E-x-p-a-n-d-e-d Learning Progressions Frameworks for K-12 Mathematics: A Companion Document to the *Learning Progressions Frameworks Designed for Use with The Common Core State Standards in Mathematics K-12*

*Part 1 - Grades K-4**

This document presents expanded grade-level views of the *Learning Progressions Frameworks for K-4 Mathematics* to show how smaller learning progressions can be drawn from the Learning Progressions Framework (LPF) using the Progress Indicators and highlighted links to the Common Core State Standards (CCSS) in mathematics. Unlike the original K-12 LPF document, this document is organized by smaller grade spans (K-2 and 3-4). (*Two other expanded version documents are also available: Part 2 for middle school/ grades 5-8, and Part 3 for high school/grades 9-12.) Each of these expanded versions displays all six LPF mathematics strands "unpacked" for ease of use by teachers and curriculum and assessment developers. The original document upon which they are based - *Learning Progressions Frameworks Designed for Use with the Common Core State Standards in Mathematics K-12,* including a more complete explanation of the research-based rationale and conceptual underpinnings - can be found at http://www.nciea.org/publications/Math_LPF_KH11.pdf *or at www.naacpartners.org*.
Other support materials *and related publications can be found at www.nciea.org or at www.naacpartners.org*.

Karin K. Hess, Editor
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Table of Contents

Two other LPF expanded version mathematics documents are available for middle school/ grades 5-8, and high school/grades 9-12, the original document on which they are based, *Learning Progressions Frameworks Designed for Use with the Common Core State Standards in Mathematics K-12,* and other support materials *and related publications* can be found at *www.nciea.org*.

Important CCSS note: All glossary, table, and footnote references embedded in the CCSS standards refer back to the original *Common Core State Standards for Mathematics* document which can be located at http://www.corestandards.org/the-standards/mathematics .

Overview: The Learning Progressions Frameworks (LPFs) Development Process
The approach used to identify the content progressions and specific standards within the *Common Core State Standards* (CCSS) considered three important dimensions. First, national content experts and researchers in mathematics were asked to identify specific content strands that represented a way to organize essential learning for all students, K-12. Next, the committee was asked to describe the "enduring understandings" (as defined by Wiggins and McTighe, 2005) for each particular content strand, as well as articulate what the learning targets would look like if students were demonstrating achievement of the enduring understandings at the end of each grade span (K-4, 5-8, and 9-12). The grade span learning targets for each strand are stated as broader performance indicators (e.g., Use equations and expressions involving basic operations to represent a given context; Build flexibility with whole numbers and fractions to understand the nature of number and number systems). The larger grained grade span learning targets are designed to describe progressively more complex demonstrations of learning across the grade spans and use wording similar to what one might see in performance level descriptors for a given grade or grade span. (See the original K-12 LPF document for the grade span learning targets - http://www.nciea.org/publications/Math_LPF_KH11.pdf.)

In mathematics, **six major LPF strands** were established. Below is a brief description of the six strands identified by the LPF content committee. "For each content area, these essential threads [strands] interact to build greater understanding of the discipline over time. Identifying a small number of essential threads makes the learning progression manageable for the classroom teacher in terms of tracking ongoing progress in the classroom" (Hess, 2008, p.5). It is not the intent that skills/concepts from a particular strand be taught in isolation in a linear sequence, but rather be integrated among strands, such as in a problem solving situation where students are demonstrating their understanding of measurement concepts while applying their knowledge of numbers and operations and using symbolic expression. *In other words, the LPFs should be thought of as a general map for learning, not a single route to a destination.*

- **Symbolic Expression (SE)** – Symbolic Expression, presented in this document as the first strand, is a reminder NOT to teach symbolic representations before students have begun to demonstrate conceptual understanding of what the symbols or procedures actually mean (e.g., what joining together (+) and taking apart (-) sets means; understanding relative magnitude of part-whole; that "equivalence" (=) means different names for the same number). Progress indicators for the Symbolic Expression strand should be taught in conjunction with skills and concepts described in PIs from other strands and introduced with building conceptual understanding in mind.
- **The Nature of Numbers & Operations (NO)** – The skills and concepts within the Nature of Numbers and Operations strands form the foundation - and often are the prerequisite skills and concepts - for many of the other mathematics strands. Local curriculum development efforts should always consider how the skills and concepts described in the Numbers & Operations

progress indicators can be introduced, practiced, and extended with skills/concepts found in the other strands. The third N&O strand focuses on mathematical reasoning and problem solving. These progress indicators can be integrated with many CCSS standards at each grade level using problem solving contexts. While listed under the Nature of Numbers & Operations strand, the skills and concepts described in these progress indicators could apply to concepts in different mathematics strands, such as when developing proofs in Geometry.

- **Measurement (ME)** – Progress indicators are organized under two key learning targets for each grade span**.**
- **Patterns, Relations, & Functions (PFR)** – Progress indicators are organized under two key learning targets for each grade span.
- **Geometry (GM)** – Progress indicators are organized under one key learning target for each grade span.
- **Data Analysis, Probability, & Statistics (DPS)** – Progress indicators are organized under two key learning targets for each grade span. There is minimal emphasis in the CCSS on Data Analysis, Probability, & Statistics at grades K-6. Because many of the DPS mathematics skills and concepts are essential to science and social studies instruction at these grade levels, progress indicators are included in the DPS strand to guide unit development where organizing and interpreting data is important. However, you will not find many links to the CCSS mathematics standards in this strand at the lower grade levels.

Once the content committee had established the broader grade span learning targets for each strand, they were asked to identify and describe the essential skills and concepts needed to achieve the grade span expectations; use research syntheses to establish a general order of how those skills and concepts emerge for most students; and further break down the descriptors into smaller grades spans: K-2, 3-4, 5-6, 7-8, and high school. The descriptors of related skills and concepts became what we now call the **progress indicators** and the ordering/numbering used (1a, 1b, 1c, etc.) reflects the research base used to establish a general learning continuum. This means that descriptions of earlier skills build the foundation for later skills (e.g., later within a grade level, later at the next grade level/span).

The final step in the LPF development process was to identify alignment with specific CCSS mathematics content standards in order to create guidance for a cohesive curriculum experience across grades. Sometimes multiple standards from within the smaller grade spans could be linked to the same progress indicator (PI); sometimes there was only one or no standard that aligned. For example, in some strands and grade spans you will see PI descriptors that do not link (align) with an existing CCSS standard; however, the research review identified critical learning at certain stages during the learning process or skills that may be essential for conceptual understanding and for demonstrating progress. Therefore, progress indicators with no CCSS links are also included in the LPF to guide instruction, formative assessment, and progress monitoring.

Reading and Interpreting the LPF format, progress indicators, and related standards for a Grade Span – See next page. 

***N&O**: Understandings of number - "how many" or "how much" – and number types extend applications of arithmetic properties, operations, and number systems and guide use of computational strategies and algorithms.*

The statement of enduring understanding across grade spans states WHY learning the skills and concepts (and standards) listed below are important and how they are generally applied.

