Area and Perimeter of Regular and Irregular Polygons

An ADE Mathematics Lesson

Days 1-5

Author	ADE Content Specialists
Grade Level	5^{th} grade
Duration	Five days

Aligns To

Mathematics:
Strand 1: Number and Operations
Concept 2: Numerical Operations
PO 2. Multiply multi-digit whole numbers.
PO 5. Simplify numerical expressions (including fractions and decimals) using the order of operations with or without grouping symbols.
Concept 3: Estimation
PO 1. Make estimates appropriate to a given situation or computation with whole numbers, fractions, and decimals.

Strand 4: Geometry and Measurement
Concept 4: Measurement
PO 4. Solve problems involving the area of 2-dimensional figures by using the properties of parallelograms and triangles.
PO 5. Solve problems involving area and perimeter of regular and irregular polygons using reallotment of square units.

Strand 5: Structure and Logic
Concept 2: Logic, Reasoning, Problem Solving, and Proof
PO 3. Select and use one or more strategies to efficiently solve the problem and justify the selection.
PO 4. Determine whether a problem to be solved is similar to previously solved problems, and identify possible strategies for solving the problem.
PO 5. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols.
PO 6. Summarize mathematical information, explain reasoning, and draw conclusions.
PO 7. Analyze and evaluate whether a solution is reasonable, is mathematically correct, and answers the question.

Connects To

Mathematics:
Strand 1: Number and Operations
Concept 2: Numerical Operations
PO 4. Apply the associative, commutative, and distributive properties to solve numerical problems.

Strand 4: Geometry and Measurement
Concept 1: Geometric Shapes
PO 1. Draw and label 2-dimensional figures given specific attributes including angle measure and side length.

Strand 5: Structure and Logic
Concept 1: Algorithms and Algorithmic Thinking
PO 2. Develop an algorithm or formula to calculate areas and perimeters of simple polygons.
Concept 2: Logic, Reasoning, Problem Solving, and Proof
PO 2. Identify relevant, missing, and extraneous information related to the solution to a problem.

Overview
The area of many figures can be found by breaking the figures into parts like triangles and parallelograms.

Purpose
In this lesson you will find the areas of two-dimensional figures by breaking them apart into smaller figures. This will help prepare you to finding areas, surface areas, and volumes of more complicated two- and three-dimensional figures.

Materials
- Area and perimeter worksheets
- Ruler
- Graph paper

Objectives
Students will:
- Solve problems involving the area and perimeter of 2-dimensional figures by using the properties of parallelograms and triangles.
- Solve problems involving area and perimeter of regular and irregular polygons using reallotment of square units.

Lesson Components

Prerequisite Skills: In 3rd and 4th grade mathematics, you learned to find the area and perimeter of rectangles and triangles. You described the change in perimeter or area when one attribute (length or width) of a rectangle changes.

Vocabulary: *perimeter, area, side, angle, attribute, array, parallelogram, rectangle, triangle, hexagon, grid*

Session 1 (2 days)

1. Students find the area and perimeter of 2-dimensional figures by using properties of parallelograms and triangles.

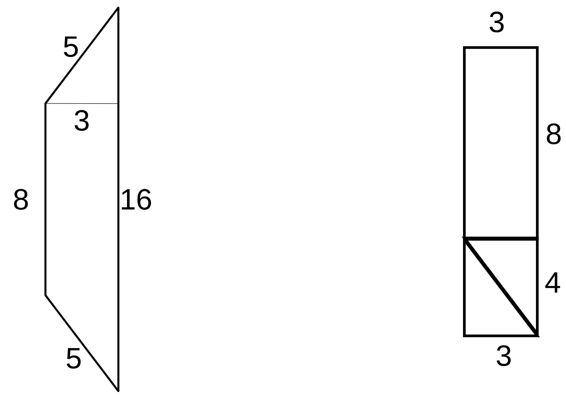
Session 2 (3 days)

1. Students solve problems involving area and perimeter of regular and irregular polygons using reallotment of square units.

Assessment
There are two assessments one after each session that will help pinpoint misconceptions about area and perimeter of 2-dimensional figures before moving on to more complex comparisons.

Area and Perimeter of Polygons
Session 1

We can determine the area of the figures listed below by applying what we know about finding the area of triangles and parallelograms.

