GEOMETRY
An ADE Mathematics Lesson
Days 1-10

Author	ADE Content Specialists
Grade Level	6th grade
Duration	Ten days

Aligns To

Mathematics
Strand 1: Number and Operations
Concept 2: Numerical Operations

PO 1. Apply and interpret the concepts of addition and subtraction with integers using models.
PO 3. Divide multi-digit whole numbers and decimals by decimal divisors with and without remainders.

Concept 3: Estimation

PO 2. Make estimates appropriate to a given situation and verify the reasonableness of the results.

Strand 3: Patterns, Algebra, and Functions
Concept 2: Functions and Relationships

PO 1. Use an algebraic expression to represent a quantity in a given context.
PO 4. Evaluate an expression involving the four basic operations by substituting given fractions and decimals for the variable.

Concept 3: Algebraic Representations

PO 1. Use an algebraic expression to represent a quantity in a given context.
PO 2. Create and solve two-step equations that can be solved using inverse properties with fractions and decimals.

Strand 4: Geometry and Measurement
Concept 1: Geometric Shapes

PO 1. Define π (pi) as the ratio between the circumference and diameter of a circle and explain the relationship among the diameter, radius, and circumference.
PO 2. Solve problems using properties of supplementary, complementary, and vertical angles.

Concept 3: Coordinate Geometry

PO 1. Graph ordered pairs in any quadrant of the coordinate plane.
PO 2. State the missing coordinate of a given figure on the coordinate plane using geometric properties to justify the solution.

Connects To

Mathematics
Strand 3: Patterns, Algebra, and Functions
Concept 2: Functions and Relationships

PO 1. Recognize and describe a relationship between two quantities, given by a chart, table, or graph, using words and expressions.
PO 3. Translate both ways between a verbal description and an algebraic expression or equation.

Strand 4: Geometry and Measurement
Concept 4: Measurement

PO 3. Estimate the measure of objects using a scale drawing or map.

Strand 5: Structure and Logic
Concept 2: Logic, Reasoning, Problem Solving, and Proof

PO 4. Apply a previously used problem-solving strategy in a new context.

Aligns To	Connects To
Strand 5: Structure and Logic **Concept 2: Logic, Reasoning, Problem Solving, and Proof** **PO 1.** Analyze a problem situation to determine the question(s) to be answered. **PO 2.** Identify relevant, missing, and extraneous information related to the solution to a problem. **PO 3.** Analyze and compare mathematical strategies for efficient problem solving; select and use one or more strategies to solve a problem. **PO 5.** Represent a problem situation using multiple representations, describe the process used to solve the problem, and verify the reasonableness of the solution.	

Overview

In this lesson, you will learn to work with radius and diameter to find the circumference of a circle. You will work with complementary, supplementary and vertical angles. You will also work with plotting points in all four quadrants of the graph.

Purpose

You will work with circles, angles, and graphing in all quadrants in your work from now on. You will need to have a very good understanding of these properties.

Materials

- Circle worksheets
- Compass
- Ruler
- String
- Angle worksheets
- Graph paper

Objectives

Students will:

- Define π (pi) as the ratio between the circumference and diameter of a circle and explain the relationship among the diameter, radius, and circumference.
- Solve problems using properties of supplementary, complementary, and vertical angles.
- Graph ordered pairs in any quadrant of the coordinate plane.
- State the missing coordinate of a given figure on the coordinate plane using geometric properties to justify the solution.

Lesson Components

Prerequisite Skills: In Kindergarten through Grade 5, you learned to identify the attributes of 2-dimensional figures. In Grade 4, you began to classify angles. In Grade 4, you plotted points in the 1st quadrant and recognized missing points when 2-dimensional figures were put on the coordinate plane.

Vocabulary: *circle, radius, diameter, circumference, π (pi), complementary angles, supplementary angles, vertical angles, quadrants, coordinate point, ordered pair*

Session 1 (3 days)

1. Students define π (pi) as the ratio between the circumference and diameter of a circle and explain the relationship among the diameter, radius, and circumference.

Session 2 (3 days)

1. Students solve problems using properties of supplementary, complementary, and vertical angles.

Session 3 (4 days)

1. Students graph ordered pairs in any quadrant in the coordinate plane.

2. Students state the missing coordinate of a given figure on the coordinate plane using geometric properties to justify the solution.

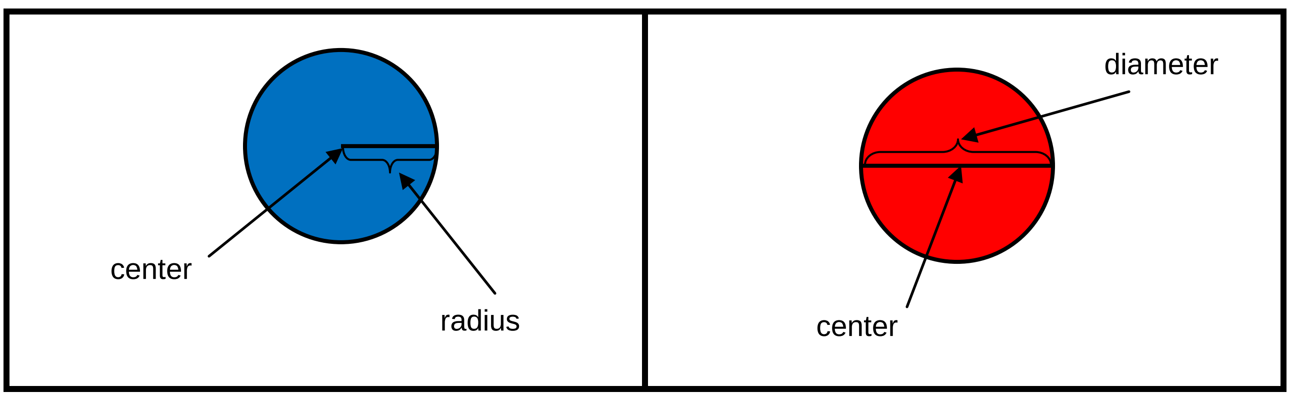
Assessment

There is one assessment after each session that will help pinpoint misconceptions about circles, angles, and graphing in any quadrant before moving on to more complex comparisons.