(K-4) Elementary School Learning Targets

E.NO-2 Build an understanding of computational strategies and algorithms:

- *Fluently add, subtract, multiply, divide, and estimate;*
- *Perform and represent operations with whole numbers, fractions, and mixed numbers;*
- *Identify multiples and factors of whole numbers.*

K-4 Elementary School Grade Span Learning Targets

- **By the end of grade 4, students demonstrate and apply the skills and concepts related to Numbers & Operations in a variety of situations or problem solving contexts.**
- **Learning targets are the more general/broad performance descriptors associated with specific skills and concepts at each grade level.**

Larger grade spans are then broken into smaller grade spans

Grades K-2	**Grades 3-4**
Build understanding and fluency with operations...	Build understanding and fluency with operations...
E.NO.2a representing addition and subtraction in multiple ways (composing/decomposing numbers, diagrams, using objects, arrays, equations, number lines), including regrouping K.OA-1, 2, 3, 4; K.NBT-1 } 1.OA-1, 2, 5, 6; 1.NBT-4, 5, 6 2.OA-1, 4; 2.NBT-7	E.NO.2d modeling multiplication (equal-sized groups, arrays, area models, equal-sized jumps on number lines, multiplicative comparisons) and division (successive subtraction, partitioning, sharing) of whole numbers 3.OA-1, 2, 3, 4, 5 4.OA-1, 2, 3; 4.NBT- 5, 6
E.NO.2b explaining or modeling the relationship between addition and subtraction 1.OA-3, 4 2.NBT-5, 7, 9	E.NO.2e describing relationships between addition-multiplication; multiplication-division; addition-subtraction; why commutativity does not apply to subtraction or division 3.OA-7, 9; 3.NBT-2 4.OA-2
E.NO.2c working flexibly with common addition and subtraction situations K. OA-2 1. OA-3, 5, 6, 8 2.OA-1, 2; 2.NBT- 2, 5, 7	E.NO.2f identifying factors and multiples of numbers 3.OA-6 4.OA-4
	E.NO.2g recognizing fractions as one number/one quantity, rather than two numbers (numerator and denominator) and using number lines to represent magnitude of fractions 3.NF-1, 2, 3a, 3c

What you see articulated in this sample LPF strand:

- **"E" denotes all Elementary (K-4) progress indicators.**
- **Most LPF progress indicators are stated in a more general way (e.g., using many related strategies; using both addition and subtraction) than a single CCSS standard; therefore progress indicators (PIs) often align with several CCSS standards at different grade levels within the grade span. This multi-standard alignment can provide insights into potential "mini progressions" for lesson design.**
- **Numerous CCSS standards align with the first descriptor under K-2 and can be interpreted that this progress indicator embodies many important foundational skills for all three grade levels, K, 1, and 2. Teachers at all of these grades may need to revisit lower grade level skills (and standards) for students needing reinforcement/ extra work on these prerequisite skills.**
- **K students would spend most of their school year working on CCSS standards: K.OA-1, 2, 3, 4; and K.NBT-1 (linked to the first PI), while grades 1 and 2 would be addressing all three PIs and the associated CCSS standards in this general/a-b-c order.**

Text in blue denotes links to CCSS standards:
2.OA-1,2 means grade 2, Operations & Algebraic Thinking, standards 1 and 2 (See p. 19 of CCSS for mathematics)

The highlighting in the expanded version of the LPF shows potential smaller learning progressions (LPs) and parts of the CCSS standards that link with progress indicators.

Elementary School Learning Targets

Data Analysis, Probability, and Statistics (DPS)- *DPS-1 Gather and interpret data to answer questions related to a particular/single context. Formulate questions, gather data, and build representations; Identify and describe variation in data, and describe and compare shapes of distributions and measures of central tendency.*

Progress Indicators for Grades K-2	Grade K CCSS standards	Grade 1 CCSS standards	Grade 2 CCSS standards
E.DPS.1a posing questions of interest that can be answered by counting or collecting data (e.g., concrete comparisons about students, classroom materials, science topics) *with teacher guidance*	**K.CC-5** **5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.** **K.CC-6** **6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1]** **([1] Include groups with up to ten objects.)**	**1-MD-1** **1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.**	**2.MD-2** **2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.** **2.MD- 5** **5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.**
E.DPS.1b identifying and sorting data/attributes; identifying rules for classifying data/attributes	**K.MD-1** **1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.** **K.MD-2** **2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*** **K.MD-3** **3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[1]** **K.G-2** **2. Correctly name shapes regardless of their orientations or overall size.** **K.G-4** **4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts & attributes**	**1.MD-1** **1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.** **1.MD-4** **4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.** **1.G-1** **1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.**	**2.G-1** **1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[1] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.** **2.MD-10** **10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems[1] using information presented in a bar graph.**

Highlighting indicates links among the Progress Indicator & one or more CCSS standard or parts of the standard(s).

The highlighting in 2 colors here illustrates two possible smaller learning progressions (LPs) for instruction & assessment for this PI. This document does not show multiple LPs with different highlighting for PIs, but different LPs may exist if you look for them by matching the highlighted CCSS language with the PI description.

Expanded Learning Progressions Frameworks for K-12 Mathematics

Elementary School Strands

Grades K - 2

Elementary (K-4) School Learning Targets, Progress Indicators, & Common Core Standards

Symbolic Expression (SE): The use and manipulation of symbols and expressions provide a variety of representations for solving problems and expressing mathematical concepts, relationships, and reasoning.

E.SE-1 *Use equations and expressions involving basic operations to represent a given context: Represent numerical relationships using combinations of symbols (=, >, <) and numbers to form expressions and equations; Solve for unknown in simple number binary number sentences (e.g., ____ + 4 = 7); Write equations showing inverse operations and related operations (e.g., addition-multiplication).*

Progress Indicators for Grades K-2	Grade K CCSS standards	Grade 1 CCSS standards	Grade 2 CCSS standards
E.SE.1a recognizing that symbols correspond to specific quantities (e.g., matching symbols to sets of quantities above three) **K.CC-3**	**K.CC-3** **3.** Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).		
E.SE.1b using oral and then written numerals and symbols to express quantities **K.CC-3, 4** **1.NBT-1** **1.MD-3** **2.NBT-3** **2.MD-7, 8**	**K.CC-3, 4** **3.** Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). **4.** Understand the relationship between numbers and quantities; connect counting to cardinality.	**1.NBT-1** **1.** Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. **1.MD-3** **3.** Tell and write time in hours and half-hours using analog and digital clocks.	**2.NBT-3** **3.** Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. **2.MD-7, 8** **7.** Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. **8.** Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?
E.SE.1c using symbols (=, >, <) to compare whole number quantities, write equations, and determine if equations are true **1.OA-7** **1.NBT-3** **2.OA-3, 4** **2.NBT-4**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.* *The concepts of greater than and less than are more important to understand at this time than the use of the symbols.*	**1.OA-7** **7.** Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2. **1.NBT-3** **3.** Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.	**2.OA-3, 4** **3.** Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends. **4.** Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. **2.NBT-4** **4.** Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

Progress Indicators for Grades K-2	Grade K CCSS standards	Grade 1 CCSS standards	Grade 2 CCSS standards
E.SE.1d representing addition and subtraction in multiple formats, including expressions **K.OA-1** **1.OA-1, 2, 3, 4** **2.OA-1, 4**	**K.OA-1** **1.** Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.	**1.OA-1, 2, 3, 4** **1.** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[1] ([1] See Glossary, Table 1.) **2.** Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. **3.** Apply properties of operations as strategies to add and subtract.[2] *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)* **4.** Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8. Add and subtract within 20.*	**2.OA-1, 4** **1.** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1] ([1] See Glossary, Table 1) **4.** Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.
E.SE.1e demonstrating understanding of finding the unknown in addition and subtraction equations (e.g., when all but 1 of 3 numbers is known- what makes this true?) **K.OA-1, 3, 4** **1.OA-7, 8** **2.MD-5** **2.OA- 4**	**K.OA-1, 3, 4** **1.** Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. **3.** Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). **4.** For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.	**1.OA-7, 8** **7.** Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2. **8.** Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = _ – 3, 6 + 6 = _.*	**2.MD-5** **5.** Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. **2.OA- 4** **4.** Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Elementary (K-4) School Learning Targets, Progress Indicators, & Common Core Standards **The Nature of Numbers and Operations (NO):** Understandings of number - "how many" or "how much" – and number types extend applications of arithmetic properties, operations, and number systems and guide the use of computational strategies and algorithms. ***E.NO-1*** *Build flexibility using whole numbers, fractions, and decimals to understand the nature of number and number systems: Count, model, and estimate quantities; Compare, represent, and order numbers; Apply place value concepts and expanded notation to compose and decompose whole numbers.*			
Progress Indicators for Grades K-2	**Grade K CCSS standards**	**Grade 1 CCSS standards**	**Grade 2 CCSS standards**
E.NO.1a showing mastery of the prerequisite core skills of cardinality, constancy, and 1:1 correspondence **K.CC-2, 4**	**K.CC-2, 4** **2.** Count forward beginning from a given number within the known sequence (instead of having to begin at 1). **4.** Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger.	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*
E.NO.1b developing an understanding of number and principles of quantity (e.g., hold up 5 fingers at once to show 5, locate things in 2s without counting; using number words to indicate small exact numbers or relative change in quantity - more, small) **K.CC-4b, 4c** **K.OA-4**	**K.CC-4b, 4c** **4.** Understand the relationship between numbers and quantities; connect counting to cardinality. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger. **K.OA-4** **4.** For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*