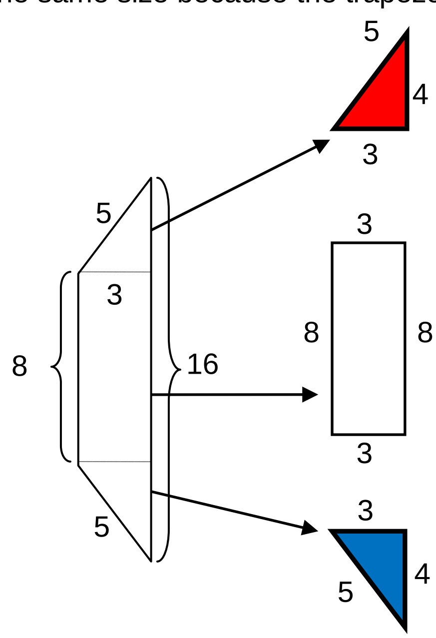


Example 1

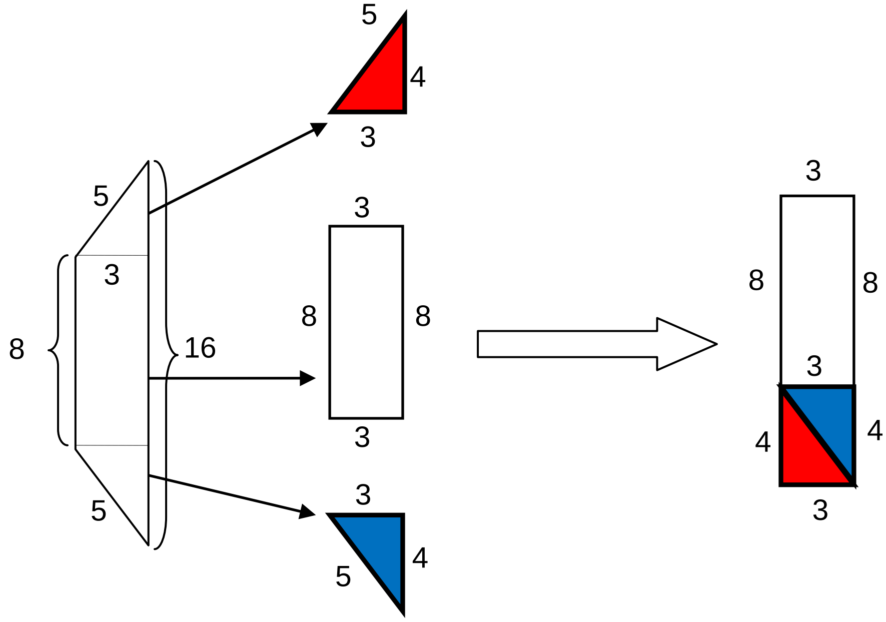
Find the area of the trapezoid on the left.

Solution

Let's divide the trapezoid into two triangles and one rectangle. The rectangle and triangles are represented by the figures to the right of the trapezoid. The rectangle has a length of 8 and width of 3. To figure out the measurement of the triangles, we take the measurement of the long side of the trapezoid (16) and subtract the length of the rectangle (8). 16 – 8 = 8. The difference between these two measurements is 8. The measurement of each triangle is half of 8 or 4. The triangles are the same size because the trapezoid is symmetrical.



These three shapes (rectangle and two triangles) can be rearranged into the figure shown on the right by flipping the red triangle and placing it at the bottom of the figure with the other triangle.



The two congruent triangles together (shown in red and blue at the bottom of the figure) form a rectangle with a base of 4 and a height of 3. This area can be calculated and added to the area of the rectangle with a base of 8 and a height of 3 to get the total area of the trapezoid.

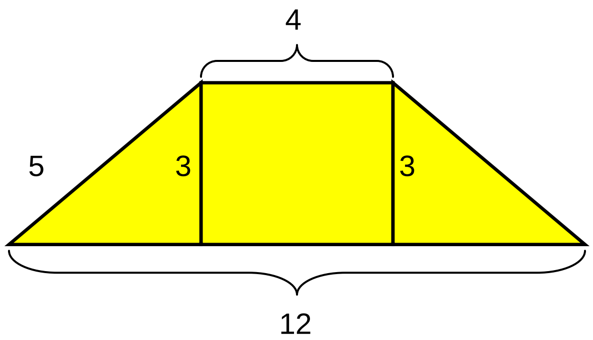
Area of the rectangle: $8 \bullet 3 = 24$ square units

Area of the rectangle formed by the two triangles: $4 \bullet 3 = 12$ square units

Total Area of the trapezoid: $24 + 12 = 36$ square units

Example 2

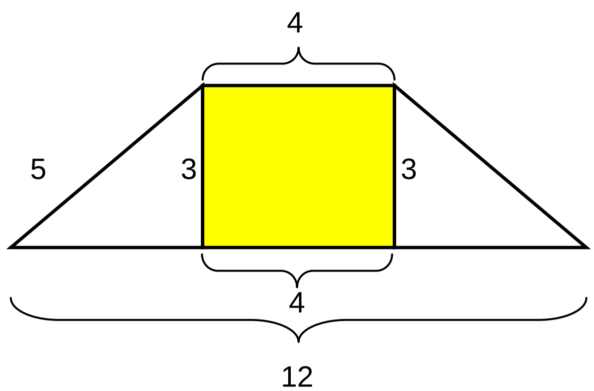
Find the area of the trapezoid shown below.