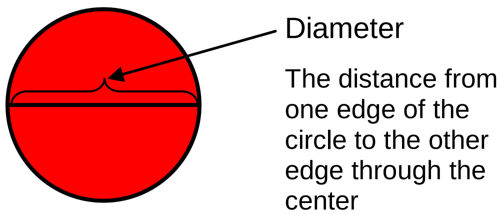
Geometric Properties
Session 1 – Circles

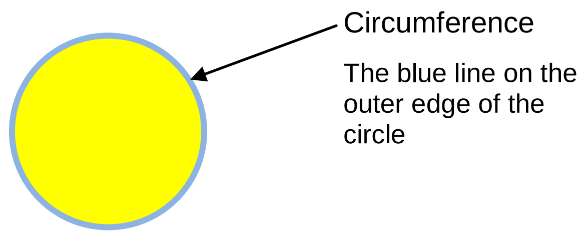
In previous work, you have worked many times with circles. Draw two circles in the space provided. On the first circle, indicate the **center** and **radius** of the circle. On the second circle, indicate the **center** and **diameter** of the circle.

Did your circles look something like these?



Measure the diameter and circumference of five circular objects in your home. Remember that the circumference of a circle is the distance around it. Use a string to measure the diameter and circumference. Place the string on the diameter and then take it off and measure it using a ruler or tape measure. Place the string around the circumference of your objects and then take the string off and measure it. Add your measurements to the data chart.





Name of object	Diameter	Circumference

Describe the pattern that you see in the data. Is there any relationship between the circumference and the diameter of a circle?

Rewrite the table in terms of the radius versus the circumference. Remember that the length of the radius is half the diameter. Describe the pattern that you see in the data.

Name of object	Radius	Circumference

Describe the pattern that you see in the data. Is there any relationship between the circumference and the radius of a circle?

Write a paragraph about the relationship between the diameter, radius, and circumference of a circle.

If you divide each circumference by the diameter of the circle, you will always get about the same answer, 3.14. This is a very special number that we call π. We call it pi. This number is a non-repeating decimal which means that it never ends. Written as a fraction, it is approximately equal to $\frac{22}{7}$. The circumference of a circle can be found by multiplying the diameter of the circle by π. We represent this by the formula $C = \pi \bullet d$ where C represents the circumference of the circle and d represents the diameter of the circle.

Example 1

Find the circumference of a circle whose diameter is 7 cm. Use $\frac{22}{7}$ as an approximation for π.

Solution

$$C = \pi \bullet d$$

$$C = \frac{22}{7} \bullet 7 = 22 \text{ cm.}$$

Since the diameter is equal to two radii we can also use the formula $C = 2 \bullet \pi \bullet r$ when the radius of the circle is given rather than its diameter.

Example 2

Find the circumference of a circle whose radius is 12 inches. Use 3.14 as an approximation for π.

Solution

$$C = 2 \bullet \pi \bullet r$$

$$C = 2 \bullet \pi \bullet 12$$

$$C = 24\pi$$

$$C = 24 \bullet 3.14$$

$$C = 75.36 \text{ inches}$$

We have used two different formulas but they will both yield the same answer.

To find the circumference of a circle
$C = \pi \bullet d$ or $C = 2 \bullet \pi \bullet r$
$\pi \cong \frac{22}{7}$ or $\pi \cong 3.14$

If you look it up in a book or on the Internet, you will find that

- Pi $(\pi)$is Greek and has been around for over 2000 years!
- Pi (π) is defined as the ratio of the circumference of the circle to its diameter (the numerical value of pi is 3.141 592 653 589 793...).

Example 3

Maria has a window in her house that is a circle. She knows that the greatest distance across the window is 3 feet. What is the circumference of the window?

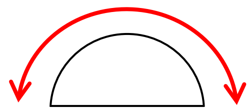
Solution

Since the greatest part of the window is 3 feet across, the diameter of the window is 3 feet. The formula for the circumference of a circle that uses the diameter is $C = \pi \bullet d$.

$C = \pi \bullet d$ means that $C = \pi \bullet 3$. Since we were not told which representation to use for π, we can use either. Let's use the decimal approximation. Then, $C = 3.14 \bullet 3 = 9.42$ feet.

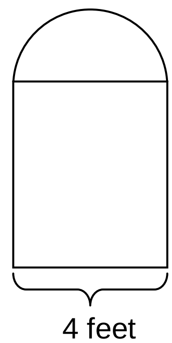
The circumference of the window is 9.42 feet.

A semicircle is one half of a circle. Many things are shaped like semicircles. Sometimes we want to find the length of a semicircular arc (the curved part of the semicircle) when we know either the radius or diameter of the semicircle. This is very similar to finding the circumference of the circle.



Example 4

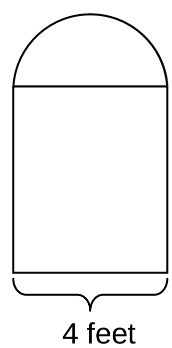
The top of a door has a window that is a semicircle on top of it. The width of the door is 4 feet. What is the length of the semicircular arc?



Solution

The window at the top of the door is a semicircle. A semicircle is half of a circle. The circumference of the window can be found by finding the circumference of the whole circle shown by the dashed line. The circumference of the whole circle is dividing by two to find the length of the semicircular arc.

Use $\frac{22}{7}$ to represent π.



$$C \text{ circle} = \pi \bullet d$$

$$C \text{ circle} = \frac{22}{7} \bullet 4$$

$$C \text{ circle} = \frac{4 \bullet 22}{7} = \frac{88}{7} = 12\frac{4}{7} \text{ feet}$$

$$C \text{ semicircle} = \frac{1}{2} \bullet 12\frac{4}{7} = 6\frac{2}{7} \text{ feet}$$

Example 5

The radius of a circle is 8.5 centimeters. Find the circumference of the circle.

Solution

$$C = 2 \bullet \pi \bullet r$$

$$C = 2 \bullet 3.14 \bullet 8.5$$

$$C = 6.28 \bullet 8.5$$

$$C = 53.38 \text{ cm}$$

It is always good to estimate and check if the answer is reasonable. Since π is about 3 and

2 × 8.5 = 17, the answer needs to be around 3 × 17 or 51. 53.38 is a reasonable answer.

Describe how to find the circumference of a circle given the radius or the diameter of the circle.