E.NO.1c developing number line skills (linear representations) using 0 to 20, and later 0 to 100	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*
E.NO.1d identifying numbers (names, symbols, quantity) and the count sequence **K.CC-1, 2, 4**	**K.CC-1, 2, 4** **1.** Count to 100 by ones and by tens. **2.** Count forward beginning from a given number within the known sequence (instead of having to begin at 1). **4.** Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger.	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*
E.NO.1e reading and writing numbers; counting and estimating (e.g., how many?; skip counting by 2s, 5s, 10s; even/odd) **K.CC-1, 2, 3, 4, 5** **1.NBT-1** **2.NBT-2, 3; 2.OA-3**	**K.CC-1, 2, 3, 4, 5** **1.** Count to 100 by ones and by tens. **2.** Count forward beginning from a given number within the known sequence (instead of having to begin at 1). **3.** Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). **4.** Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger.	**1.NBT-1** **1.** Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.	**2.NBT-2 , 3** **2.** Count within 1000; skip-count by 5s, 10s, and 100s. **3.** Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. **2.OA-3** **3.** Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

	5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.		
E.NO.1f representing, ordering, and comparing whole numbers **K.CC-6, 7** **1.NBT-1, 3** **2.NBT-4**	**K.CC-6, 7** **6.** Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1] [1] Include groups with up to ten objects. **7.** Compare two numbers between 1 and 10 presented as written numerals.	**1.NBT-1, 3** **1.** Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. **3.** Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.	**2.NBT-4** **4.** Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.
E.NO.1g recognizing that numbers can be divided (represented as fractions	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*
E.NO.1h applying place value understanding to compare and order numbers, express number relationships (<, >, =), and express numbers in expanded form **1.NBT-2c, 3** **2.NBT-1,4**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.* *The concepts of greater than, less than, and equals are more important to understand at this time than the use of the symbols.*	**1.NBT-2c, 3** **2.** Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). **3.** Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.	**2.NBT-1, 4** **1.** Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens — called a "hundred." b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). **4.** Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.
E.NO.1i recognizing zero as an additive identify, origin for the number line, and representing no units as a quantity or in place value **K.CC-3** **1.NBT-2** **2.NBT-1**	**K.CC-3** **3.** Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).	**1.NBT-2** **2.** Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones — called a "ten." b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70,	**2.NBT-1** **1.** Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens — called a "hundred." b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine

		80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (& 0 ones).	hundreds (and 0 tens and 0 ones).

Elementary (K-4) School Learning Targets, Progress Indicators, & Common Core Standards

The Nature of Numbers and Operations (NO): Understandings of number - "how many" or "how much" – and number types extend applications of arithmetic properties, operations, and number systems and guide the use of computational strategies and algorithms.

E.NO-2 *Build an understanding of computational strategies and algorithms: Fluently add, subtract, multiply, divide, and estimate; Perform and represent operations with whole numbers, fractions, and mixed numbers; Identify multiples and factors of whole numbers.*

Progress Indicators for Grades K-2	Grade K CCSS standards	Grade 1 CCSS standards	Grade 2 CCSS standards
E.NO.2a representing addition and subtraction in multiple ways (composing/ decomposing numbers, diagrams, using objects, arrays, equations, number lines), including regrouping **K.OA-1, 2, 3, 4** **K.NBT-1** **1.OA-1, 2, 5, 6** **1.NBT-4, 5, 6** **2.OA-1, 4** **2.NBT-7**	**K.OA-1, 2, 3, 4** **1.** Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. [1] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.) **2.** Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. **3.** Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). **4.** For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. **K.NBT-1** **1.** Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three,	**1.OA-1, 2, 5, 6** **1.** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[1] [1] See Glossary, Table 1. **2.** Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. **5.** Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). **6.** Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). **1.NBT-4, 5, 6** **4.** Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the	**2.OA-1, 4** **1.** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1] [1] See Glossary, Table 1. **4.** Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. **2.NBT-7** **7.** Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

	four, five, six, seven, eight, or nine ones.	relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. **5.** Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. **6.** Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	
E.NO.2b explaining or modeling the relationship between addition and subtraction **1.OA-3, 4** **1.NBT-4, 6** **2.NBT-5, 7, 9**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.* *Understanding that there is a difference between 'joining together' and 'taking apart' is a precursor to understanding the relationship between addition and subtraction.*	**1.OA-3, 4** **3.** Apply properties of operations as strategies to add and subtract.[2] *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)* **4.** Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8. Add and subtract within 20.* **1.NBT-4, 6** **4.** Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. **6.** Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a	**2.NBT-5, 7, 9** **5.** Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. **7.** Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. **9.** Explain why addition and subtraction strategies work, using place value and the properties of operations.[1] [1] Explanations may be supported by drawings or objects.

		written method and explain the reasoning used.	
E.NO.2c working flexibly with common addition and subtraction situations **K.OA-2** **1. OA-3, 5, 6, 8** **1.NBT-5, 6** **2.OA-1, 2** **2.NBT- 2, 5, 7**	**K.OA-2** **2.** Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.	**1. OA-3, 5, 6, 8** **3.** Apply properties of operations as strategies to add and subtract.[2] *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)* **5.** Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). **6.** Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). **8.** Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = _ – 3, 6 + 6 = _.* **1.NBT-5, 6** **5.** Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. **6.** Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	**2.OA-1, 2** **1.** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1] [1] See Glossary, Table 1. 2. Fluently add and subtract within 20 using mental strategies.2 By end of Grade 2, know from memory all sums of two one-digit numbers. **2.NBT- 2, 5, 7** **2.** Count within 1000; skip-count by 5s, 10s, and 100s. **5.** Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. **7.** Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

The Nature of Numbers and Operations (NO): Understandings of number - "how many" or "how much" – and number types extend applications of arithmetic properties, operations, and number systems and guide the use of computational strategies and algorithms.
E.NO-3 *Use reasoning to support solutions and informal arguments and to develop metacognitive skills: Use estimation and rounding to support informal arguments; Develop both additive and multiplicative thinking; Test, model, and explain solutions.*

Progress Indicators for Grades K-2	Grade K CCSS standards	Grade 1 CCSS standards	Grade 2 CCSS standards
E.NO.3a exploring and explaining answers to questions, such as "Does this answer make sense?" **K.CC-4, 5, 6** **KOA-1** **1.OA-1, 2, 7, 8** **1.G-3** **2.OA-1** **2.NBT-7, 9**	**K.CC-4, 5, 6** **4.** Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger. **5.** Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects. **6.** Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1] [1] Include groups with up to ten objects. **KOA-1** **1.** Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. [1] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)	**1.OA-1, 2, 7, 8** **1.** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[1] [1] See Glossary, Table 1. **2.** Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. **7.** Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2. **8.** Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = _ – 3, 6 + 6 = _.* **1.G-3** **3.** Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.	**2.OA-1** **1.** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1] [1] See Glossary, Table 1. **2.NBT-7, 9** **7.** Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. **9.** Explain why addition and subtraction strategies work, using place value and the properties of operations.[1] [1] Explanations may be supported by drawings or objects.

