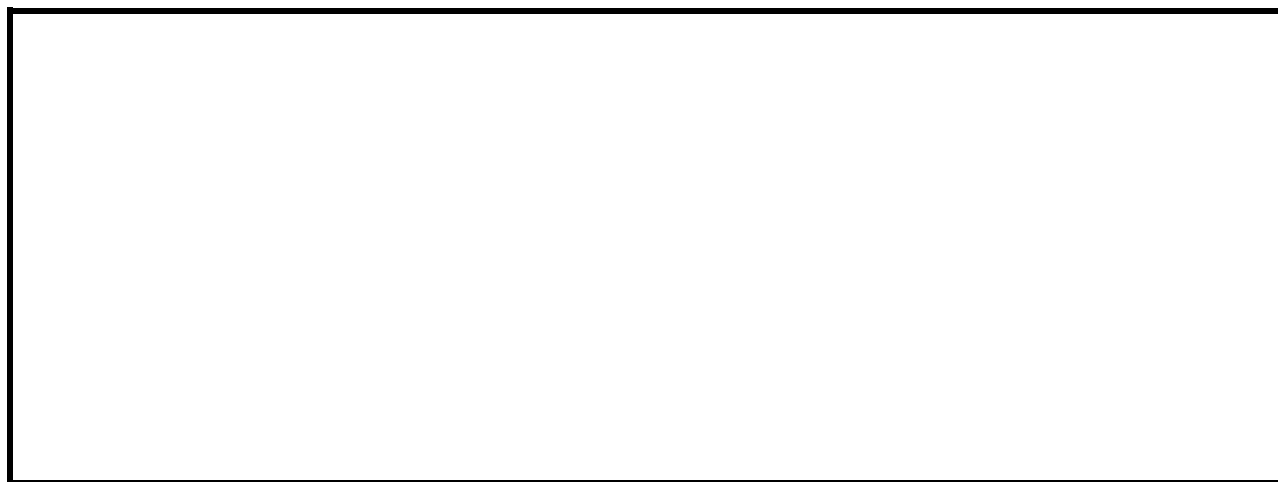
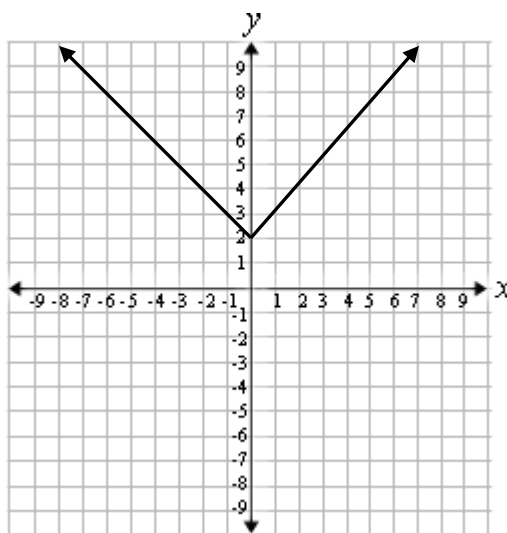
x	y
1	-1
5	3
7	5
25	23

2. $f(x) = 2x + 7$

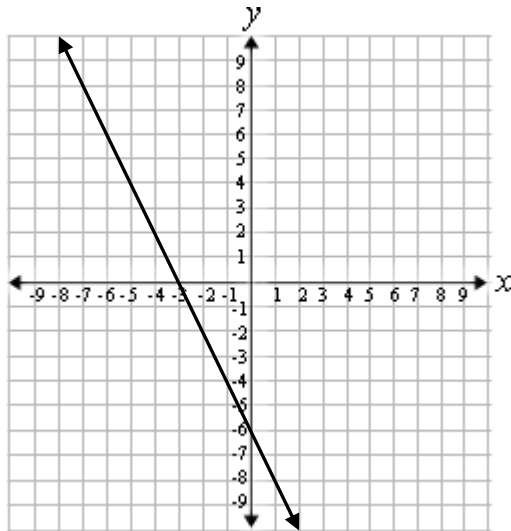
3. $f(x) = \frac{3}{4}x + 5$



4. $f(x) = |x| + 2$



5. $f(x) = -2x - 6$



Extensions

1. Look through journals, magazines, newspapers, and other printed material to apply the skills you have acquired in learning how to graph and how to understand the graphs you have created.
2. Use graphing techniques when studying data analysis. Look for statistical reports that are in print or on various media such as television or the Internet to explain the meanings of the graphical representation of the statistical reports.
3. This website helps you to explore how the graph of a linear function changes as the slope and y-intercept change.
<http://standards.nctm.org/document/eexamples/chap7/7.5/index.htm>

Sources

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2008 The Final Report of the National Mathematics Advisory Panel, p. 16, 29

1999 Bringing the NCTM Standards to Life, Exemplary Practices from High Schools, p. 9-12