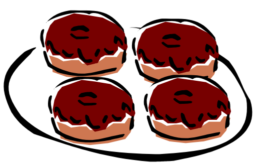
1. Find the circumference of a circle whose radius is 8 cm.

   Use 3.14 as an approximation for π.

2. Find the circumference of a circle whose radius is 14 cm.

Use $\frac{22}{7}$ as an approximation for π.

3. Avery went to the donut shop with her family. She wondered what the circumference of a typical chocolate cream donut was. She asked her mom to estimate the diameter of a donut. Her mom told her it was about 4 inches across. What is the circumference of the chocolate cream donut?



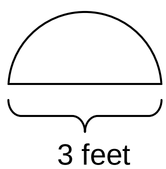
4. The radius of a circle is 5.2 inches. Find the circumference of the circle.

Use 3.14 as an approximation for π.

5. The diameter of a circle is 10 centimeters. What is the circumference of the circle?

Use $\frac{22}{7}$ as an approximation for π.

6. What is the length of the semicircular arc of a semicircular window whose diameter is 3 feet?



Geometric Properties
Assessment 1 – Circles

1. Find the circumference of a circle whose radius is 7 cm.

   Use 3.14 as an approximation for π.

2. Find the circumference of a circle whose radius is 14 inches.

   Use $\frac{22}{7}$ as an approximation for π.

3. David is going to plant a circular garden. He knows that the path cutting across the center of the garden will be 15 feet long. This is the longest distance across the garden. What is the circumference of the garden?

Geometric Properties
Session 2 – Angles

There are many special relationships between pairs of angles. We are going to concentrate on complementary angles, supplementary angles and vertical angles.

Complementary angles are two angles whose measures have a sum of 90 degrees.

Supplementary angles are two angles whose measures have a sum of 180 degrees.

Example 1

Two angles are **complementary**. The first has a measure of 25°. What is the measure of the second angle?

Solution

$\angle 1 = 25°$. $\angle 2 = 90° - 25° = 65°$. Therefore the second angle is 65°.

Example 2

Two angles are **complementary**. If both angles have the same measure, what is the measure of each angle?

Solution

$\angle 1 = \angle 2$ and $\angle 1 + \angle 2 = 90°$. Therefore $90° \div 2$ = the measure of each angle.

$90° \div 2 = 45°$. Each angle is 45°.

Example 3

Two angles are **supplementary**. If both angles have the same measure, what is the measure of each angle?

Solution

$\angle 1 = \angle 2$ and $\angle 1 + \angle 2 = 180°$. Therefore $180° \div 2$ = the measure of each angle.

$180° \div 2 = 90°$. Each angle is 90°.

Example 4

Two angles are **supplementary**. The first has a measure of 85°. What is the measure of the second angle?

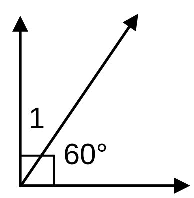
Solution

$\angle 1 = 85°$. $\angle 2 = 180° - 85° = 95°$. Therefore the second angle is 95°.

- Note that two angles that are **complementary** make a **right angle** when they are put together.
- Two angles that are **supplementary** make a **straight angle** when they are put together.

Example 5

Find the measure of $\angle 1$ in the diagram.



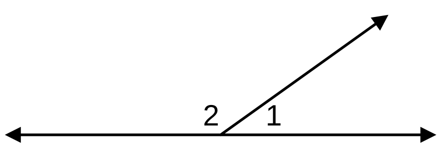
Solution

The two angles are complementary because they make a right angle.

$\angle 2 = 60°$ and $\angle 1 = 90° - 60° = 30°$. Therefore $\angle 1$ is 30°.

Example 6

If the measure of $\angle 1 = 35°$, what is the measure of $\angle 2$?



Solution

The two angles form a straight angle so they are supplementary and total 180°.

$\angle 1 = 35°$ and $\angle 2 = 180° - 35° = 145°$. Therefore, $\angle 2 = 145°$.

Example 7

Two angles are supplementary. One angle measures half of a right angle. Find the other angle.

Solution

A right angle measures 90°. Half of this is 45° which is the measure of the first angle.

The second angle is 180° - 45° = 135°.

Complete the following tables showing all your work.

Two angles are complementary. Find the missing angle in each case.

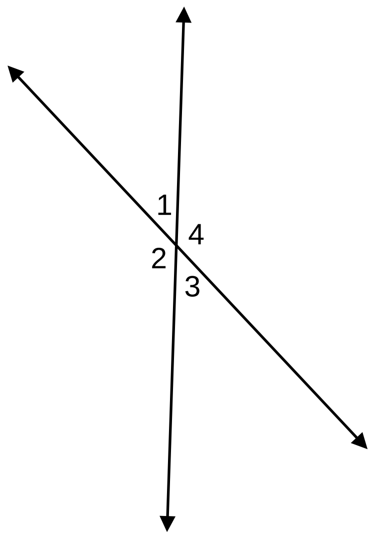
$\angle 1$	$\angle 2$	**Work**
25°		
	47°	
33°		
	68°	
71°		
	16°	

Two angles are supplementary. Find the missing angle in each case.

$\angle 1$	$\angle 2$	**Work**
125°		
	77°	
83°		
	28°	
95°		
	142°	

Vertical angles are the opposite angles formed when two lines intersect. Vertical angles have the same measure.

In the diagram, $\angle 1$ and $\angle 3$ are vertical angles and $\angle 2$ and $\angle 4$ are vertical angles.



Example 1

If the measure of $\angle 2 = 135^\circ$, what are the measures of all of the other angles? Explain the properties that you used to figure out the measures.

Solution

We are given that $\angle 2 = 135^\circ$. Since vertical angles have the same measure, $\angle 4 = 135^\circ$.

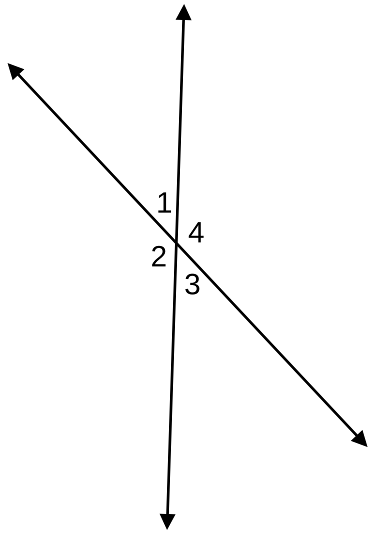
$\angle 1$ and $\angle 2$ are supplementary angles as they make a straight angle. That means that

$\angle 1 + \angle 2 = 180^\circ$. $\angle 1 = 180^\circ - 135^\circ = 45^\circ$.