E.NO.3b constructing arguments using concrete referents such as objects, diagrams, tables, actions (e.g., clapping, movement) and estimating **K.OA-1, 2** **K.G-5** **1.OA-1, 2, 7, 8** **1.G-3** **1.MD-4** **2.OA-1** **2.NBT-7, 9** **2.MD-3** **2.G-3**	**K.OA-1, 2** **1.** Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. [1] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.) **2.** Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. **K.G-5** 5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.	**1.OA-1, 2, 7, 8** **1.** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[1] [1] See Glossary, Table 1. **2.** Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. **7.** Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2. **8.** Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = _ – 3, 6 + 6 = _.* **1.G-3** **3.** Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares. **1.MD-4** **4.** Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	**2.OA-1** **1.** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1] [1] See Glossary, Table 1. **2.NBT-7, 9** **7.** Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. **9.** Explain why addition and subtraction strategies work, using place value and the properties of operations.[1] [1] Explanations may be supported by drawings or objects. **2.MD-3** **3.** Estimate lengths using units of inches, feet, centimeters, and meters. **2.G-3** **3.** Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Elementary (K-4) School Learning Targets, Progress Indicators, & Common Core Standards **Measurement (ME)**: Measurement attributes, processes, and tools help us quantify, compare, and solve problems involving objects, situations, and events. ***E.ME-1*** *Explore relationships among units, attributes, and measures within a system of measurement: Identify measurement attributes and units; Use measurement attributes to describe and compare objects, situations, or events.*			
Progress Indicators for Grades K-2	**Grade K CCSS standards**	**Grade 1 CCSS standards**	**Grade 2 CCSS standards**
E.ME.1a recognizing, identifying, and describing the measurable attributes of objects **K.MD-1**(length/height, weight) **1.MD-2** (l/h), **3** (time) **2.MD-1** (l/h/w), **7** (time), **8** (money)	**K.MD-1** **1.** Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.	**1.MD-2, 3** **2.** Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.* **3.** Tell and write time in hours and half-hours using analog and digital clocks.	**2.MD-1 , 7, 8** **1.** Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. **7.** Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. **8.** Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?
E.ME.1b comparing and ordering objects/events according to their specified attributes (using standard or non-standard units of measure), including indirectly by using a third object, or using common referents to estimate or compare **K.MD-2, 3** **1.MD-1** **2.MD-3, 4**	**K.MD-2, 3** **2.** Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.* **3.** Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[1] [1] Limit category counts to be less than or equal to 10.	**1.MD-1** **1.** Order three objects by length; compare the lengths of two objects indirectly by using a third object.	**2.MD-3, 4** **3.** Estimate lengths using units of inches, feet, centimeters, and meters. **4.** Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.
E.ME.1c recognizing that the smaller the unit, the more units are needed to measure an object ; and that units can be decomposed/ partitioned into smaller units **1.MD-2** **2.MD-2, 3**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.* *Using a variety of non-standard tools/units to measure the length of the same object is a precursor to understanding the concept described in this progress Indicator.*	**1.MD-2** **2.** Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*	**2.MD-2, 3** **2.** Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. **3.** Estimate lengths using units of inches, feet, centimeters, and meters.

Elementary (K-4) School Learning Targets, Progress Indicators, & Common Core Standards **Measurement (ME)**: Measurement attributes, processes, and tools help us quantify, compare, and solve problems involving objects, situations, and events. ***E.ME-2*** *Apply appropriate techniques (iteration and tiling), tools (standard and non-standard), and formulas (area and perimeter) to determine or estimate measurements.*			
Progress Indicators for Grades K-2	**Grade K CCSS standards**	**Grade 1 CCSS standards**	**Grade 2 CCSS standards**
E.ME.2a applying nonstandard and common standard units to measure (length, height, weight, time) **1.MD-2, 3** **2.MD-1, 2, 3, 4, 7**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.* *Using a variety of non-standard tools/units to measure length, height, or weight is a precursor to understanding the concept described in this progress indicator.*	**1.MD-2, 3** **2.** Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.* **3.** Tell and write time in hours and half-hours using analog and digital clocks.	**2.MD-1, 2, 3, 4, 7** **1.** Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. **2.** Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. **3.** Estimate lengths using units of inches, feet, centimeters, and meters. **4.** Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. **7.** Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
E.ME.2b selecting tools and using units of measures appropriately and consistently, with no gaps or overlaps in the technique of measuring **1.MD-2, 3** **2.MD-1, 3, 7**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	**1.MD-2, 3** **2.** Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.* **3.** Tell and write time in hours and half-hours using analog and digital clocks.	**2.MD-1, 3, 7** **1.** Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. **3.** Estimate lengths using units of inches, feet, centimeters, and meters. **7.** Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
E.ME.2c recognizing situations that require precision and those where an estimation or proportional matching is appropriate **2.MD-1, 3**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.* *Using a variety of non-standard tools/units to measure the length or height of the same object is a precursor to understanding the concept described in this progress indicator.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	**2.MD-1, 3** **1.** Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. **3.** Estimate lengths using units of inches, feet, centimeters, and meters.
E.ME.2d describing a unit as an amount (rather than as an object or a mark on a scale)	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*

Elementary (K-4) School Learning Targets, Progress Indicators, & Common Core Standards

Patterns, Relations, and Functions (PRF): Patterns, relations, and functions are used to represent and analyze change in various contexts, make predictions and generalizations, and provide models and explanations for real-world phenomena.

E.PRF-1 *Use concrete, pictorial, and symbolic representations to identify, describe, compare, and model situations that involve change.*

Progress Indicators for Grades K-2	Grade K CCSS standards	Grade 1 CCSS standards	Grade 2 CCSS standards
E.PRF.1a describing changes qualitatively (e.g., growing taller) and quantitatively (e.g., growing 2 inches in one year) **K.CC-6** **K.MD-2** **1.OA-1, 5** **1.NBT-3** **1.MD-1** **2.NBT-4**	**K.CC-6** **6.** Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1] ([1] Include groups with up to ten objects.) **K.MD-2** **2.** Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*	**1.OA-1, 5** **1.** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[1] ([1] See Glossary, Table 1.) **5.** Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). **1.NBT-3** **3.** Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <. **1.MD-1, 2** **1.** Order three objects by length; compare the lengths of two objects indirectly by using a third object.	**2.NBT-4** **4.** Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.
E.PRF.1b exploring and describing how addition or subtraction changes a quantity **K.OA-1, 2** **1.OA-1, 5** **1.NBT-3**	**K.OA-1, 2** **1.** Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. ([1] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)) **2.** Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.	**1.OA-1, 5** **1.** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[1] ([1] See Glossary, Table 1.) **5.** Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). **1.NBT-3** **3.** Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =,	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*

		and <.	
E.PRF.1c modeling problem-solving situations that involve addition and subtraction of whole numbers using objects, diagrams, and symbols **K.OA-1, 2** **1.OA-1, 5** **2.OA-1, 3, 4** **2.NBT-7** **2.MD-5**	**K.OA-1, 2** **1.** Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. ([1] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)) **2.** Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.	**1.OA-1, 5** **1.** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[1] ([1] See Glossary, Table 1.) **5.** Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).	**2.OA-1, 3, 4** **1.** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1] ([1] See Glossary, Table 1.) **3.** Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends. **4.** Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. **2.NBT-7** **7.** Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. **2.MD-5** **5.** Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

Elementary (K-4) School Learning Targets, Progress Indicators, & Common Core Standards

Patterns, Relations, and Functions (PRF): Patterns, relations, and functions are used to represent and analyze change in various contexts, make predictions and generalizations, and provide models and explanations for real-world phenomena.