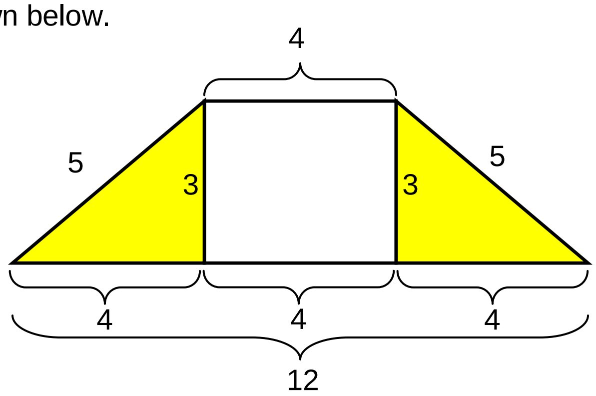
Solution

We can divide this figure into two triangles and a rectangle as we did in example 1. Since parallelograms have congruent sides, the opposite sides of the rectangle must be 4. The other opposite sides are given as 3.

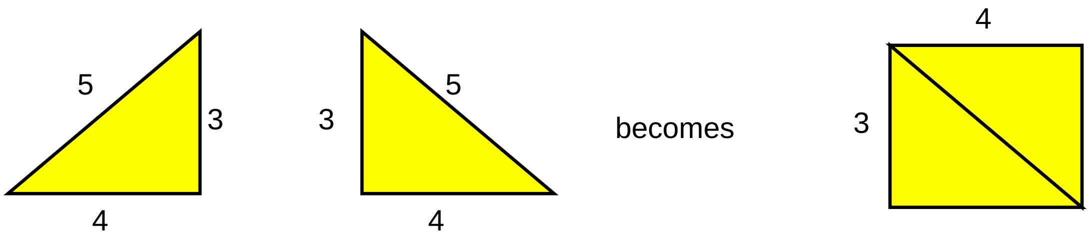
The area of this rectangle is $4 \bullet 3 = 12$ square units.



We can find the length of the other side of the triangles because we know that long side of the trapezoid is 12 and the length of the rectangle is 4. If we subtract 4 from 12 we get 8. When we divide 8 by 2 and we get 4. So, the unmarked sides of the triangle must each be 4. This gives us the two triangles drawn below.



Rearrange these two triangles to form a rectangle.



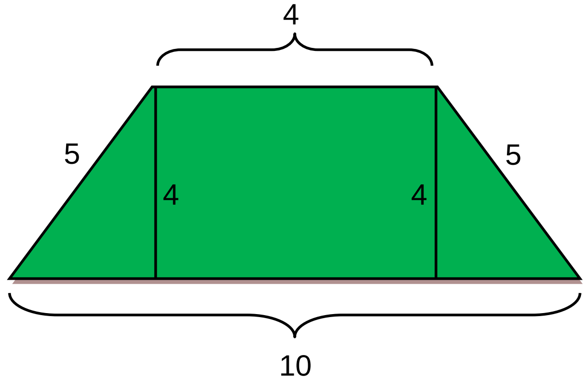
The area of this rectangle formed by the two triangles is $4 \bullet 3 = 12$ square units.

The area of the middle rectangle is also $4 \bullet 3 = 12$ square units.

Therefore the total area of the trapezoid equals $12 + 12 = 24$ square units.

Now you try to do one problem. Many hints are given to help you find the area.

Problem 1: Find the area of the trapezoid shown below. Look at the examples above to help you.



Break the trapezoid up into three figures and draw them below. Label the measurements that are shown on the trapezoid.

Can you find the area of one of the figures that you drew in the box above? Show your work in this box.

Determine the length of the unknown sides of the triangles you drew in the first box. (Hint: look at example 2).

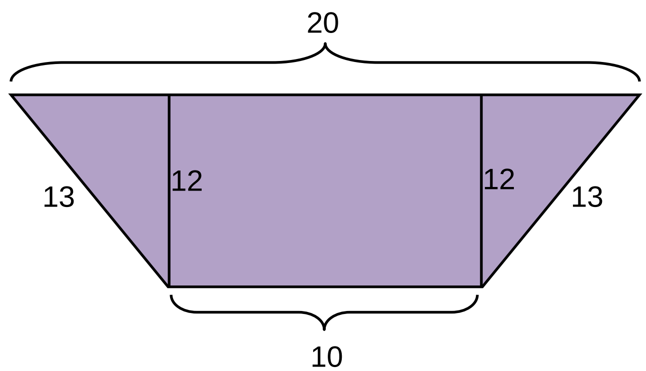
Make a rectangle from the two triangles you drew in the first box and find the area of this rectangle.

Now add the area of the two rectangles together to determine the area of the trapezoid. Don't forget to use square units for area.

Check: Did you get 28 square units? Great! You can do this lesson also by cutting out the trapezoid and cutting into shapes.

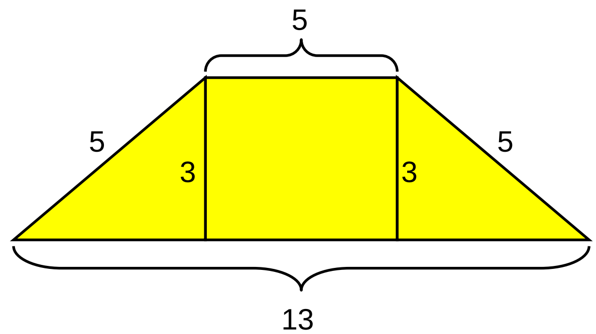
Problem 2:

Find the area of the trapezoid shown below. Show all your work in the space provided.



