

Absolute Value

An ADE Mathematics Lesson

Days 1-5

Author	ADE Content Specialists
Grade Level	9 th grade
Duration	Five days

Aligns To

Mathematics HS:

Strand 1: Number and Operations

Concept 1: Number Sense

PO 3. Express that the distance between two numbers is the absolute value of their difference.

Concept 2: Numerical Operations

PO 1. Solve word problems involving absolute value, powers, roots, and scientific notation.

Concept 3: Estimation

PO 2. Use estimation to determine the reasonableness of a solution.

Strand 5: Structure and Logic

Concept 2: Logic, Reasoning, Problem Solving, and Proof

PO 2. Solve problems by formulating one or more strategies, applying the strategies, verifying the solution(s), and communicating the reasoning used to obtain the solution(s).

Connects To

Mathematics HS:

Strand 3: Patterns, Algebra, and Functions

Concept 3: Algebraic Representations

PO 1. Create and explain the need for equivalent forms of an equation or expression.

PO 5. Solve linear equations and equations involving absolute value, with one variable.

Strand 5: Structure and Logic

Concept 2: Logic, Reasoning, Problem Solving, and Proof

PO 1. Analyze a problem situation, determine the question(s) to be answered, organize given information, determine how to represent the problem, and identify implicit and explicit assumptions that have been made.

Overview

The distance between two numbers is the absolute value of their difference.

Purpose

The purpose of this lesson is to solidify understanding of absolute value. This allows you to simplify algebraic expressions and equations that include absolute value.

Materials

- Absolute value worksheets

Objectives

Students will:

- Demonstrate that the distance between two numbers is the absolute value of their difference.
- Solve word problems involving absolute value.

Lesson Components

Prerequisite Skills: This lesson builds on grade 7 and 8 skills of modeling and solving problems involving absolute value. In prior grades, you have also learned to find or estimate the location of numbers on a number line.

Vocabulary: *absolute value, distance between two numbers*

Session 1: Absolute Value and Distance (3 days)

1. Express that the difference between two numbers is the absolute value of their difference.

Session 2: Absolute Value in Context (2 days)

1. Solve word problems involving absolute value.

Assessment

There are two assessments that will help pinpoint misconceptions before moving on to algebraic expressions and equations containing absolute value. Assessments should be completed after each session before moving onto the next session or lesson.

Absolute Value

Session 1 – Absolute Value and Distance

It is often desirable to find the distance between two numbers on a number line. Let's examine some numbers and find the distance between them on the number line.

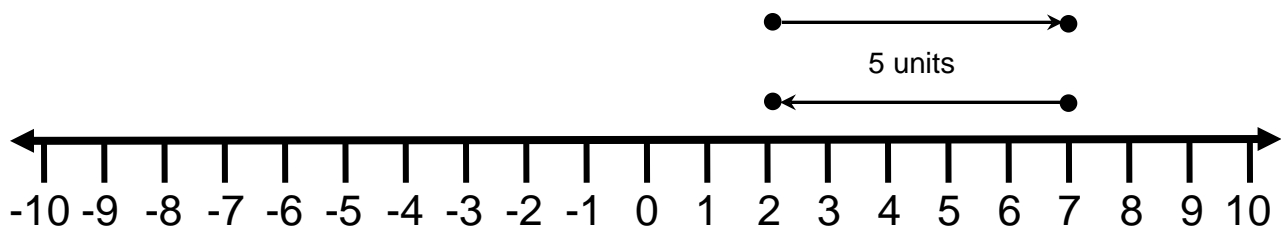
Example 1:

Find the distance between 2 and 7 on the number line.

Solution:

First place the numbers on the number line.

Count the units from 2 to 7 or from 7 to 2. Since we are trying to find distance, and distance is always positive, it does not matter from which endpoint we begin to count.



Since it does not matter in which order we count, 2 to 7 or 7 to 2, we can designate this by using an absolute value sign.