E.PRF-2 *Give examples, interpret, and analyze repeating and growing patterns and functions involving the four basic operations.*

Progress Indicators for Grades K-2	Grade K CCSS standards	Grade 1 CCSS standards	Grade 2 CCSS standards
E.PRF.2a recognizing, describing, and extending simple repeating (ABAB) and growing (A+1, A+2, A+3) patterns (e.g., colors, sounds, words, shapes, numeric – counting, odd, even) **K.CC-6** **K.OA-1, 4** **K.G-2** **1.OA-1, 5** **1.NBT-2b, 2c, 5, 6** **2.OA-3, 4** **2.NBT-1b, 2**	**K.CC-6** **6.** Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1] ([1] Include groups with up to ten objects.) **K.OA-1, 4** **1.** Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. ([1] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)) **4.** For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. **K.G-2** **2.** Correctly name shapes regardless of their orientations or overall size.	**1.OA-1, 5** **1.** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[1] ([1] See Glossary, Table 1.) **5.** Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). **1.NBT-2b, 2c, 5, 6** **2.** Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). **5.** Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. **6.** Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	**2.OA-3, 4** **3.** Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends. **4.** Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. **2.NBT-1b, 2** **1.** Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). **2.** Count within 1000; skip-count by 5s, 10s, and 100s.
E.PRF.2b creating and explaining repeating and growing patterns using objects or numbers **K.OA-1, 4** **K.NBT-1** **K.G-2**	**K.OA-1, 4** **1.** Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. ([1] Drawings need not show details, but should show the mathematics in the problem. (This	**1.OA-1, 5** **1.** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[1]	**2.OA-3, 4** **3.** Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends. **4.** Use addition to find the total number of objects

1.OA-1, 5 **1.NBT-2, 5, 6** **2.OA-3, 4** **2.NBT-1, 2**	applies wherever drawings are mentioned in the Standards.)) **4.** For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. **K.NBT-1** **1.** Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. **K.G-2** **2.** Correctly name shapes regardless of their orientations or overall size.	[1] See Glossary, Table 1.) **5.** Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). **1.NBT-2, 5, 6** **2.** Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones — called a "ten." b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (& 0 ones). **5.** Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. **6.** Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. **2.NBT-1, 2** **1.** Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens — called a "hundred." b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). **2.** Count within 1000; skip-count by 5s, 10s, and 100s.
E.PRF.2c extending and analyzing simple numeric patterns with rules that involve addition and subtraction **K.OA-1, 4** **K.NBT-1** **1.OA-1, 5** **1.NBT-2, 5, 6** **2.OA-3, 4** **2.NBT-1, 2, 8**	**K.OA-1, 4** **1.** Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. [1] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)) **4.** For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. **K.NBT-1** **1.** Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that	**1.OA-1, 5** **1.** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[1] [1] See Glossary, Table 1.) **5.** Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). **1.NBT-2, 5, 6** **2.** Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones — called a "ten." b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.	**2.OA-3, 4** **3.** Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends. **4.** Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. **2.NBT-1, 2** **1.** Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens — called a "hundred." b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven,

	these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). **5.** Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. **6.** Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	eight, or nine hundreds (and 0 tens and 0 ones). **2.** Count within 1000; skip-count by 5s, 10s, and 100s. **2.NBT-8** **8.** Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

Elementary (K-4) School Learning Targets, Progress Indicators, & Common Core Standards

Geometry (GM): Visualizations, spatial reasoning, and properties of 2- and 3-dimensional figures can be used to analyze, represent, and model geometric concepts and relationships.

E.GM-1 *Recognize that two- and three-dimensional shapes have particular attributes: Describe and compare objects and figures based on geometric attributes; Compose, decompose, and draw figures based on spatial reasoning and the properties and attributes of the shapes; Apply concepts of symmetry.*

Progress Indicators for Grades K-2	Grade K CCSS standards	Grade 1 CCSS standards	Grade 2 CCSS standards
E.GM.1a recognizing, describing (using spatial language) and naming shapes regardless of orientation or size and locating shapes in the environment **K.G-1, 2**	**K.G-1, 2** **1.** Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*. **2.** Correctly name shapes regardless of their orientations or overall size.	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*
E.GM.1b analyzing and comparing 2- (and later) 3- dimensional shapes using informal language (e.g., flat, solid, corners) to describe their differences and similarities, as well as their component parts (number of sides, vertices) and other attributes (e.g., sides of equal length) **K.G-3, 4**	**K.G-3, 4** **3.** Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid"). **4.** Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*
E.GM.1c composing two-dimensional shapes (rectangles, squares, triangles, half-circles, and quarter-circles) **K.G-6** **1.G-2**	**K.G-6** **6.** Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*	**1.G-2** **2.** Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[1] ([1] Students do not need to learn formal names such as "right rectangular prism.")	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*
E.GM.1d composing three dimensional shapes, using concrete models/ materials (cubes, prisms, cones, and cylinders) **1.G-2**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	**1.G-2** **2.** Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[1]	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*

		(1 Students do not need to learn formal names such as "right rectangular prism.")	
E.GM.1e drawing and identifying shapes with specific attributes (e.g., number of sides or equal angles) not determined by direct measuring) **2.G-1**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	**2.G-1** **1.** Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[1] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. ([1] Sizes are compared directly or visually, not compared by measuring.)
E.GM.1f partitioning shapes into 2, 3, or 4 equal parts and describing the parts (halves, quarters, fourths, thirds) **1.G-3** **2.G-2, 3**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	**1.G-3** **3.** Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.	**2.G-2, 3** **2.** Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. **3.** Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.
E.GM.1g using spatial language to describe and name more complex or atypical shapes based on their defining characteristics	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*

Elementary (K-4) School Learning Targets, Progress Indicators, & Common Core Standards

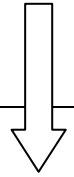
Data Analysis, Probability, and Statistics (DPS): Questions are posed and investigated by collecting data or retrieving existing data, and representing, analyzing, and interpreting data. Investigations, inferences, and predictions are used to make critical and informed decisions.

E.DPS-1 *Gather and interpret data to answer questions related to a particular/single context. Formulate questions, gather data, and build representations; Identify and describe variation in data, and describe and compare shapes of distributions and measures of central tendency.*

Progress Indicators for Grades K-2	Grade K CCSS standards	Grade 1 CCSS standards	Grade 2 CCSS standards
E.DPS.1a posing questions of interest that can be answered by counting or collecting data (e.g., concrete comparisons about students, classroom materials, science topics) *with teacher guidance* **K.CC-5, 6** **1-MD-1** **2.MD-2, 5, 9**	**K.CC-5, 6** **5.** Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects. **6.** Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1] ([1] Include groups with up to ten objects.)	**1-MD-1** **1.** Order three objects by length; compare the lengths of two objects indirectly by using a third object.	**2.MD-2, 5, 9** **2.** Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. **5.** Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. **9.** Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
E.DPS.1b identifying and sorting data/attributes; identifying rules for classifying data/attributes **K.MD-1, 2, 3** **K.G-2, 4** **1.MD-1, 4** **1.G-1** **2.G-1** **2.MD-10**	**K.MD-1, 2, 3** **1.** Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. **2.** Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.* **3.** Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[1] ([1] Limit category counts to be less than or equal to 10.) **K.G-2, 4** **2.** Correctly name shapes regardless of their orientations or overall size. **4.** Analyze and compare two- and three-	**1.MD-1, 4** **1.** Order three objects by length; compare the lengths of two objects indirectly by using a third object. **4.** Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. **1.G-1** **1.** Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.	**2.G-1** **1.** Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[1] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. **2.MD-10** **10.** Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems[1] using information presented in a bar graph.

	dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).		
E.DPS.1c collecting and organizing/ representing data (e.g., picture graphs, tally charts, bar graphs) **K.CC-5** **K.MD-1, 2, 3** **1-MD-4** **2.MD-1, 2, 4, 5, 6, 9, 10**	**K.CC-5** **5.** Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects. **K.MD-1, 2, 3** **1.** Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. **2.** Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.* **3.** Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[1] ([1] Limit category counts to be less than or equal to 10.)	**1-MD-4** **4.** Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	**2.MD-1, 2, 4, 5, 6, 9, 10** **1.** Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. **2.** Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. **4.** Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. **5.** Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. **6.** Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram. **9.** Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. **10.** Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems[1] using information presented in a bar graph.
E.DPS.1d recognizing that data can take on different values **1-MD-4** **2.MD-3, 9, 10**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	**1-MD-4** **4.** Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	**2.MD-3, 9, 10** **3.** Estimate lengths using units of inches, feet, centimeters, and meters. **9.** Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot,

			where the horizontal scale is marked off in whole-number units. **10.** Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems[1] using information presented in a bar graph.
E.DPS.1e describing and comparing data and beginning to identify what the data do or do not show (e.g., bar graphs, line plots, picture graphs) **1-MD-4** **2.MD-2, 4, 5, 6, 9, 10**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	**1-MD-4** **4.** Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	**2.MD-2, 4, 5, 6, 9, 10** **2.** Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. **4.** Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. **5.** Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. **6.** Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram. **9.** Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. **10.** Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems[1] using information presented in a bar graph.