Area and Perimeter of Polygons
Assessment 1

Find the area of the trapezoid shown below. Show your work in the space provided.

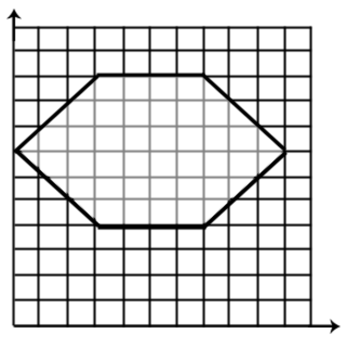


Area and Perimeter of Polygons
Session 2

We can also determine the area of 2-dimensional figures by drawing them on a grid, cutting them out, and rearranging the pieces into a common shape that we can find the area.

Example 1

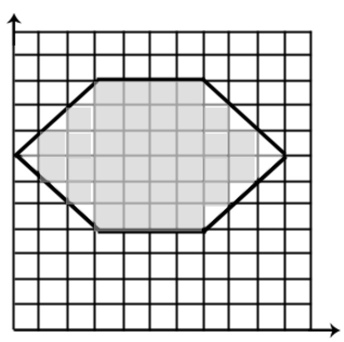
Find the area of the hexagon shown below.



Solution

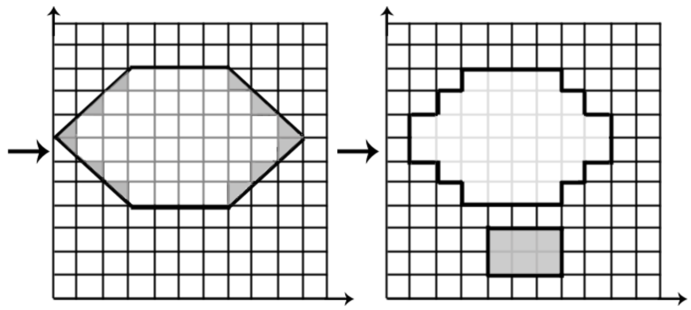
Step 1

We can count the number of squares that are totally in the hexagon where none of the sides of the square are cut off. The figure below shows these squares highlighted.



Step 2:

We cut off the triangular parts of the hexagon that are not complete squares and rearrange them to make figures where we can count the squares.



Note that the shaded parts of the diagram on the left have been cut and rearranged to form the shaded square on the bottom of the diagram on the right.

Step 3:

Count up all the squares to determine the area of the original hexagon.

How many complete squares do you see in the figure in Step 1?

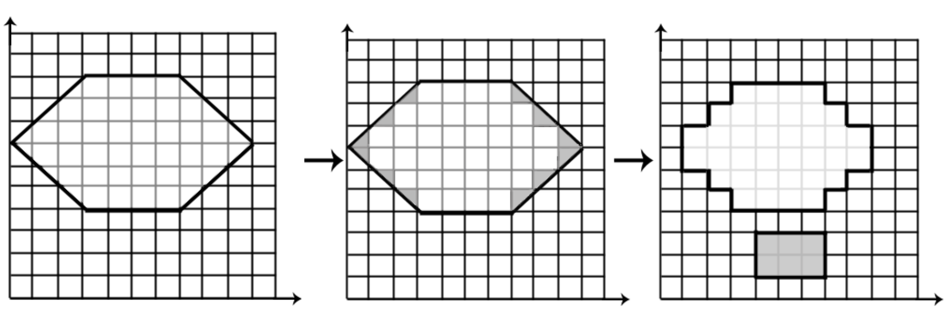
How many squares do you count in the shaded rectangle made from the triangular parts in Step 2?

How many total squares do you have?

This is the area of the original hexagon.

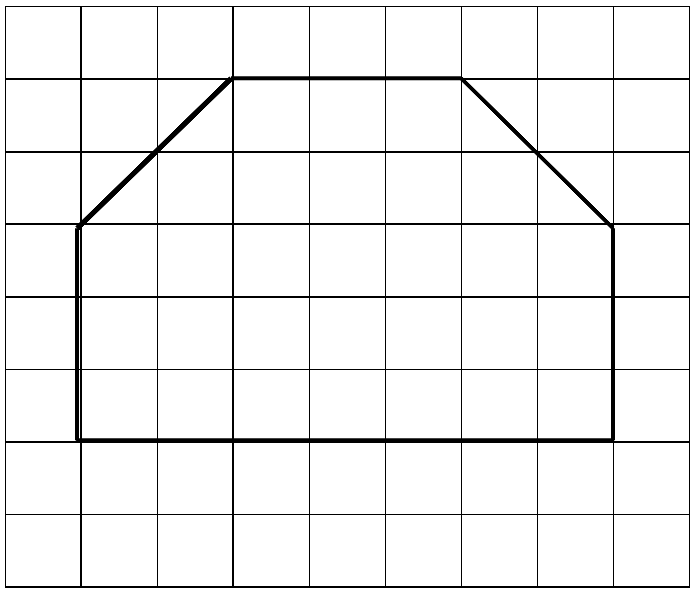
Did you get 36 squares in the figure in Step 1 and 6 squares in the shaded rectangle from Step 2 for a total area of 42 square units?