$\angle 1$ and $\angle 3$ are vertical angles and so have the same measure. Since $\angle 1 = 45^\circ$,

$\angle 3 = 45^\circ$.

Example 2: In the diagram, $\angle 1 = 70°$. Find the measures of all the remaining angles. Justify your answer.



Solution

In the diagram, $\angle 1$ and $\angle 3$ are vertical angles. Vertical angles have the same measure. $\angle 1 = 70°$, so $\angle 3 = 70°$.

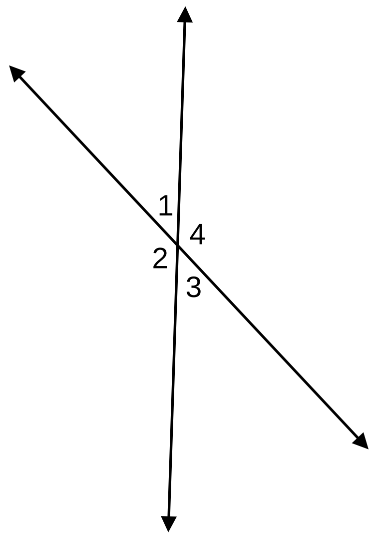
In the diagram, $\angle 1$ and $\angle 4$ are supplementary angles which means their measures must have a sum of 180°. Since $\angle 1 = 70°$, find $\angle 4$ by subtracting 70° from 180°. Then, $\angle 4 = 110°$.

Since $\angle 4$ is a vertical angle to $\angle 2$, their measures must be equal and $\angle 2 = 110°$ as well.

Therefore

$\angle 1 = 70°$ $\quad$ $\angle 2 = 110°$ $\quad$ $\angle 3 = 70°$ $\quad$ $\angle 4 = 110°$.

The problems that follow the diagram all refer to the same diagram. Given one angle, find the missing other three angles. Show your work in the space provided.



1. If ∠1 = 50°, find the measures of ∠2, ∠3, and ∠4.

2. If ∠2 = 100°, find the measures of ∠1, ∠3, and ∠4.

3. If $\angle 3 = 65°$, find the measures of $\angle 1$, $\angle 2$, and $\angle 4$.

4. If $\angle 4 = 95°$, find the measures of $\angle 1$, $\angle 2$, and $\angle 3$.

Geometric Properties
Assessment 2 – Angles

1. Define the following terms.

    a. Complementary angles

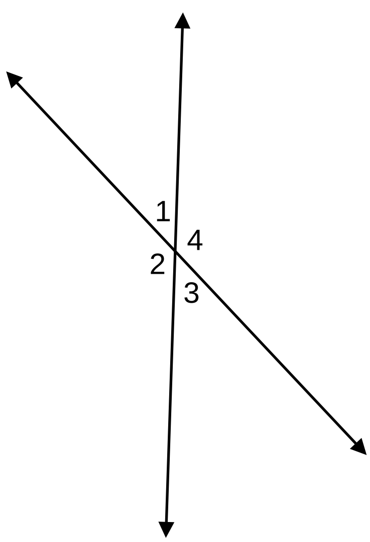
    b. Supplementary angles

    c. Vertical angles

2. If two angles are complementary and one of the angles is 55°, find the other angle.

3. If two angles are supplementary and one of the angles is 117°, find the other angle.

4. In the diagram, $\angle 3 = 48°$. Find the measures of the other three angles. Justify your work.



Geometry
Session 3 – The Coordinate Plane

Let's review some basic terms in graphing points.

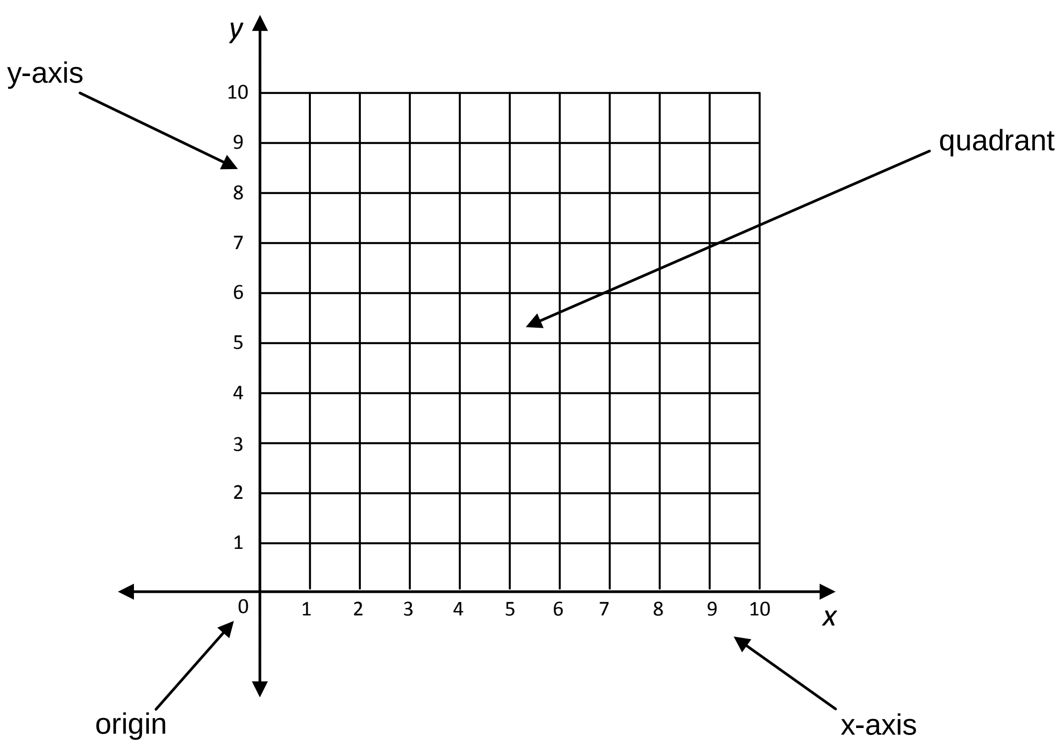
The **coordinate plane (or coordinate grid)** is a plane containing points identified by their distance from the origin in ordered pairs along two perpendicular lines referred to as axes (note: also referred to as Cartesian coordinate system and rectangular coordinate plane).