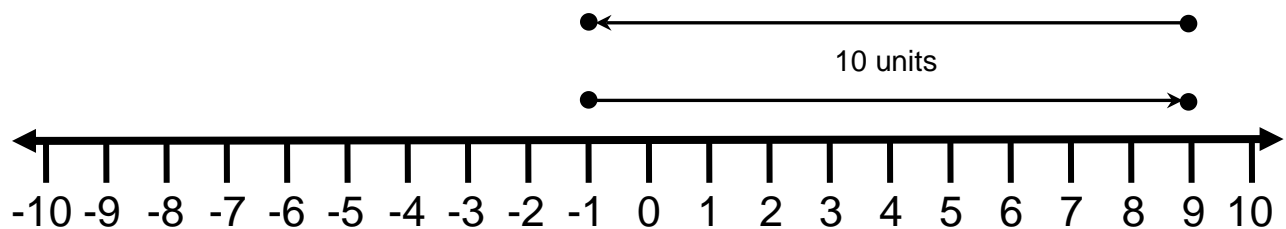
Remember that **absolute value** is defined by a number's distance from zero on a number line (e.g., the absolute value of -4 is 4, the absolute value of 4 is 4; symbolically, $|-4| = 4$ and $|4| = 4$).

- The **distance between two numbers** is the absolute value of their difference.
- The difference between two numbers a and b is $|a - b|$. It makes no difference which number we designate by “ a ” and which number we designate by “ b ”.

In Example 1, the difference between 2 and 7 is $|2 - 7| = |-5| = 5$. Note that we can also say that the difference between 2 and 7 is $|7 - 2| = |5| = 5$. In either case, we find that the difference between 2 and 7 is 5 as shown in our number line above.

Example 2:

Find the distance between 9 and -1 using a number line.



Solution: In Example 2, the difference between 9 and -1 is $|9 - (-1)| = |10| = 10$. We could also say that the difference between 9 and -1 is $|-1 - 9| = |-10| = 10$. In both cases, we find that the difference between 9 and -1 is 10.

Because we are taking the absolute value of the difference of the numbers, we will always end up with a positive number and order will not matter.

It is not always convenient to draw a number line so often we find the distance between two numbers by finding the absolute value of their difference.

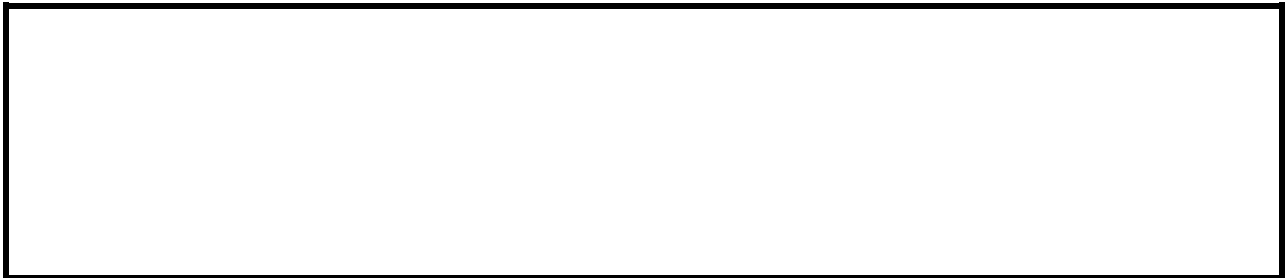
Example 3:

Find the distance between 2 and 30.

Solution:

$$|2 - 30| = |-28| = 28 \text{ or } |30 - 2| = |28| = 28$$

It is not necessary to find the distance two ways. This is done to show you that the order does not matter. Can you indicate this problem using a number line in the space provided?

**Example 4:**

Find the distance between -15 and 29.

Solution:

$$|-15 - 29| = |-44| = 44$$

Can you find the distance between -15 and 29 in another way in the space provided?



Example 5:

Find the difference between -101 and -123.

Solution:

$$|-101 - (-123)| = |-101 + 123| = 22$$

Can you find the distance between -101 and -123 in another way in the space provided?

In questions 1 – 3, draw a number line in the space provided to help you solve the problem.

1. Using a number line, find the distance between 1 and 8. Justify your answer.

2. Using a number line, find the distance between -5 and 12. Justify your answer.

3. Using a number line, find the distance between -10 and -1. Justify your answer.

For the remaining problems, find the difference using absolute value and without using a number line. Show your work in the space provided.