Summary



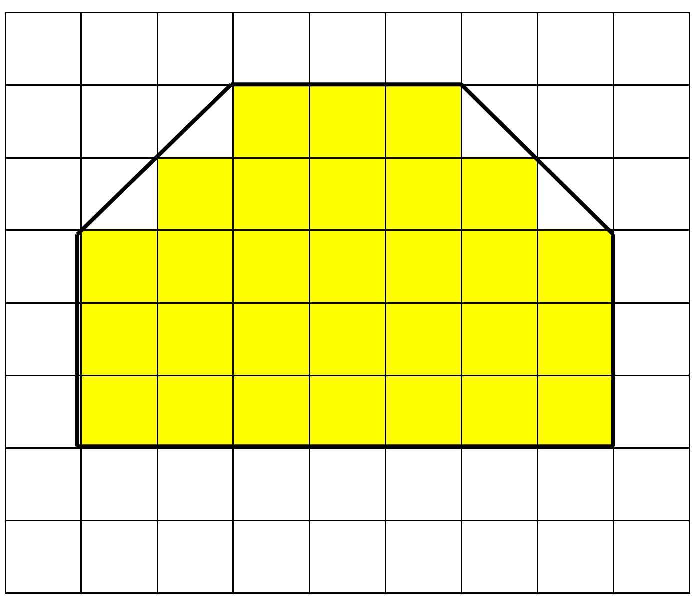
Example 2

Find the area of the trapezoid shown below.



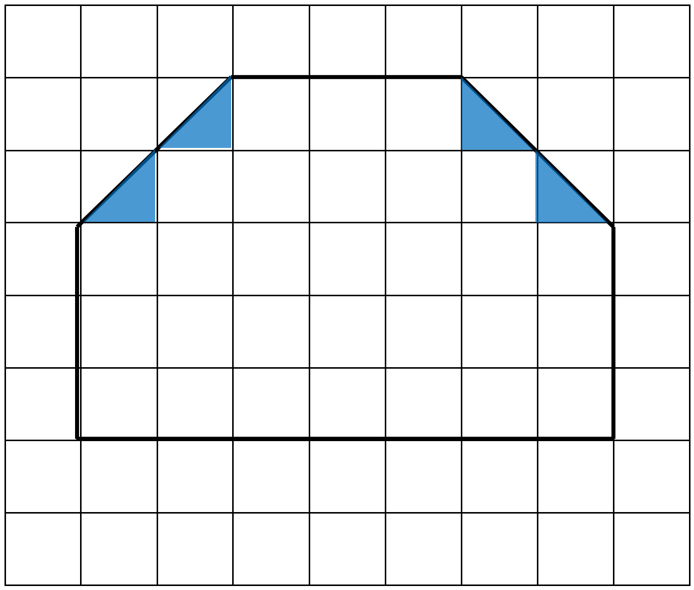
Follow the steps outlined in example 1.

Step 1: Count the number of squares that are complete within the trapezoid. How many are there that are highlighted yellow?

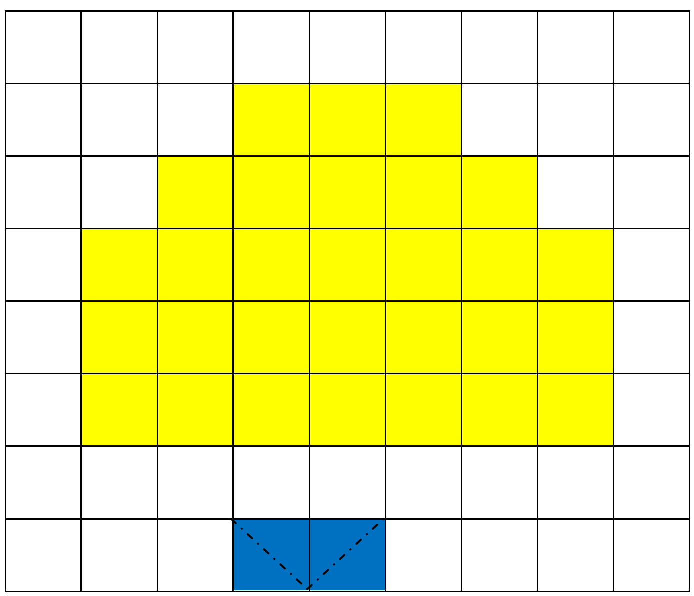


Step 2:

We cut off the triangular parts of the triangle that are shaded and rearrange them to make figures where we can count the squares



When we rearrange the diagram, we have two figures that look like the figures below.