An **ordered pair** is a pair of numbers used to locate and describe points in the coordinate plane in the form (x, y).

An **axis** (plural is axes) is one of two perpendicular number lines used to form a coordinate system. The horizontal axis is called the **x-axis**. The vertical axis is called the **y-axis**.

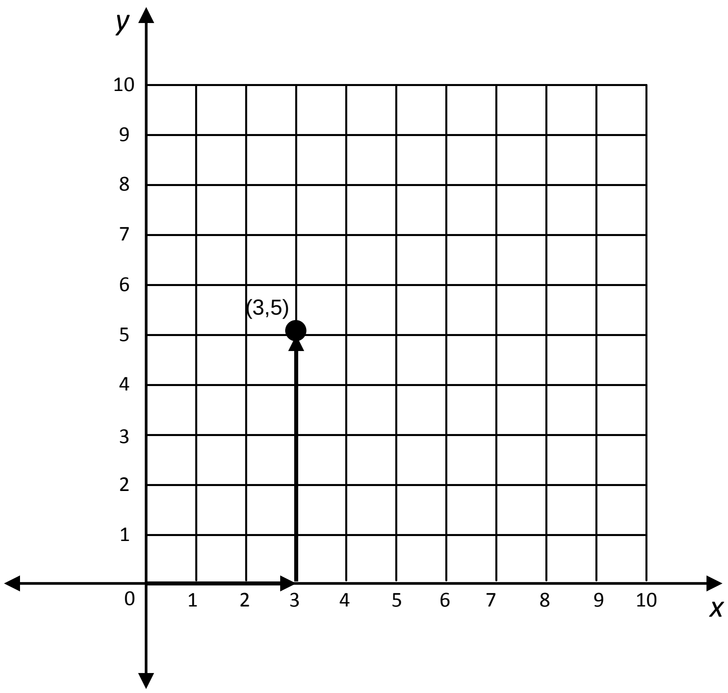
A **quadrant** is one of the four sections into which the coordinate plane is divided by the *x*- and *y*-axes.

The **origin** is the intersection of the axes in a coordinate grid, often defined as (0, 0) in two-dimensions.



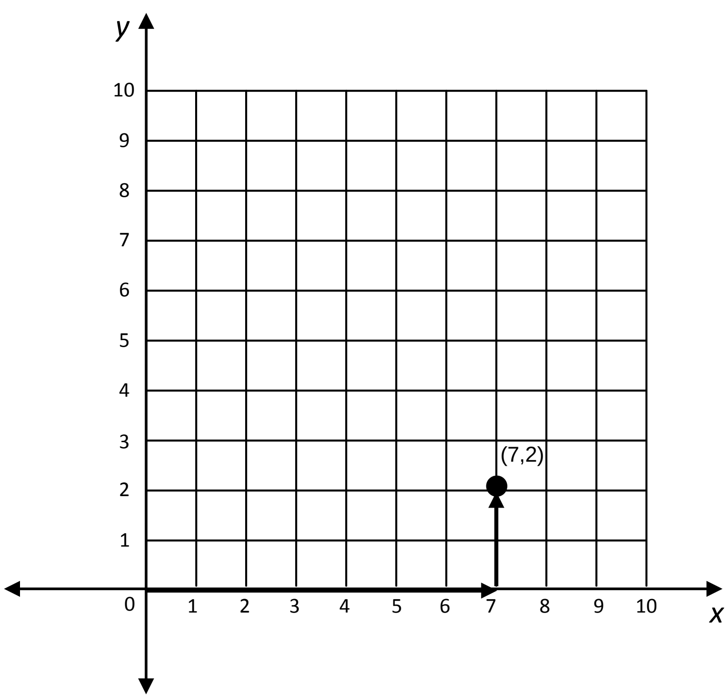
Review of graphing in the 1st quadrant (both the x and y are positive numbers)

Example 1: To graph a point, we move along the x-axis first. Graph the point, (3, 5) means that we want first to find 3 on the x-axis and then to move up 5 spaces from there.



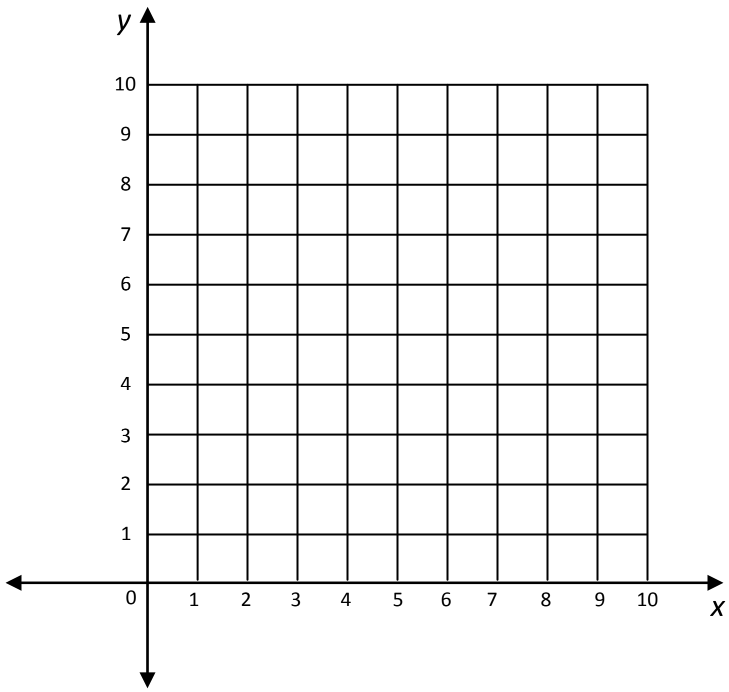
Example 2: Graph the point (7, 2) on the coordinate grid.

To graph the point, (7, 2) we first move 7 units right and then move 2 units up.