4. Find the distance between 5 and 44.

5. Find the distance between -11 and 78.

6. Find the distance between -53 and 2.

7. Find the distance between 8 and -19.

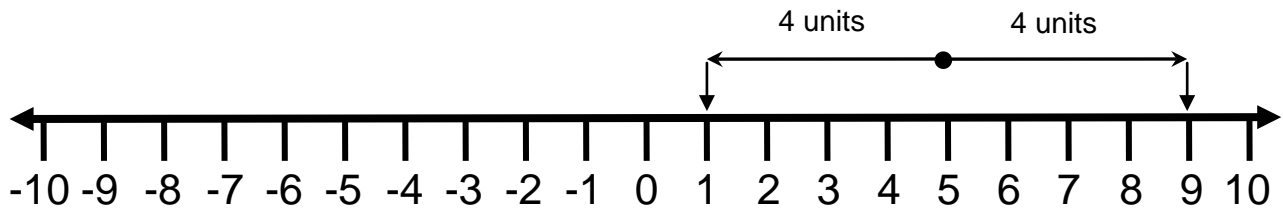
8. Find the distance between -111 and -222.

9. Find the distance between 21 and -1.

10. Find the distance between -30 and -44.

Example 6: The distance between two numbers is 4. One number is 5. What are the possibilities for the other number?

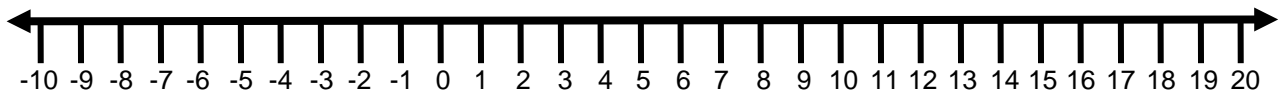
Solution: You may see problems like this. Keep in mind that there are two possible answers for this problem. The distance may be to the right or left of 5. In this case, the other number may be 1 or 9.



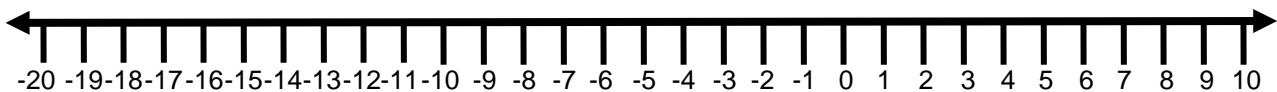
Check: $|9 - 5| = 4$ or $|1 - 5| = |-4| = 4$

For the remaining problems, find the other endpoint given an endpoint and length of the line segment. Use a number line to help you.

11. The distance between two numbers is 12. One number is 7. What are the possibilities for the other number?



12. The distance between two numbers is 9. One number is -7. What are the possibilities for the other number?



Absolute Value Assessment 1

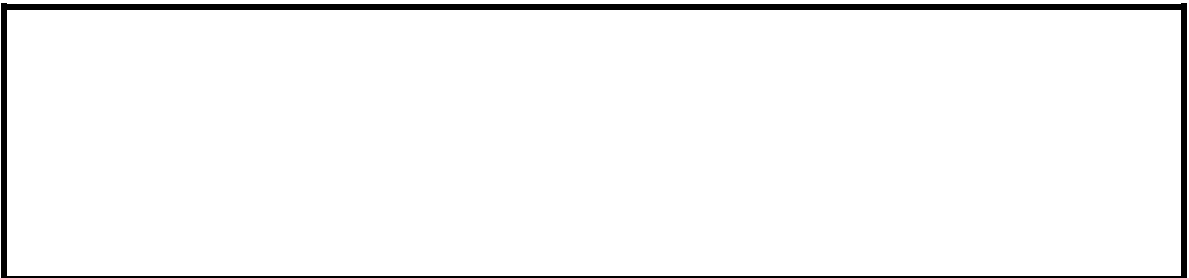
1. Using a number line, find the distance between 1 and 10. Justify your answer.