Elementary (K-4) School Learning Targets, Progress Indicators, & Common Core Standards **Data Analysis, Probability, and Statistics (DPS):** Questions are posed and investigated by collecting data or retrieving existing data, and representing, analyzing, and interpreting data. Investigations, inferences, and predictions are used to make critical and informed decisions. ***E.DPS-2*** *Conduct simple probability experiments and characterize the outcomes in words, diagrams, or numerically.*			
Progress Indicators for Grades K-2	**Grade K CCSS standards**	**Grade 1 CCSS standards**	**Grade 2 CCSS standards**
E.DPS.2a describing the probability of events as being possible or impossible	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, these skills/concepts represent the "hypothesized" learning continuum.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, these skills/concepts represent the "hypothesized" learning continuum.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, these skills/concepts represent the "hypothesized" learning continuum.*
E.DPS.2b describing the probability of events as being certain, likely, unlikely, or impossible			
E.DPS.2c representing all possible outcomes for expectations of varied results (e.g., using words, drawings, tree diagrams to show all different combinations for making sandwiches from a choice of ingredients)			

Expanded Learning Progressions Frameworks for K-12 Mathematics

Elementary School Strands

Grades 3 – 4

Elementary (K-4) School Learning Targets, Progress Indicators, & Common Core Standards

Symbolic Expression (SE): The use and manipulation of symbols and expressions provide a variety of representations for solving problems and expressing mathematical concepts, relationships, and reasoning.

E.SE-1 *Use equations and expressions involving basic operations to represent a given context: Represent numerical relationships using combinations of symbols (=, >, <) and numbers to form expressions and equations; Solve for unknown in simple number binary number sentences (e.g., ____ + 4 = 7); Write equations showing inverse operations and related operations (e.g., addition-multiplication).*

Progress Indicators for Grades 3-4	Grade 3 CCSS standards	Grade 4 CCSS standards
E.SE.1f extending finding unknown numbers in equations using multiplication and division, including using letters for unknown quantities **3.OA-4, 8** **4.OA-3**	**3.OA-4, 8** **4.** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?* **8.** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[3] [3] This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order.)	**4.OA-3** 3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
E.SE.1g using symbols (=, >, <) to compare whole numbers, fractions, or decimals; write equations; and express inverse or related operations **3.NF-3d** **4.OA-1** **4.NF-2, 7**	**3.NF-3d** **3.** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	**4.OA-1** **1.** Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. **4.NF-2, 7** **2.** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. **7.** Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.
E.SE.1h expressing whole numbers as fractions, and fractions as equivalent decimals; recognizing that a fraction is one number, not two **3.NF-3c**	**3.NF-3c** **3.** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the*	**4.NF-6** **6.** Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

4.NF-6	*same point of a number line diagram.*	

Elementary (K-4) School Learning Targets, Progress Indicators, & Common Core Standards **The Nature of Numbers and Operations (NO):** Understandings of number - "how many" or "how much" – and number types extend applications of arithmetic properties, operations, and number systems and guide the use of computational strategies and algorithms. ***E.NO-1*** *Build flexibility using whole numbers, fractions, and decimals to understand the nature of number and number systems: Count, model, and estimate quantities; Compare, represent, and order numbers; Apply place value concepts and expanded notation to compose and decompose whole numbers.*		
Progress Indicators for Grades 3-4	**Grade 3 CCSS standards**	**Grade 4 CCSS standards**
E.NO.1j applying place value concepts to: read, write, and compare whole numbers up to 100,000; use expanded form; and round numbers to a given place **3.NBT-1** **4.NBT-2, 3**	**3.NBT-1** **1.** Use place value understanding to round whole numbers to the nearest 10 or 100.	**4.NBT-2, 3** **2.** Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **3.** Use place value understanding to round multi-digit whole numbers to any place.
E.NO.1k explaining the meaning of place value (that a digit in one place represents 10 times what it represents in the place to its right) **4.NBT-1**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	**4.NBT-1** **1.** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*
E.NO.1l identifying and locating fractions on the number line or as regions, or parts of a set or unit, and recognizing that whole numbers are a subset of rational numbers **3.NF-1, 2, 3a, 3c**	**3.NF-1, 2, 3a, 3c** **1.** Understand a fraction 1/*b* as the quantity formed by 1 part when *a* whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by a parts of size 1/*b*. **2.** Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. b. Represent a fraction *a*/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line. **3.** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. c. Express whole numbers as fractions, and recognize fractions that are	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*

	equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*	
E.NO.1m composing and representing equivalent fractions in the form a/b **3.NF-1, 2, 3a, 3b** **4.NF-1, 2**	**3.NF-1, 2, 3a, 3b** **1.** Understand a fraction 1/*b* as the quantity formed by 1 part when *a* whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by a parts of size 1/*b*. **2.** Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. b. Represent a fraction *a*/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line. **3.** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.	**4.NF-1, 2** **1.** Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. **2.** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
E.NO.1n comparing and modeling fractions, including with different denominators **3.NF-2, 3a, 3b, 3c, 3d** **4.NF-1, 2**	**3.NF-2, 3a, 3b, 3c, 3d** **2.** Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. b. Represent a fraction *a*/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line. **3.** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.* d. Compare two fractions with the same numerator or the same	**4.NF-1, 2** **1.** Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. **2.** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

	denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	
E.NO.1o rewriting fractions as equivalent decimals **4.NF-5, 6**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	**4.NF-5, 6** **5.** Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[2] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* **6.** Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*
E.NO.1p using number words to indicate decimal values (tenths, hundredths	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.* *Working with, writing, and comparing money values is a precursor to understanding this Progress Indicator.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*
E.NO.1q using and comparing decimals to the hundredths **4.NF-7**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level* *Working with, writing, and comparing money values is a precursor to understanding this Progress Indicator.*	**4.NF-7** **7.** Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

Elementary (K-4) School Learning Targets, Progress Indicators, & Common Core Standards

The Nature of Numbers and Operations (NO): Understandings of number - "how many" or "how much" – and number types extend applications of arithmetic properties, operations, and number systems and guide the use of computational strategies and algorithms.

E.NO-2 *Build an understanding of computational strategies and algorithms: Fluently add, subtract, multiply, divide, and estimate; Perform and represent operations with whole numbers, fractions, and mixed numbers; Identify multiples and factors of whole numbers.*

Progress Indicators for Grades 3-4	Grade 3 CCSS standards	Grade 4 CCSS standards
E.NO.2d modeling multiplication (equal-sized groups, arrays, area models, equal-sized jumps on number lines, multiplicative comparisons) and division (successive subtraction, partitioning, sharing) of whole numbers **3.OA-1, 2, 3, 4, 5** **4.OA-1, 2, 3** **4.NBT-5, 6**	**3.OA-1, 2, 3, 4, 5** **1.** Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.* **2.** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.* **3.** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1] [1] See Glossary, Table 2. **4.** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?* **5.** Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*	**4.OA-1, 2, 3** **1.** Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. **2.** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1] [1] See Glossary, Table 2. **3.** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **4.NBT- 5, 6** **5.** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **6.** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
E.NO.2e describing relationships between addition-multiplication; multiplication-division; addition-subtraction; why commutativity does not apply to subtraction or division **3.OA-5, 7, 9** **3.NBT-2** **4.OA-2**	**3.OA-5, 7, 9** **5.** Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* **7.** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 ×	**4.OA-2** **2.** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1] [1] See Glossary, Table 2.