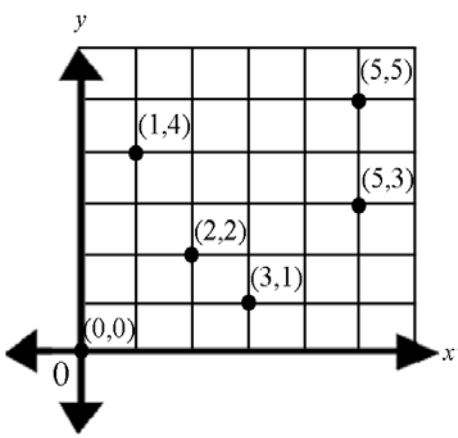


Example 3

Graph the following points on the coordinate grid: (0, 0), (1, 4), (5, 5), (5, 3), (2, 2) and (3, 1).

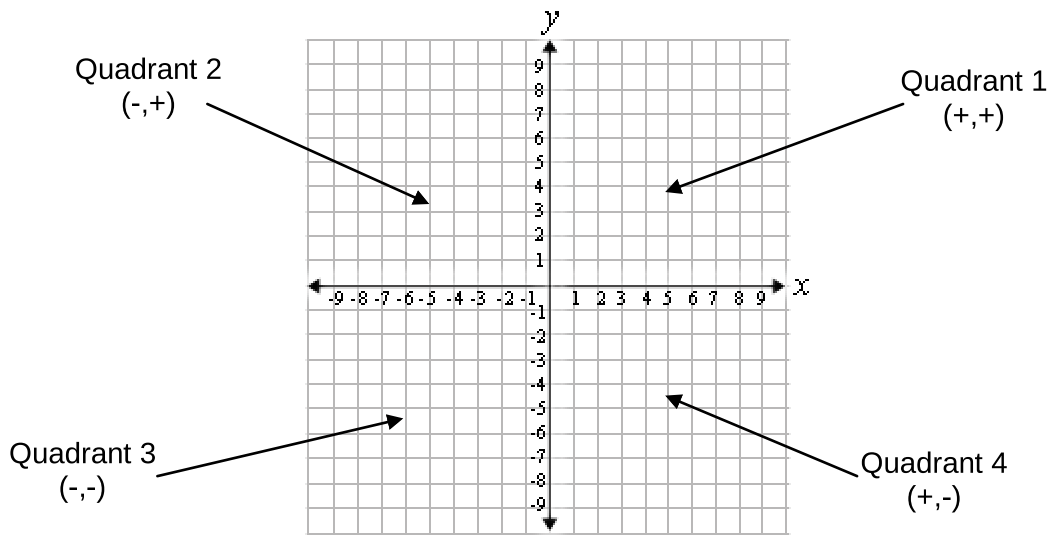


Solution



GRAPHING POINTS IN ANY QUADRANT

A coordinate grid contains four quadrants. Notice the signs of the x and y coordinates in the grid and table below.



The table lists points and in which quadrant they would be located.

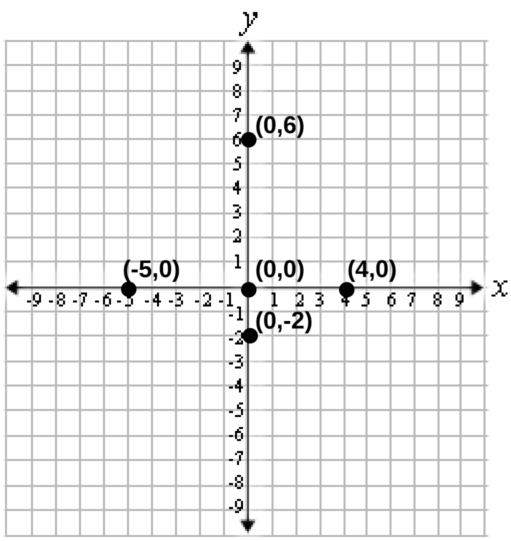
Quadrant						
1 **(+, +)**	**(1, 1)**	**(3, 4)**	**(2, 5)**	**(5, 1)**	**(8, 9)**	**(6, 6)**
2 **(-, +)**	**(-1, 1)**	**(-3, 4)**	**(-2, 5)**	**(-5, 1)**	**(-8, 9)**	**(-6, 6)**
3 **(-, -)**	**(-1, -1)**	**(-3, 4)**	**(-2, -5)**	**(-5, -1)**	**(-8, -9)**	**(-6, -6)**
4 **(+, -)**	**(1, -1)**	**(3, -4)**	**(2, -5)**	**(5, -1)**	**(8, -9)**	**(6, -6)**

Complete this table by placing the following points in the proper quadrants.

(2, 3), (-3, 5), (-2, -2), (4, -7), (8, 1), (9, -1), (-7, -4), (4, -7), (1, 2), (-5, 9), (-3, 1), (-8, -5)

Quadrant			
1			
2			
3			
4			

Sometimes the points lie along the x-axis or the y-axis. In this case, the ordered pair will have an x-value of 0 or a y-value of 0. If the x-value is 0, the point is on the y-axis. If the y-value is 0, the point is along the x-axis.



The point (0, 0) is called the origin. The points (4, 0) and (-5, 0) lie on the x-axis.

The points (0, 6) and (0, -2) lie on the y-axis.

Graph the following points on the coordinate plane: (1, 5), (-2, -3). (8, -3), (0, 4), (-5, 6), (0, 0) and (7, 0).

If an x-value is positive, move to the right.

If an x-value is negative, move to the left.

If a y-value is positive, move up.

If a y-value is negative, move down.

For each point, begin at the origin and then

To graph (1, 5), move 1 unit to the right and then 5 units up.

To graph (-2, -3), move 2 units to the left and then 3 units down.

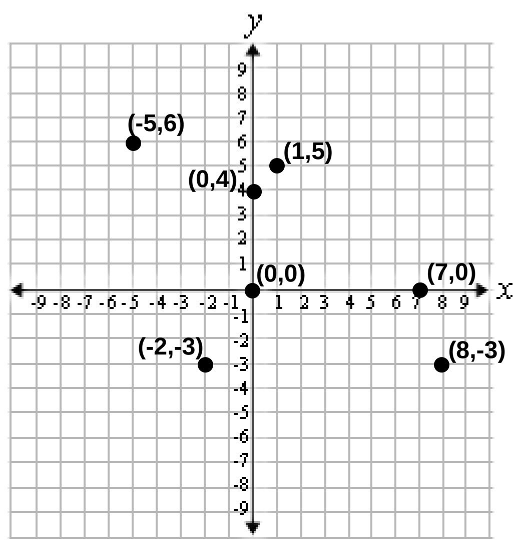
To graph (8, -3), move 8 units to the right and then 3 units down.