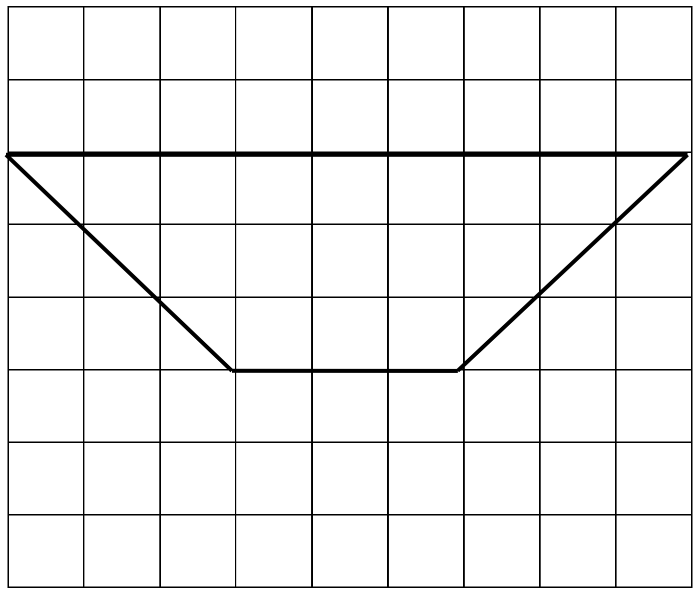
What is the total amount of yellow squares and blue squares?

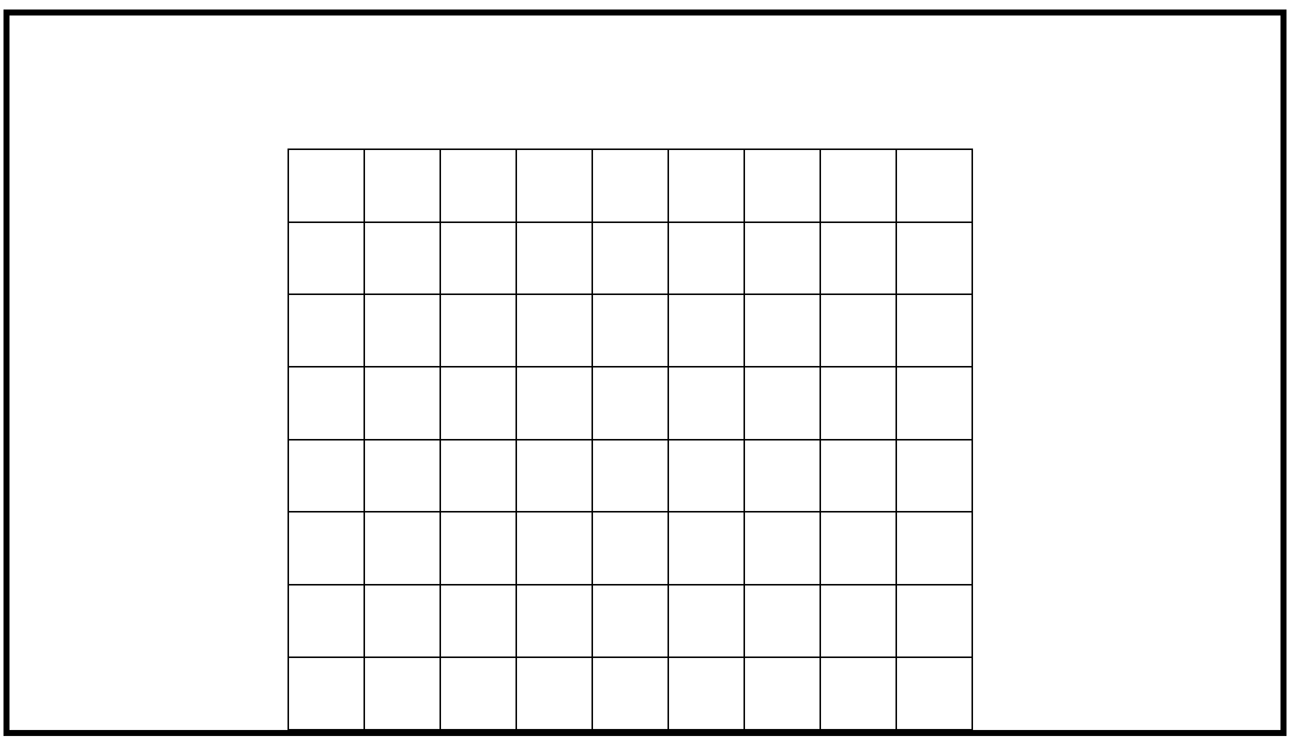
29 yellow squares + 2 blue squares = 31 square units which is the area of the original trapezoid.

Now you try to work a problem.