2. Using a number line, find the distance between -8 and 9. Justify your answer.



3. Without using a number line, find the distance between -5 and -12. Justify your answer.



4. Without using a number line, find the distance between 21 and -19. Justify your answer.



5. Using a number line, find the distance between 120 and -55. Justify your answer.



6. Using a number line, find the distance between -63 and -121. Justify your answer.



Absolute Value

Session 2 – Absolute Value in Context

Many times, word problems involve finding the distance between two quantities. It is necessary to use absolute value to find the distance. Study the following examples.

Example 1:

Determine the difference in altitude between Death Valley (86 meters below sea level) and the summit of Mount St. Helens (2,549 meters above sea level).

Solution:

We can represent the elevation of Death Valley by -86 meters. We can represent the elevation of Mount St. Helens by 2549 meters.



To find the difference between the two elevations, find the absolute value of their difference.

$$|-86 - 2549| = |-2635| = 2635 \text{ meters}$$

Remember that it does not make any difference in what order the numbers are placed within the absolute value signs.

$$|2549 - (-86)| = |2635| = 2635 \text{ meters}$$

We end up with the same answer. The difference between the elevation of Death Valley and Mount St. Helens is 2635 meters.

Example 2:

Determine the difference in altitude between Long Beach, California (7 feet below sea level) and Denver, Colorado (5130 feet above sea level).

**Solution:**

We can represent the elevation of Long Beach by -7 feet. We can represent the elevation of Denver by 5130 feet.

To find the difference between the two elevations, find the absolute value of their difference.

$$|-7 - 5130| = |-5137| = 5137 \text{ feet}$$

Remember that it does not make any difference in what order the numbers are placed within the absolute value signs.

$$|5130 - (-7)| = |5137| = 5137 \text{ feet}$$

We end up with the same answer. The difference between the elevation of Long Beach and Denver is 5137 feet.

Example 3:

On June 13, 2005, a scuba diver, Juno Gomes, reached a depth of 318.25 meters. Many people such as Kazgi Sherpa and Mark Batard have reached the summit of Mt. Everest at 8840 meters. What is the difference in distance between the depth of Gomes' scuba dive and the summit of Mt. Everest?

**Solution:**

We can represent the dive's depth by -318.25 meters. We can represent the summit of Mount Everest by 8840 meters.

To find the difference between the two elevations, find the absolute value of their difference.

$$|-318.25 - 8840| = |-9158.25| = 9158.25 \text{ meters}$$

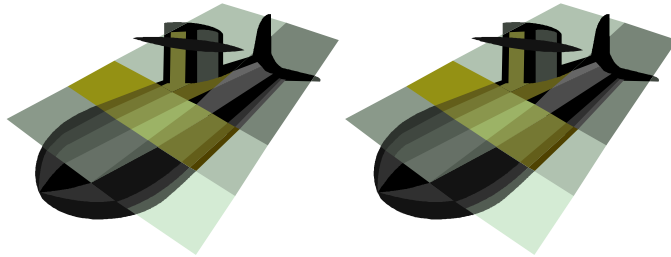
Remember that it does not make any difference in what order the numbers are placed within the absolute value signs.

$$|8840 - (-318.25)| = |9158.25| = 9158.25 \text{ meters}$$

We end up with the same answer. The difference between depth of the dive and the summit of Mt. Everest is 9158.25 meters.

Example 4:

On July 29th, 2008, Russian mini-submarines, Mir-1 and Mir-2, descended 5500 feet to the bottom of Siberia's Lake Baikal. The Concorde airplane, retired on November 26, 2003 rapidly climbed to an altitude of 60,000 feet a very short time in some of its flights. What is the distance between the descent of Mir-1 and Mir-2 and the altitude of Concorde?

**Solution:**

We can represent the depth of the mini-submarines' dive by -5500 feet. We can represent the altitude of Concorde by 60,000 feet.