To graph (0, 4), start at 0 and move 4 units up.

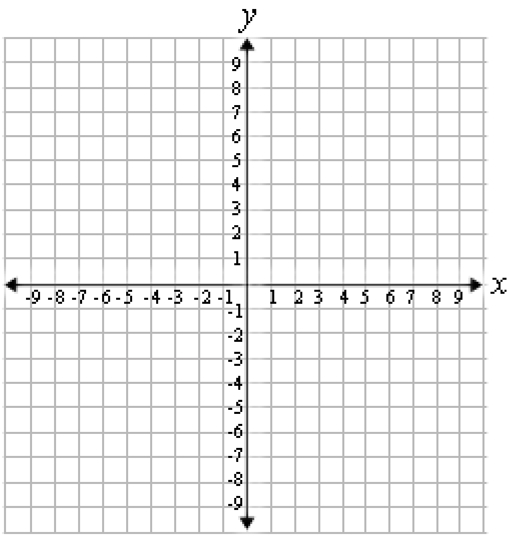
To graph (-5, 6), move 5 units to the left and then 6 units up.

To graph (0, 0), start at the origin and stay there.

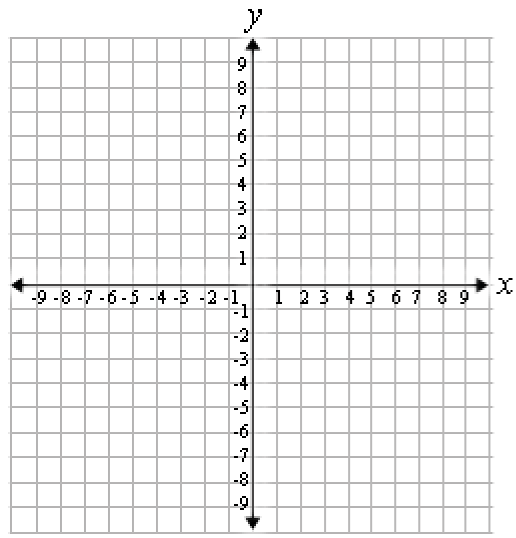
To graph (7, 0), move 7 units to the right and 0 units up.



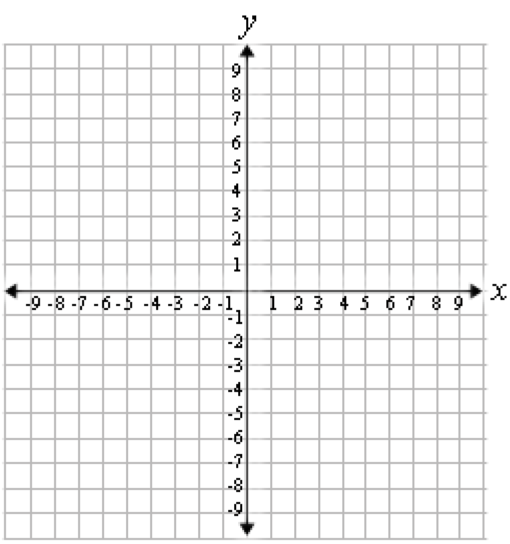
Graph each point below the graphs on the graphs above them. Label each point with the coordinates.



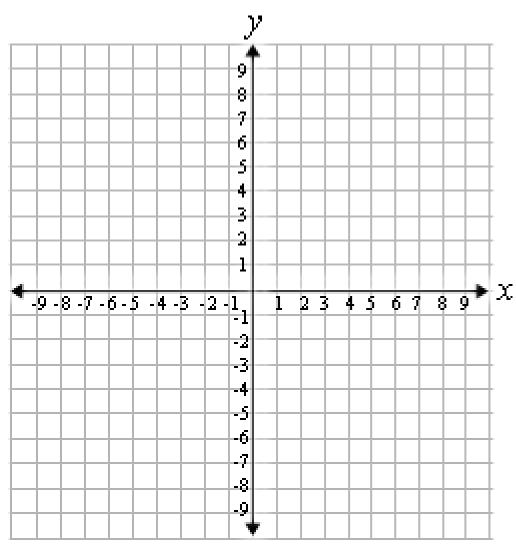
(0, 0), (4, 0), (0, 8), (-5, 0)



(2, 3), (5, -6), (4, 7), (8, -1)



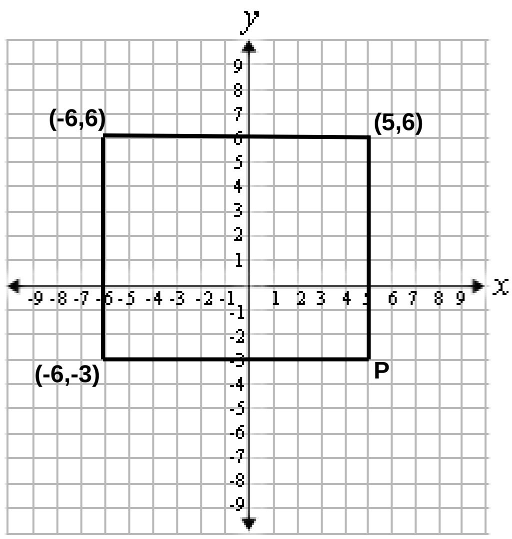
(3, 3), (-5, -5), (6, 6), (-2, -2)



(2, -4), (-3, -6), (-5, 8), (-4, 3)

Sometimes a geometric shape is drawn on a coordinate grid and we are asked to find the missing point.