Problem 1

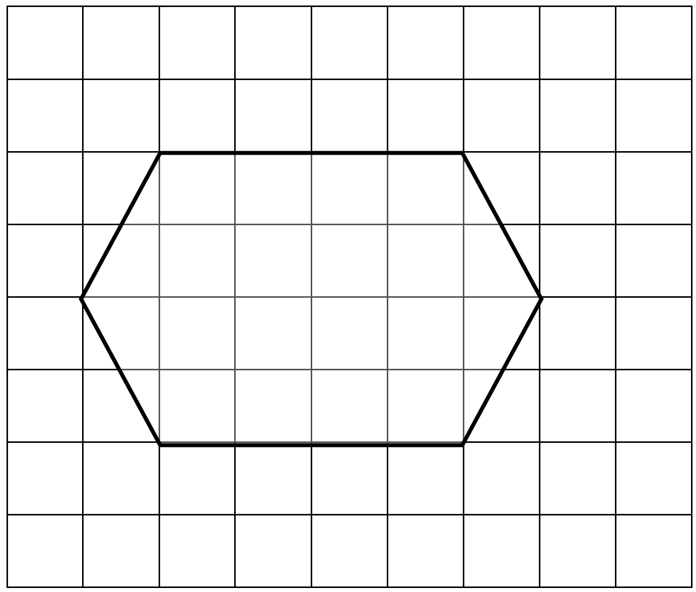
Find the area of the trapezoid shown below. There is a second grid so that you can show your work on it when you rearrange the parts of the trapezoid.

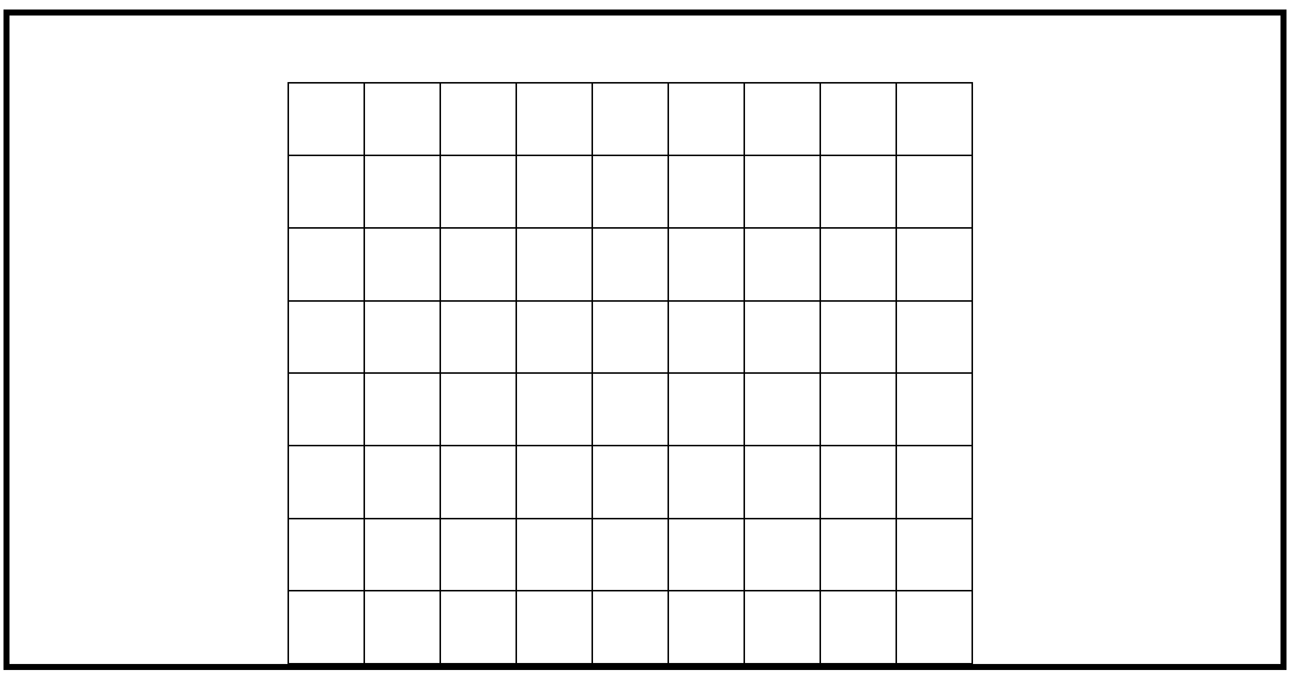




Area and Perimeter of Polygons
Assessment 2

1. Find the area of the hexagon below showing all your work in the space provided.





2. Describe the process for finding the area of a trapezoid or hexagon using squares on a graph or grid.

Extensions

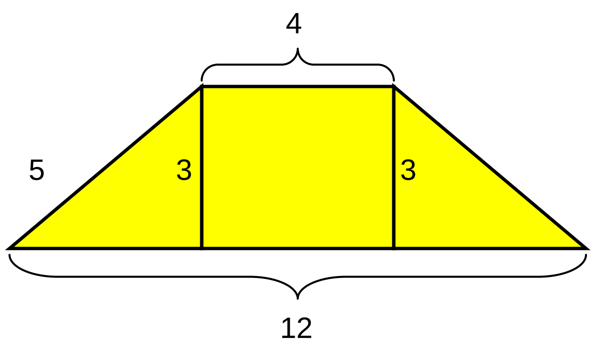
1. Locate 2-dimensional figures in your everyday environment and find the area and perimeter of these items. Examples of this include finding the areas of floors or ceilings, windows, gardens, etc.
2. Find the area of a trapezoid using the formula for trapezoid. This enrichment lesson is included on the next page.

Area and Perimeter of Polygons
Extension – Area of Trapezoids

In Session One, we found the area of a trapezoid by dividing the trapezoid into two triangles and a rectangle. We can also find the area of a trapezoid by using a formula based on the bases of the trapezoid and the height of the trapezoid.