	5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers **9.** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.* **3.NBT-2** **2.** Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	
E.NO.2f identifying factors and multiples of numbers **3.OA-6** **4.OA-4**	**3.OA-6** **6.** Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*	**4.OA-4** **4.** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.
E.NO.2g recognizing fractions as one number (one quantity), rather than two numbers (numerator and denominator) and using number lines to represent magnitude of fractions **3.NF-1, 2, 3a, 3c**	**3.NF-1, 2, 3a, c** **1.** Understand a fraction 1/*b* as the quantity formed by 1 part when *a* whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by a parts of size 1/*b*. **2.** Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. b. Represent a fraction *a*/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line. **3.** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*
E.NO.2h adding, subtracting, and multiplying fractions, including mixed numbers **4-NF-3, 4**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	**4-NF-3, 4** **3.** Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same

		denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. **4.** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction *a/b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

Elementary (K-4) School Learning Targets, Progress Indicators, & Common Core Standards

The Nature of Numbers and Operations (NO): Understandings of number - "how many" or "how much" – and number types extend applications of arithmetic properties, operations, and number systems and guide the use of computational strategies and algorithms.

E.NO-3 *Use reasoning to support solutions and informal arguments and to develop metacognitive skills: Use estimation and rounding to support informal arguments; Develop both additive and multiplicative thinking; Test, model, and explain solutions.*

Progress Indicators for Grades 3-4	Grade 3 CCSS standards	Grade 4 CCSS standards
E.NO.3c evaluating the reasonableness of answers using mental computation, arithmetic patterns, and estimation strategies, including rounding to the nearest 10 or 100 **3.OA-5, 8, 9** **3.NBT-1, 2, 3** **3.MD-2** **4.OA-3, 4, 5** **4.NBT-3**	**3.OA-5, 8, 9** **5.** Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* **8.** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[3] [3] This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order. **9.** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.* **3.NBT-1, 2, 3** **1.** Use place value understanding to round whole numbers to the nearest 10 or 100. **2.** Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. **3.** Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations. **3.MD-2** **2.** Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[1] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to	**4.OA-3, 4, 5** **3.** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **4.** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. **5.** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.* **4.NBT-3** **3.** Use place value understanding to round multi-digit whole numbers to any place.

	represent the problem.[2] [2] Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Glossary, Table 2).	
E.NO.3d constructing arguments and explaining reasonableness of outcomes using a variety of concrete supports (e.g., models, diagrams, tables) **3.OA-8, 9** **3.MD-1, 3, 4, 7b, 7c, 7d, 8** **4.OA-3, 5** **4.NBT- 5, 6** **4.NF-1, 2, 3b, 3d, 4a, 6, 7** **4.MD-4**	**3.OA-8, 9** **8.** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[3] [3] This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order. **9.** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.* **3.MD-1, 3, 4, 7b, 7c, 7d, 8** **1.** Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. **3.** Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.* **4.** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters. **7.** Relate area to the operations of multiplication and addition. b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.	**4.OA-3, 5** **3.** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **5.** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.* **4.NBT- 5, 6** **5.** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **6.** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **4.NF-1, 2, 3b, 3d, 4a, 6, 7** **1.** Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. **2.** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. **3.** Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using

	8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	visual fraction models and equations to represent the problem. **4.** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* **6.** Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* **7.** Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model. **4.MD-4** **4.** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

Elementary (K-4) School Learning Targets, Progress Indicators, & Common Core Standards

Measurement (ME): Measurement attributes, processes, and tools help us quantify, compare, and solve problems involving objects, situations, and events.

E.ME-1 *Explore relationships among units, attributes, and measures within a system of measurement: Identify measurement attributes and units; Use measurement attributes to describe and compare objects, situations, or events.*

Progress Indicators for Grades 3-4	Grade 3 CCSS standards	Grade 4 CCSS standards
E.ME.1d describing and demonstrating: unit attributes, iterating, tiling, identical units, number line intervals, standardization, proportionality, additivity, and origin **3.MD-1, 4, 5, 6, 7a, 7c, 7d** **4.MD-7**	**3.MD-1, 4, 5, 6, 7a, 7c, 7d** **1.** Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. **4.** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. **5.** Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. **6.** Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). **7.** Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* × *b* and *a* × *c*. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.	**4.MD-7** **7.** Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.
E.ME.1e justifying the need for measuring with standard units as compared to non-standard Units	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*
E.ME.1f selecting the appropriate unit for measuring a given attribute (length, area, mass, liquid volume, size of angle), recognizing that a unit must have the	**3.MD-2,5** **2.** Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[1] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such	**4.MD-1, 2, 6** **1.** Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column

same attributes as the object (e.g., unit of length must measure an object that has length) **3.MD-2,5** **4.MD-1, 2, 6**	as a beaker with a measurement scale) to represent the problem.[2] [1] Excludes compound units such as cm3 and finding the geometric volume of a container.[2] Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Glossary, Table 2). **5.** Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.	table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ..* **2.** Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. **6.** Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
E.ME.1g exploring what happens to 2-dimensional measurements (perimeter or area) when the dimensions of the figure are changed **3.MD-7, 8** **4.MD-3**	**3.MD-7, 8** **7.** Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* × *b* and *a* × *c*. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. **8.** Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	**4.MD-3** **3.** Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

Elementary (K-4) School Learning Targets, Progress Indicators, & Common Core Standards **Measurement (ME):** Measurement attributes, processes, and tools help us quantify, compare, and solve problems involving objects, situations, and events. ***E.ME-2*** *Apply appropriate techniques (iteration and tiling), tools (standard and non-standard), and formulas (area and perimeter) to determine or estimate measurements.*		
Progress Indicators for Grades 3-4	**Grade 3 CCSS standards**	**Grade 4 CCSS standards**
E.ME.2e selecting and applying appropriate customary or metric units and tools to measure or estimate (liquid volume, mass, perimeter, area, time, and angles) **3.MD-1, 2, 4, 6, 7a, 8** **4.MD-5a, 6**	**3.MD-1, 2, 4, 6, 7a, 8** **1.** Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. **2.** Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[1] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[2] [2] Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Glossary, Table 2). **4.** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters **6.** Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). **7.** Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. **8.** Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	**4.MD-5a, 6** **5.** Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. **6.** Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
E.ME.2f recognizing relative sizes of units of measure and making simple conversions within systems when solving problems (e.g., 12 in.= 1 ft) **4.MD-1**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	**4.MD-1** **1**. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*
E.ME.2g recognizing situations that require precision (money, time, distances, fractions, decimals) and those where an	**3.MD-2** **2.** Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[1] Add, subtract, multiply, or divide to solve one-step word problems involving masses or	**4.MD-2** **2.** Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems

estimation is appropriate **3.MD.2** **4.MD-2**	volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[2] [2] Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Glossary, Table 2).	that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
E.ME.2h using a variety of strategies (decomposing complex shapes, using counting strategies, arrays, formulas) to estimate or measure area and perimeter (including irregular shapes/objects) **4.MD-3**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	**4.MD-3** **3.** Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*
E.ME.2i selecting and using benchmarks to estimate measurements **3.MD-2**	**3.MD-2** **2.** Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[1] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[2] [2] Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Glossary, Table 2).	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*

Elementary (K-4) School Learning Targets, Progress Indicators, & Common Core Standards

Patterns, Relations, and Functions (PRF): Patterns, relations, and functions are used to represent and analyze change in various contexts, make predictions and generalizations, and provide models and explanations for real-world phenomena.