To find the difference between the two elevations, find the absolute value of their difference.

$$|-5500 - 60,000| = |-65,500| = 65,500 \text{ feet}$$

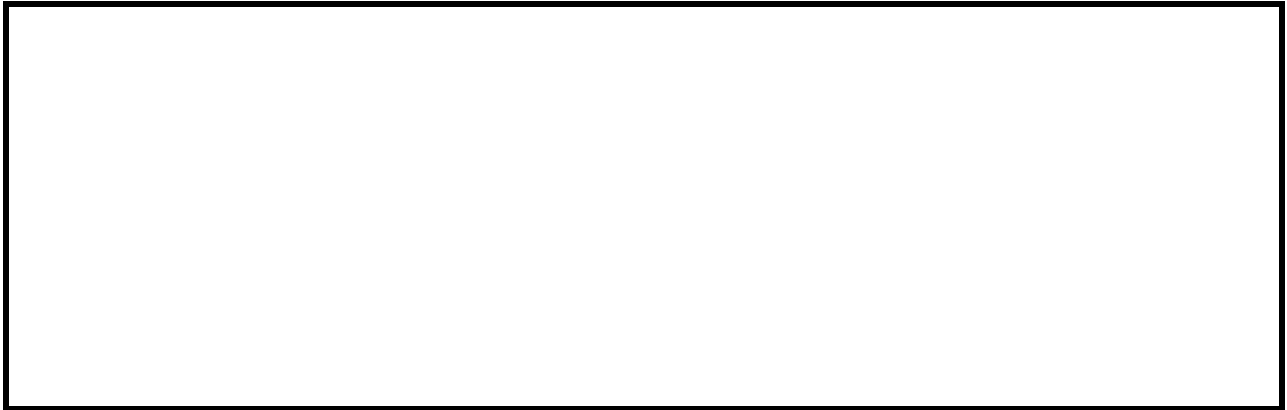
Remember that it does not make any difference in what order the numbers are placed within the absolute value signs.

$$|60,000 - (-5500)| = |65,500| = 65,500 \text{ feet}$$

We end up with the same answer. The difference between depth of the dive and the altitude of Concorde is 65,500 feet.

Solve the following problems showing all your work in the space provided.

1. Determine the difference in altitude between Death Valley (86 meters below sea level) and the Alaska's Mount McKinley (6194 meters above sea level).



2. Determine the difference in altitude between New Orleans, Louisiana (8 feet below sea level and Rapid City, South Dakota (3202 feet above sea level).



3. The lowest point in the Dominican Republic is Lago Enriquillo at 46 meters below sea level. The highest point in that country is Pico Duarte at 3175 meters. Determine the difference between the lowest point and the highest point in the Dominican Republic.



4. The lowest point in the Egypt is Qattara Depression at 133 meters below sea level. The highest point in that country is Mount Catherine at 2629 meters. Determine the difference between the lowest point and the highest point in Egypt.



Absolute Value Assessment 2

Use the information in the following table to answer the questions that follow it.

City or mountain	State or area	Elevation	Feet or Meters	Below or Above Sea Level
Mariana Trench	South Pacific Ocean	10924	meters	below
Death Valley	CA	86	meters	below
Long Beach	CA	7	feet	below
Columbus	OH	685	feet	above
Tucson	AZ	2250	feet	above
Denver	CO	5130	feet	above
Mt. St. Helens	OR	2549	meters	above
Mt. McKinley	AK	6194	meters	above
Mt. Everest	Nepal	8840	meters	above

1. Determine the difference in elevation between Columbus and Tucson.

2. Determine the difference in elevation between Long Beach and Denver.

3. Determine the difference in elevation between the Mariana Trench and Death Valley.

4. Determine the difference in elevation between the Mariana Trench and Mt. Everest.

Extensions

This website provides an activity and applet for you to further explore absolute value on a number line. There is also a link to absolute value functions if you are interested in exploring further.

<http://www.analyzemath.com/Definition-Absolute-Value/Definition-Absolute-Value.html>

Sources

2008 AZ Mathematics Standards

2000 NCTM Principles and Standards

Information for elevations of points:

<http://egsc.usgs.gov/isb/pubs/booklets/elvadist/elvadist.html#Highest>

<http://www.scubarecords.com/>

<http://en.rian.ru/russia/20080729/115171121.html>

<http://www.mnteverest.net/history.html>

<http://classic.mountainzone.com/news/everestspeed.html>

<http://en.wikipedia.org/wiki/Concorde>

<http://www.altimeters.net/cityaltitudes2.html>

<http://www.worldatlas.com/aatlas/infopage/highlow.htm>

<http://geology.com/records/deepest-part-of-the-ocean.shtml>