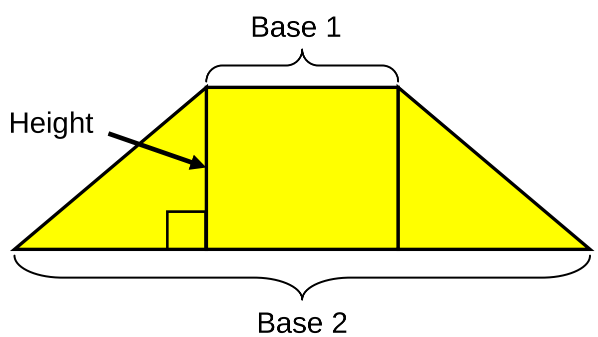
Let’s consider the examples shown in Session 1.

Example 1: Find the area of the trapezoid shown below.



Solution

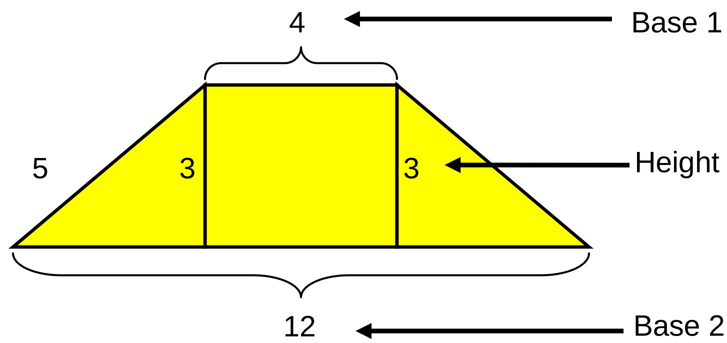
First let’s review what a trapezoid is. A trapezoid has two bases and these bases are parallel. A trapezoid also has a height. These bases and heights are marked in the trapezoid shown below.



There is a formula to find the area of a trapezoid when you know the height and the two bases.

Area of trapezoid = $\frac{1}{2} \bullet$ **height** $\bullet$ **(sum of base one plus base two). Remember that you must add the bases together before you multiply this sum by** $\frac{1}{2} \bullet$ **height.**

We know from Example 2 in Session One that the height of the trapezoid is 3. One base is 4 and one base is 12.



Add the bases together. 4 + 12 = 16.

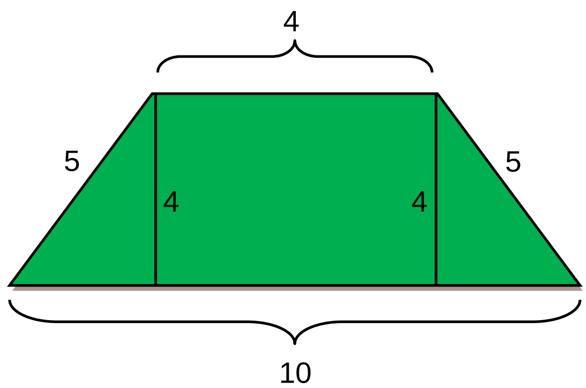
Multiply this sum by the height. 16 $\bullet$ 3 = 48.

Now take one half of this number. $\frac{1}{2} \bullet 48 = 24$ square units.

Is this the same answer that we obtained for Example 2 in Session 1? Yes it is. Check the Example and see if you can see any connection between the formula and the work done in Example 2 in Session 1. Write down what you notice in the box below.

Example 2

Find the area of the trapezoid shown below using the formula for the area of the trapezoid. Look at the work you did in Session 1 to help you.



What is the formula to find the area of a trapezoid?

What is the height of this trapezoid?

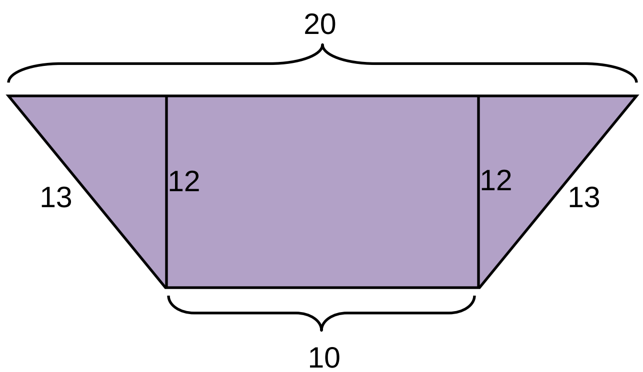
What is the measurement of each base?

Now, calculate the area of the trapezoid showing all your work below.

Does your answer match the answer you got for the area of this trapezoid when you worked this problem in Session 1?

Problem 1:

Find the area of the trapezoid shown below showing all your work in the space provided.



Area of trapezoid = $\frac{1}{2} \bullet$ height $\bullet$ (sum of base one plus base two).

Sources

2008 AZ Mathematics Standards
2006 NCTM Curriculum Focal Points
2000 NCTM Principles and Standards
2008 The Final Report of the National Mathematics Advisory Panel