E.PRF-1 *Use concrete, pictorial, and symbolic representations to identify, describe, compare, and model situations that involve change.*

Progress Indicators for Grades 3-4	Grade 3 CCSS standards	Grade 4 CCSS standards
E.PRF.1d describing and modeling how addition, subtraction, multiplication, or division changes a quantity, including with fractions **3.OA-1, 2, 7** **4.OA-1** **4.NBT-1** **4.NF-3, 4**	**3.OA-1, 2, 7** **1.** Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.* **2.** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.* **7.** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.	**4.OA-1** **1.** Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. **4.NBT-1** **1.** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* **4.NF-3, 4** **3.** Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. **4.** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed?*

		Between what two whole numbers does your answer lie?
E.PRF.1e using representations (tables, graphs, equations) to show how values of one quantity are related to values of another and to draw conclusions **3.OA-9** **3.MD-3, 4** **4.OA- 2**	**3.OA-9** **9.** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.* **3.MD-3, 4** **3.** Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.* **4.** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.	**4.OA-2** **2.** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1] ([1] See Glossary, Table 2.)
E.PRF.1f representing and explaining equivalence concretely, graphically, and symbolically (equations, rules) **3.MD-1, 7c** **4.OA-5** **4.NBT-5, 6** **4.MD-4**	**3.MD-1, 7c** **1.** Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. **7.** Relate area to the operations of multiplication and addition. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.	**4.OA-5** **5.** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.* **4.NBT-5, 6** **5.** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **6.** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **4.MD-4** **4.** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*
E.PRF.1g identifying situations with constant or varying rates of change (with two quantities)	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*

Elementary (K-4) School Learning Targets, Progress Indicators, & Common Core Standards **Patterns, Relations, and Functions (PRF):** Patterns, relations, and functions are used to represent and analyze change in various contexts, make predictions and generalizations, and provide models and explanations for real-world phenomena. ***E.PRF-2*** *Give examples, interpret, and analyze repeating and growing patterns and functions involving the four basic operations.*		
Progress Indicators for Grades 3-4	**Grade 3 CCSS standards**	**Grade 4 CCSS standards**
E.PRF.2d representing and analyzing patterns and rules (e.g. doubling, adding 3) using words, tables, graphs, and models **3.OA-3, 5, 7, 9** **4.OA-4, 5**	**3.OA-3, 5, 7, 9** **3.** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1] ([1] See Glossary, Table 2.) **5.** Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* **7.** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. **9.** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*	**4.OA-4, 5** **4.** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. **5.** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*
E.PRF.2e extending, translating, and analyzing numeric patterns and their rules using addition, subtraction, multiplication, and division **3.OA-3, 5, 7, 9** **3.NBT-3** **4.OA-4, 5**	**3.OA-3,5,7,9** **3.** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1] ([1] See Glossary, Table 2.) **5.** Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* **7.** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.	**4.OA-4, 5** **4.** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. **5.** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

	9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.* **3.NBT-3** **3.** Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.	

Elementary School Learning Targets **Geometry (GM):** Visualizations, spatial reasoning, and properties of two- and three-dimensional figures can be used to analyze, represent, and model geometric concepts and relationships. ***E.GM-1*** *Recognizing that two- and three-dimensional shapes have particular attributes: Describe and compare objects and figures based on geometric attributes; Compose, decompose, and draw figures based on spatial reasoning and the properties and attributes of the shapes; Apply concepts of symmetry.*		
Progress Indicators for Grades 3-4	**Grade 3 CCSS standards**	**Grade 4 CCSS standards**
E.GM.1h describing, analyzing, comparing, and classifying two dimensional figures (triangles, quadrilaterals) using shared attributes **3.G-1** **4.G-2**	**3.G-1** **1.** Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.	**4.G-2** **2.** Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
E.GM.1i partitioning shapes into equal parts with equal areas and recognizing that each part is a unit fraction of the whole **3.G-2**	**3.G-2** **2.** Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*
E.GM.1j recognizing and drawing points, lines, line segments, rays, angles, and perpendicular and parallel lines and identifying these in plane figures **4.G-1**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	**4.G-1** **1.** Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
E.GM.1k recognizing and drawing lines of symmetry in a variety of figures **4.G-3**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	**4.G-3** **3.** Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

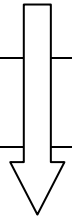
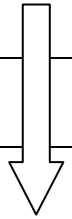
Elementary (K-4) School Learning Targets, Progress Indicators, & Common Core Standards

Data Analysis, Probability, and Statistics (DPS): Questions are posed and investigated by collecting data or retrieving existing data, and representing, analyzing, and interpreting data. Investigations, inferences, and predictions are used to make critical and informed decisions.

E.DPS-1 *Gather and interpret data to answer questions related to a particular/single context. Formulate questions, gather data, and build representations; Identify and describe variation in data, and describe and compare shapes of distributions and measures of central tendency.*

Progress Indicators for Grades 3-4	Grade 3 CCSS standards	Grade 4 CCSS standards
E.DPS.1f formulating questions and designing investigations (defining measures and variables) **3.MD-2** **4.MD-2**	**3.MD-2** 2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[1] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[2] [(1] Excludes compound units such as cm3 and finding the geometric volume of a container.[2] Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Glossary, Table 2).)	**4.MD-2** **2.** Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
E.DPS.1g collecting data and representing data (e.g., bar graphs, frequency tables, line plots) **3. MD-1, 2, 3, 4** **4.MD-1, 2, 4**	**3. MD-1, 2, 3, 4** **1.** Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. **2.** Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[1] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[2] [(1] Excludes compound units such as cm3 and finding the geometric volume of a container.[2] Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Glossary, Table 2).) **3.** Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.* **4.** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.	**4.MD-1, 2, 4** **1.** Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...* **2.** Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. **4.** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

E.DPS.1h recognizing and identifying sources of variability in the data (measurement variabiity and natural variability) **3.MD-2, 3, 4** **4.MD-2**	**3.MD-2, 3, 4** **2.** Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[1] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[2] ([1] Excludes compound units such as cm3 and finding the geometric volume of a container.[2] Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Glossary, Table 2).) **3.** Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.* **4.** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.	**4.MD-2** **2.** Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
E.DPS.1i describing data shapes and what the data representations do and do not show (bar graphs, picture graphs, frequency tables, line plots, circle graphs) including the attributes used	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*
E.DPS.1j identifying clumps, gaps, trends, or central tendency (mode, median) in the data	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*
E.DPS.1k using data to make and support claims and interpretations (e.g., making comparisons among individuals, between individuals and the group, and among groups) **3. MD-1, 3, 4**	**3. MD-1, 3, 4** **1.** Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. **3.** Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.* **4.** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*

Elementary (K-4) School Learning Targets, Progress Indicators, & Common Core Standards **Data Analysis, Probability, and Statistics (DPS):** Questions are posed and investigated by collecting data or retrieving existing data, and representing, analyzing, and interpreting data. Investigations, inferences, and predictions are used to make critical and informed decisions. ***E.DPS-2*** *Conduct simple probability experiments and characterize the outcomes in words, diagrams, or numerically.*		
Progress Indicators for Grades 3-4	**Grade 3 CCSS standards**	**Grade 4 CCSS standards**
E.DPS.2d describing the probability of events as being certain, likely, equally likely, unlikely, or impossible.	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, these skills/concepts represent the "hypothesized" learning continuum.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, these skills/concepts represent the "hypothesized" learning continuum.*
E.DPS.2e identifying expectations for varied results in situations involving randomness (e.g., using coin tosses, spinners, dice, playing cards)		
E.DPS.2f representing all possible outcomes for expectations of varied results (e.g., using words, tree diagrams)		
E.DPS.2g conducting repeated trials of simple probability experiments, using displays (e.g., tables, tree diagrams, histograms) to understand results and explain variations		

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