Example 1



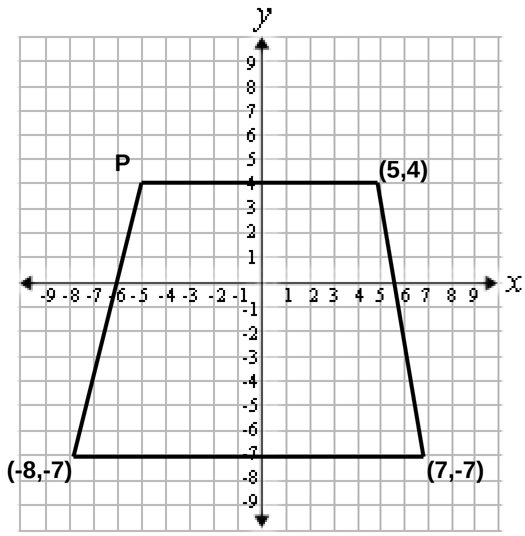
What is the missing coordinate for the rectangle?

Solution

We see from the graph that the missing coordinate has the same x-value as the point (5, 6) and the same y-value as the point (-6, -3). So the missing coordinate labeled point P is (5, -3)

Example 2

Find the missing coordinate in the trapezoid on the coordinate plane. Justify your answer.

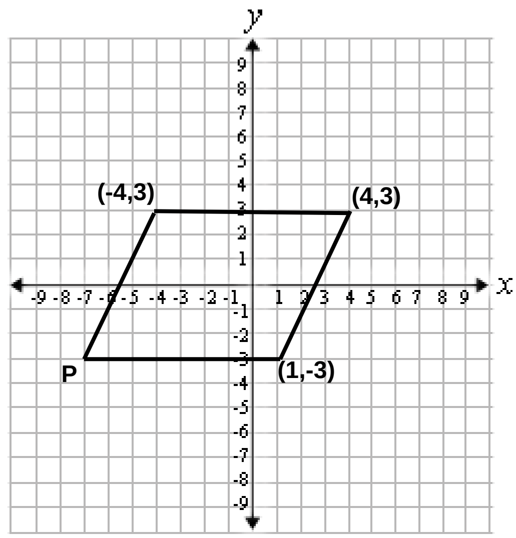


What is the missing coordinate for the trapezoid?

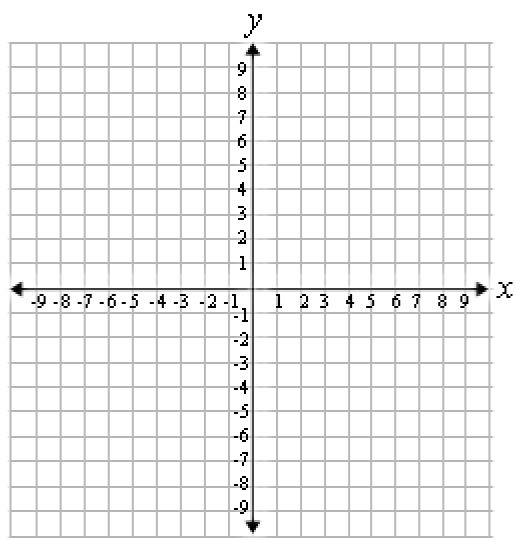
Solution

Because the figure is a trapezoid, we know that the x-value is going to be -5 from reading the graph. We know that the y-value will be the same as the point (5, 4). So the missing coordinate is (-5, 4).

Find the missing coordinate in the parallelogram on the coordinate plane. Justify your answer.



Draw a hexagon on the graph and label all of the coordinate points.

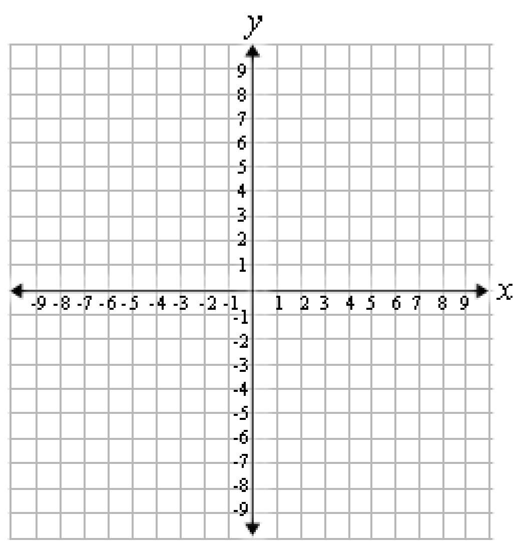


Geometry
Assessment 3 – The Coordinate Plane

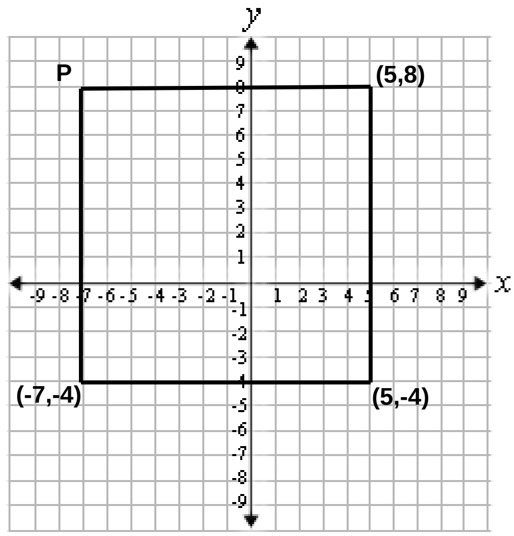
1. Describe the process used to graph a point on a coordinate grid in any quadrant.

2. Graph the points given on the given coordinate plane

(0, -2), (-3, 4), (5, -6), (-7, -7), (1, 0), (-8, 9), (0, 3), and (0, 0).



3. Find the missing coordinate in the rectangle. Justify your answer.



Extensions

1. Explore the relationship between diameter, radius and circumference using an interactive applet. The website contains an explanation, investigation, and problems.
   http://illuminations.nctm.org/ActivityDetail.aspx?id=116

2. Explore complementary, supplementary, and vertical angles with the explanations and interactive applets.

   Complementary angles - http://www.mathopenref.com/anglecomplementary.html
   Supplementary angles - http://www.mathopenref.com/anglesupplementary.html
   Vertical angles - http://www.mathopenref.com/anglesvertical.html

3. Practice graphing points on the coordinate grid.
   Billy Bug Game - http://www.oswego.org/ocsd-web/games/BillyBug2/bug2.html
   Coordinate Game - http://www.shodor.org/interactivate/activities/GeneralCoordinates/
   Maze Game- http://www.shodor.org/interactivate/activities/MazeGame/
   Graph Mole - http://funbasedlearning.com/algebra/graphing/default.htm

Sources

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